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Heat Flow Estimation Using Sequential Method and Transfer Function Based on Green's Functions

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Abstract. *The main objective of this article is to use two different methods of inverse heat conduction problems, to estimate the heat flux, the first is called Sequential algorithm and the second one is the Transfer Function based on Green's Function in a classical heat conduction problem which is subjected to a heat flux at one face and isolated to another face, this problem can be called X22 in Green's Function notation. As the main results, it was observed that the heat flux can be estimated with both methods and those algorithms can be used to estimate heat flow near to the region of application with satisfactory results but in regions far from heat flow application in both cases, the results are worse.*

Keywords: *Inverse Problem, Heat Conduction, Heat flow, Sequential algorithm, ill-posed problems*

1. INTRODUCTION

According to Beck *et al.* (1985), the inverse heat conduction problem is one of many ill-posed problems. The notion of a well-posed or "correctly set" mathematical problem made its debut in the 1923 discussions. One of the difficulties of ill-posed problems is in defining what is meant by a solution because the solution does not satisfy general conditions of existence, uniqueness, and stability.

An analytical solution of temperature was made based on Green's functions (GF), and specific boundary condition nomenclature was used Cole *et al.* (2010). Using GF is possible to find solutions to heat conduction problems of the most varied and complex types, such as three-dimensional, transient, heat-generating and non-uniform problems and which may still be subject to non-homogeneous boundary conditions varying with time and space.

Fernandes (2013) and Fernandes *et al.* (2015) studied the procurement and application of analytical solutions based on Green Functions (GF) in techniques of inverse problems. Their study is focused on the problem arising from the unknown heat source as the heat generation due to the friction present in the machining process and the measurement of thermal properties using optimization techniques and parameter estimates.

2. DIRECT PROBLEM - ANALYTICAL SOLUTION

The problem of a plate being subjected to a heat flow on a surface and isolated on the opposite side is also one of the classic problems in conducting heat with application in thermal models to obtain thermal properties. Figure 1 presents this case.

This is a general one-dimensional, transient heat conduction problem with no internal heat generation that can be described by the heat diffusion equation Eq. 1, where T is temperature, α is thermal diffusivity, t is time and x is the length of the plate.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (1)$$

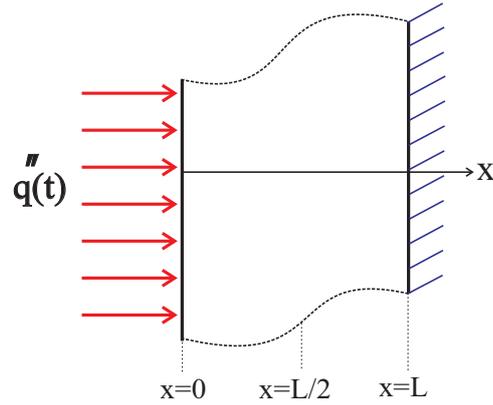


Figure 1. One-dimensional thermal problem G_{X22} .

Equation (2) shows one plate subject to the boundary conditions in $x = 0$.

$$-k \left. \frac{\partial T}{\partial x} \right|_{x=0} = q''(t) \quad (2)$$

where k is thermal conductivity and q'' is the heat flux. In $x = L$ the boundary condition is isolated, represented by Eq. (3).

$$\left. \frac{\partial T}{\partial x} \right|_{x=L} = 0 \quad (3)$$

and the initial condition represented by Eq. (4)

$$T(x, 0) = F(x) = T_0 \quad (4)$$

The solution for this case in Green's Function can be found at Fernandes (2009) and is represented by Eq.(5) :

$$T(0, t) = T_0 + \frac{\alpha}{k} \frac{1}{L} \sum_{n=1}^{N-1} q_n''(t)(t_{n+1} - t_n) + \frac{\alpha}{k} \left(\frac{L}{m\pi} \right)^2 \frac{2}{L\alpha} \sum_{n=1}^{N-1} q_n''(t) \left[\exp^{-\left(\frac{m\pi}{L}\right)^2 \alpha (t-t_{n+1})} - \exp^{-\left(\frac{m\pi}{L}\right)^2 \alpha (t-t_n)} \right] \quad (5)$$

The Equation (5) were implemented in *MATLAB*. The simulated material had the following thermal properties: thermal conductivity, $k = 0.15$ (W/mK), thermal diffusivity, $\alpha = 1,17e^{-7}$ (m^2/s), initial temperature, $T_0 = 25$ °C, and as a dimension length, $L = 5e^{-3}$ (m), time interval, $dt = 0.5$ (s) and the heat flux is show in the Fig. 2.

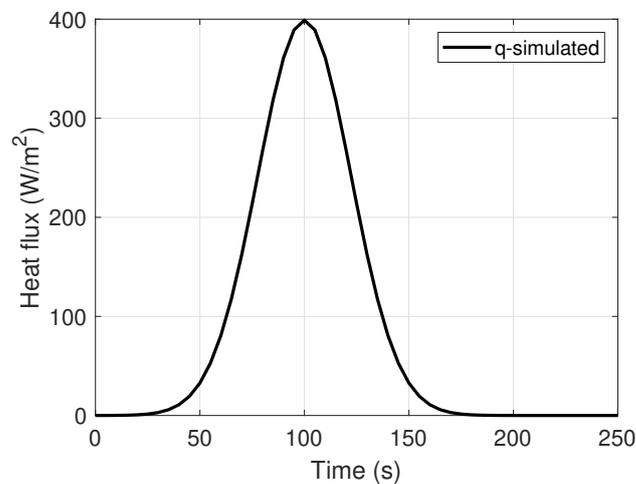


Figure 2. Heat flux simulated.

The temperature profile over time is shown in Fig 3. As can be seen in the Figure, the temperature increases first in the region in which the heat flux is subjected, $x = 0$, also, it is observed that the furthest position, $x = L$ is the region less affected by the heat flux.

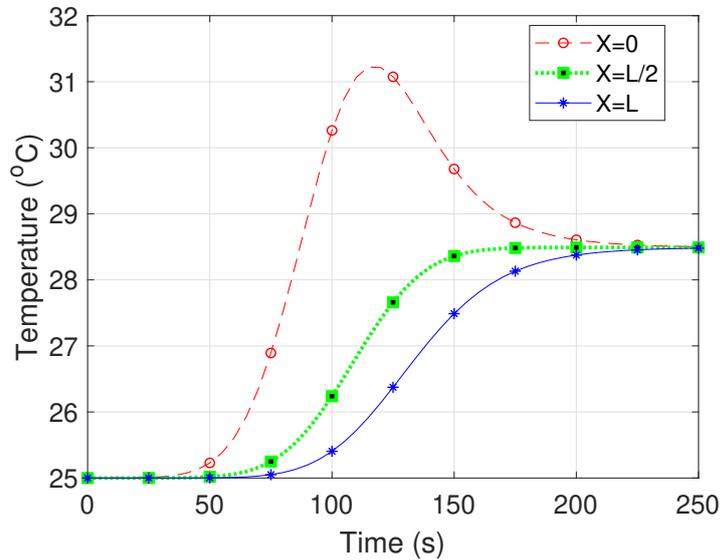


Figure 3. Temperature for the position $x = 0$, $x = L/2$ and $x = L$.

3. INVERSE PROBLEM

In this work, the Transfer function based on Green's Function (TFBGF) Fernandes *et al.* (2015) and the sequential method Beck *et al.* (1985) were used as inverse problem to estimate a heat flux in 3 different positions $x = 0$, $x = L/2$ and $x = L$.

3.1 Transfer function based on Green's Function

According to Fernandes *et al.* (2015), the impulsive response (transfer function) of the X22 problem and inverse problem solver is given by.

$$h(x, t) = \frac{\alpha}{k} G_{X22}(x, t|0, t) = \frac{\alpha}{kL} + \frac{2\alpha}{kL} \sum_{m=1}^M e^{-\left(\frac{m\pi}{L}\right)^2 \alpha t} \cos\left(\frac{m\pi x}{L}\right) \quad (6)$$

Inverse problem solver

Figure 4 shows the direct problem, which has input the heat flux and output the temperature, if we have the heat flux, and the (TFBGF) model, it's easy to predicted the temperature.

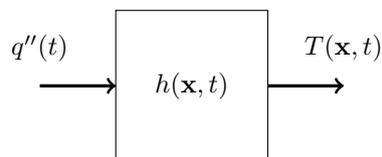


Figure 4. Dynamic system: direct problem.

$$T(\mathbf{x}, t) = h(\mathbf{x}, t) \cdot q''(t). \quad (7)$$

$$T(\mathbf{x}, s) = H(\mathbf{x}, s) \cdot q''(s), \quad H(\mathbf{x}, s) = \mathcal{L}\{h(\mathbf{x}, t)\}. \quad (8)$$

$$T(\mathbf{x}, t) = \mathcal{L}^{-1}\{H(\mathbf{x}, s) \cdot q''(s)\} = h(\mathbf{x}, t) * q''(t) = \int_0^t h(\mathbf{x}, t - \tau) q''(\tau) d\tau. \quad (9)$$

The impulse response to the analyzed positions is given by the Fig.5.

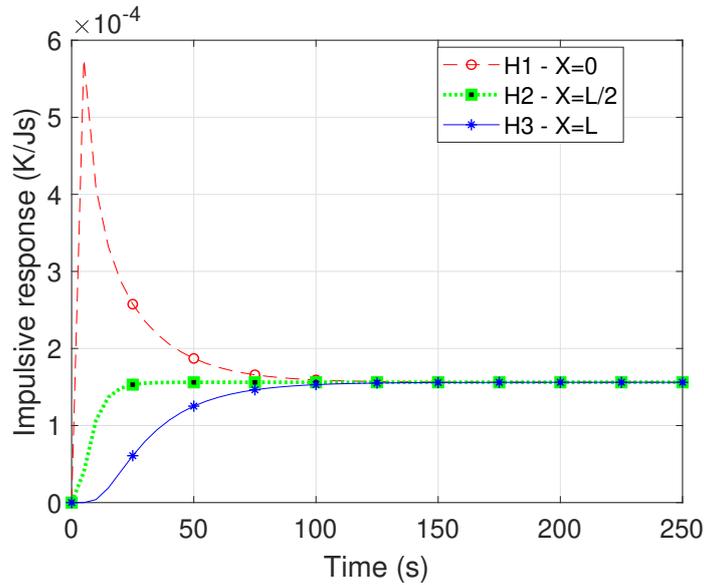


Figure 5. Impulsive response for the position $x = 0$, $x = L/2$ and $x = L$.

According to Fernandes *et al.* (2015), the inverse problem is given by Eq.12. Figure 6 represents the dynamic system, with the inverse problem, which has temperature input signal, the inverse Laplace transform is applied and the heat flux is obtained as a result.

$$q''(s) = T(\mathbf{x}, s)/H(\mathbf{x}, s). \quad (10)$$

$$q''(t) = \mathcal{L}^{-1}\{q(s)\}. \quad (11)$$

so

$$q''(t) = T(\mathbf{x}, t) * \mathcal{L}^{-1}\{1/H(\mathbf{x}, s)\}. \quad (12)$$

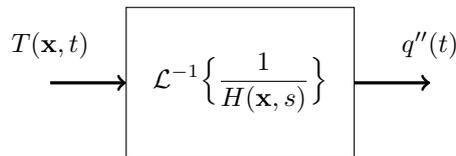


Figure 6. Dynamic system: inverse problem.

3.2 Sequential Method

A single heat flux component was estimated at each time step, producing a *sequential* algorithm. These techniques were restricted to a single temperature sensor. The nature of transient heat conduction is "diffusive"; that effect of heat input at time zero on an interior location is both lagged and damped for the surface.

The basic principles are developed in a form that can utilize integral models such as Duhamel's integral and also difference equation procedures such as finite differences or elements.

$$q''_M = \sum_{i=1}^r \frac{(Y_{M+i-1} - \hat{T}_{M+i-1}|_{q_M=\dots=0}) \Phi_1}{\sum_{i=1}^r \Phi_1^2} \quad (13)$$

With the Eq. (13) it's possible and easy to estimate the heat flux in overtime.

4. RESULTS AND DISCUSSION

In this section we will do a comparison between the results about the estimation of heat flux using two different methods: Sequential algorithm and Transfer Function Based on Green's Functions (TFBGF).

4.1 Comparison between the methods: Sequential algorithm and TFBGF

Figures 7 and 8 represents respectively, the heat flux simulated and the heat flux estimated based in the Sequential algorithm and using the method Transfer Function Based on Green's Function for the positions $x = 0$, $x = L/2$ and $x = L$.

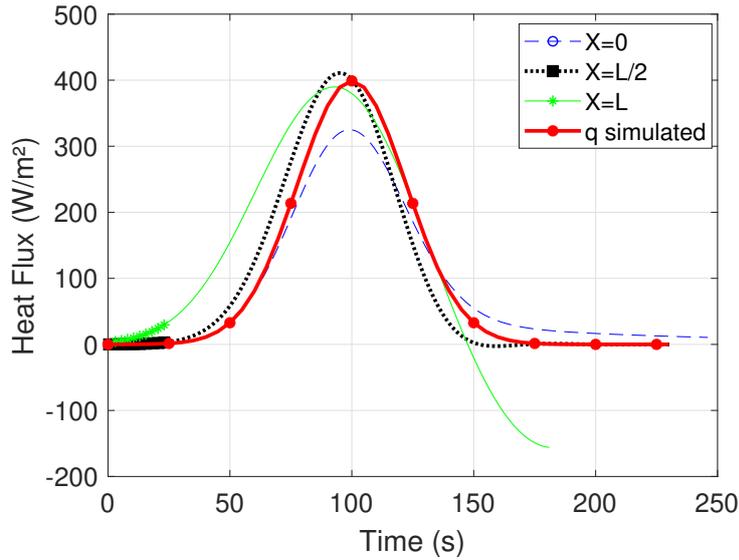


Figure 7. Estimation of heat flux in all positions using the Sequential method.

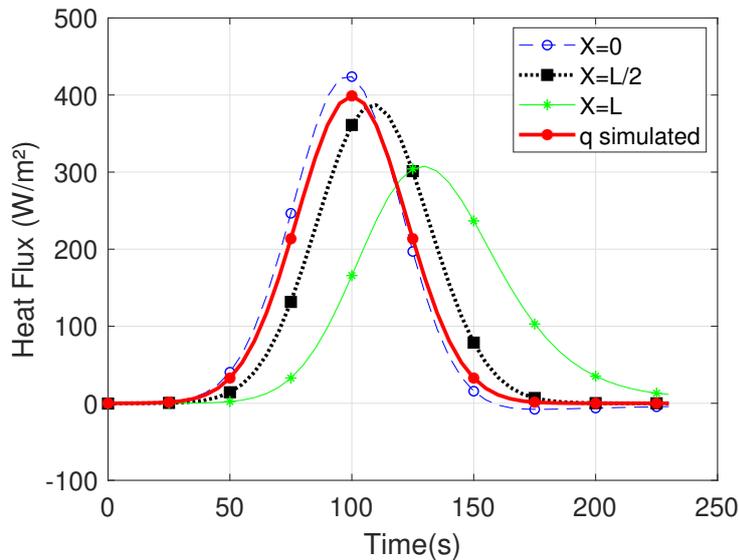


Figure 8. Estimation of heat flux in all positions using the TFBGF method.

Figure 9 shows the simulated heat flux and the estimated heat flux over time using the Sequential algorithm and transfer function based on Green's function (TFBGF) for the position $x = 0$. In the red line is represented the simulated heat flux, the blue line shows the estimated heat flux using TFBGF, and the black line represents the heat flux estimated using the Sequential algorithm. As can be observed in the Figure, the method TFBGF presents a smaller error compared to the simulated heat flux for this position.

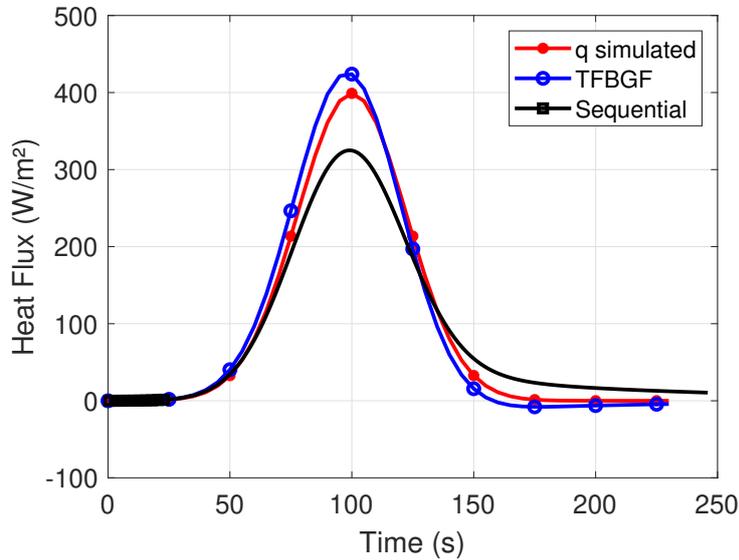


Figure 9. Heat Flux estimated for the position $x = 0$.

Figure 10 shows the simulated heat flux and the estimated heat flux over time. It can be observed that for the position $x = L/2$ the estimated heat flux in the Sequential method and the TFBGF, in both methods the results are satisfactory.

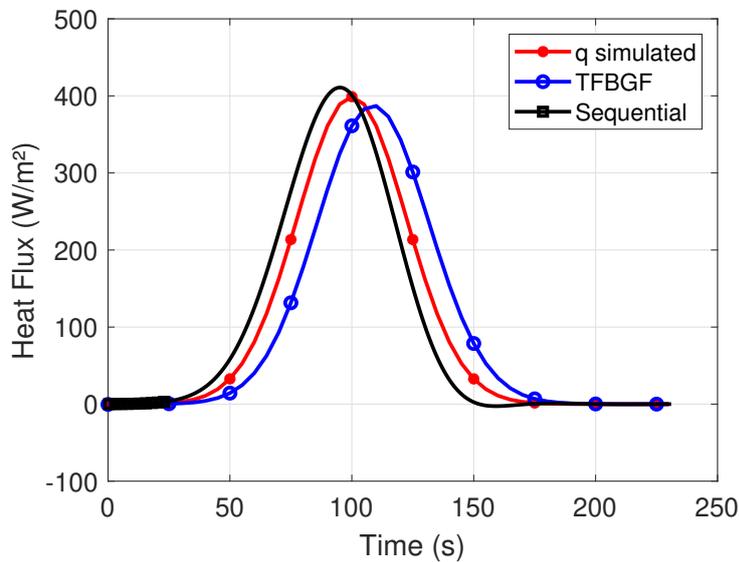


Figure 10. Heat Flux estimated for the position $x = L/2$.

Figure 11 shows the simulated heat flux and the estimated heat flux over time. It can be observed that for the position $x = L$ the estimated heat flux in both methods the results have no error.

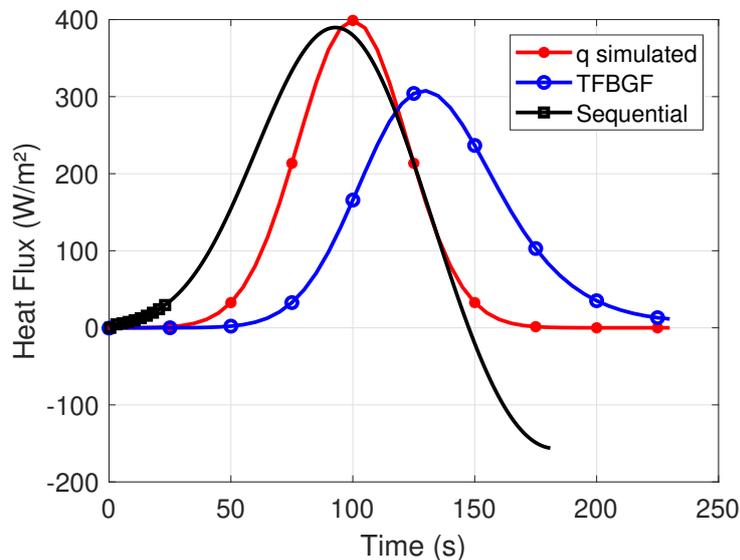


Figure 11. Heat Flux estimated for the position $x = L$.

5. CONCLUSIONS

The Sequential methods and Transfer Function based on Green's Function allow the estimation of the heat flux applied in different position, but it must be observed that the transfer function does not use any method of explicit regularization, allowing a greater representation of reality, even with good results, since it can be observed that in the sequential method there are negative values that are not coherent with the simulated ones.

Another interesting point to note is that even if the transfer function method presents a greater error in $L/2$, it can not differ much from the result as we move away from the region where the heat flow is applied, since the sequential method begins to present large errors in larger L , but in medium regions has a better fitting.

6. ACKNOWLEDGMENTS

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