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FLUTTER ANALYSIS TOOLS IN A NONLINEAR STRUCTURAL-FLIGHT DYNAMICS NUMERICAL PLATFORM

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Abstract. *A numerical model of an airplane done with Nonlinear Flight Dynamics– Nonlinear Structural Dynamics–Strain-based formulation (NFNS_s) methodology is used as a basis to create numerical tools to extract sufficient data to support a flutter physical analysis, which is the main objective of this research. The numerical code was updated to store the matrices of eigenvalues and eigenvectors at each trimmed condition. Furthermore, it is possible to filter the aeroelastic modes of interest, calculate the results and plot the charts of frequency, damping ratio, amplitude, and phase. These results provide support for future analysis about the flutter physical mechanism and solutions that can be applied to avoid the phenomenon and improve aircraft's design.*

Keywords: *flexible aircraft, numerical model, code update, flutter, aeroelasticity.*

1. INTRODUCTION

The continuous advances of using new materials, such as glass and carbon fibers, always focused on the improvement of aircraft's performance, have enabled higher-aspect-ratio wings and slenderer fuselages. As a result, the level of flexibility has increased significantly (Guimarães Neto, 2014). So, aeroelasticity performs an important role in the new developments of the aerospace industry.

The term aeroelasticity designates the field of study interested in evaluating the interactions that are established between the disciplines of aerodynamics, elasticity and dynamics (Wright and Cooper, 2007). Collar (1946) proposed a scheme very representative and useful that summarizes the concept of aeroelasticity. This representation explains in a very subtle way the sub-relationships between the disciplines that form the field of study of aeroelasticity: fluid mechanics (aerodynamic forces), solids mechanics (elastic forces) and dynamics (inertial forces). The areas resulting from the interaction of the different subjects cover the fields of stability and control, static aeroelasticity and structural dynamics. Figure 1 shows an adapted representation of the Collar's triangle.

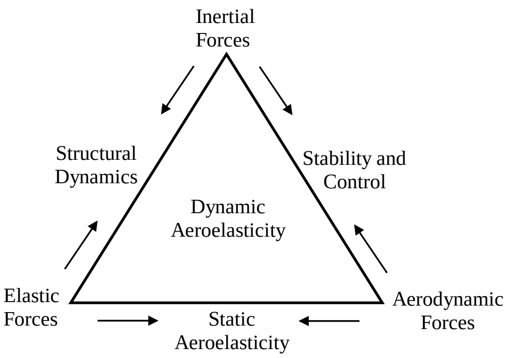

Figure 1. Collar's Triangle.
Adapted from: Collar (1946).

Inside the field of dynamic aeroelasticity, one of the phenomena that carries most attention is flutter. According to Garrick and Reed III (1981), flutter is an unstable self-excited vibration in which the structure extracts energy from the air stream and often results in catastrophic structural failure. Because of that, flutter is the most important of all the aeroelastic phenomena and is the most difficult to predict too. Although the importance to study flutter was detected in the beginning of the last century, the complexity of this phenomenon creates a wide field of studies yet.

According to Bisplinghoff and Ashley (1975), the insights that aeroelasticians have are largely mathematical. Although it was pronounced more than forty years ago, it is not common to find a physical explanation that links the models with what is happening in the structure, since the majority of the actual methodologies are purely mathematical (Jinwu *et al.*, 2013; Guimarães Neto, 2014; Buttini, 2014; Marqui, 2017; Sousa *et al.*, 2017). Normally, the approaches that use eigenvalues to find the instability are aimed to frequency and damping ratio charts. These charts illustrate the moment that the aircraft finds a negative damping (or flutter velocity) and it can varies depending on the configurations of the model, such as the aerodynamic model used (Wright and Cooper, 2007).

A numerical platform developed initially in Ribeiro (2011), called AEROFLEX, was adapted in Sousa (2013) for a medium size jet airplane with similar properties to Embraer EMB-190/195 and Boeing 737-200/300. The AEROFLEX initial code was able to find the matrices of eigenvalues and eigenvectors at a trimmed condition. But, there is not any function to process the results in an aeroelastic viewpoint. So, it was modified to find essential information (frequency, damping ratio, amplitude and phase) about what is occurring in each part of the structure.

2. METHODOLOGY

The methodology used to elaborate the numerical code in Ribeiro (2011) and Sousa (2013) is named as Nonlinear Flight Dynamics– Nonlinear Structural Dynamics (NFNS). The NFNS methodology uses beam formulation to capture geometrically nonlinearities of structural deformations. Basically, there are three different branches to implement beam formulation, each one with a different group of independent variables (Su and Cesnik, 2011). The numerical code in Sousa (2013) was done with the strain-based formulation (NFNS_s). This specific methodology is referenced in Sousa (2013) as NFNS_s and is capable to compute large deformations and inertial coupling between elastic and rigid generalized coordinates (Shearer, 2006; Su and Cesnik, 2011; Ribeiro, 2011; Sousa, 2013). The degrees of freedom of the rigid aircraft, the Euler angles, the longitudinal and lateral positions, the altitude and the strains of all structural elements are the model's generalized coordinates. The NFNS_s formulation considers beams formed by elements, each one with three nodes and four local strains: extension in X axis (ε_x), twist in X axis (k_x), bending in Y axis (k_y) and bending in Z axis (k_z) (Sousa, 2013). Table 1 illustrates an airplane discretization example with the quantity of degrees of freedom for an aircraft with five elements in each wing, 2 elements for each horizontal tail and one element for vertical tail. It is important to note that the quantity is multiplied by 2 because the deformation rate is calculated too. It can be also noted that the mathematical model involves a high quantity of degrees of freedom and requires a high computational burden.

Table 1. Dimension calculus of eigenvalue and eigenvector matrices.

Member	Number of Elements x Number of Degrees of Freedom	Dimension
Right Wing	5 x 4	20
Left Wing	5 x 4	20
Right Horizontal Tail	2 x 4	8
Left Horizontal Tail	2 x 4	8
Vertical Tail	1 x 4	4
SUBTOTAL		60
TOTAL		120

The equations of motion were obtained by using the Principle of Virtual Work and the Hamilton's Principle (Brown, 2003; Shearer, 2006; Su, 2008). Basically, the sum of all virtual work associated with internal and external forces of all elements must be zero. The virtual displacements are arbitrary and the virtual work of elastic members is produced by inertial forces, internal structural elastic forces, external forces and moments (Sousa *et. al*, 2017). Figure 2 shows a scheme about how the equations of motion were obtained.

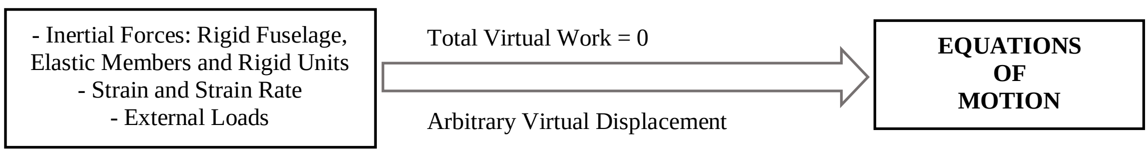

Figure 2. Scheme used to find the equations of motion.
Available from: Sousa *et al.* (2017).

The NFNS_s methodology is described in details in Brown (2003), Shearer (2006), Su (2008), Su and Cesnik (2010), Ribeiro (2011) and Sousa (2013). More details about the model developed can be achieved in Sousa (2013) and Sousa *et al.* (2017).

2.1 Code Update

The modifications were made in order to add functionalities that would make possible the acquisition and treatment of the aeroelastic information obtained at each trimmed condition. The changes were basically the development of new routines to treat the matrices of eigenvalues and eigenvectors that the AEROFLEX code itself already provides. Figure 3 presents, briefly, the changes done.

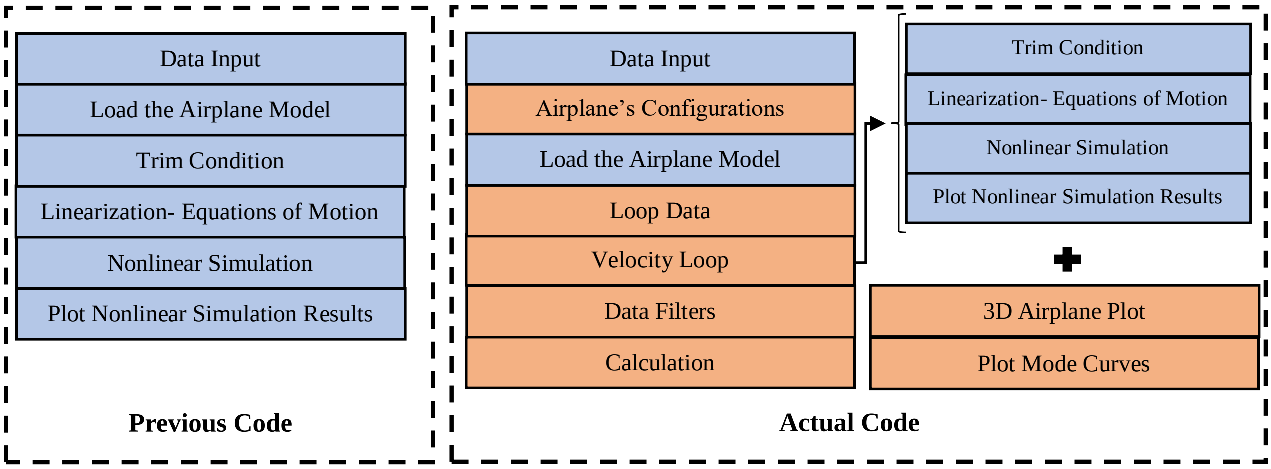

Figure 3. Comparison between the previous code and the actual one.

Originally, AEROFLEX required a single simulation velocity, from which the nonlinear temporal simulation was done. In this way, the original AEROFLEX routine was modified to include a velocity loop. So, the nonlinear temporal simulation, which was previously done naturally, can be now enabled or disabled in the main routine, which depends on the type of analysis the user wants to do. New features were incorporated in the main code too, e.g. the possibility to choose different stiffness matrices for each member (wings, horizontal tails and vertical tail) and to alter the mass distribution of the wings.

The aircraft model is loaded and trimmed at each step (velocity) of the simulation to generate new linearized matrices, from which the eigenvalues and eigenvectors are calculated. These eigenvalues and eigenvectors are stored to be the basis for the two subsequent processes: data filters to select the data based on the real and imaginary parts of eigenvalues and the calculation that permits to achieve the final results (frequency, damping ratio, amplitude and phase). Furthermore, two auxiliary codes were created to work and plot the final results, which are basically the three-dimensional chart and the curves for each degree of freedom (ε_x, k_x, k_y and k_z) of a selected aeroelastic mode.

2.1.1 Data Filters

The development of the data filters included four distinct subroutines:

- Data cleaning: subroutine used to select eigenvalues and eigenvectors of interest. In this subroutine, two limit values are used by the code to select the eigenvalues. It is done based on the imaginary and real part. The first filter sets the value from which the code selects only the eigenvalues with the module of the imaginary part greater than that input number. The second is similar and corresponds to the value of the real part. By selecting the eigenvalues of interest, the code filters the eigenvectors associated automatically;
- Eigenvalues and eigenvectors ordering: responsible for the primary ordering of the data based on the real part of eigenvalues;
- Extraction: the total set of eigenvalues and eigenvectors obtained is composed of conjugate complex pairs. As usual, only one of these eigenvalues is used to extract the information related to frequency and damping ratio (Ogata, 2010) — the eigenvalue with positive imaginary part was chosen;
- Ordering check: subroutine used to check and reorder eigenvalues and eigenvectors, if necessary. In short, this code calculates the order that eigenvalues and eigenvectors need to be set to maximize the smooth behavior of the curves.

Inside the ordering check subroutine, one of the problems that was solved and requires a special attention is the mode tracking. According to Eldred *et al.* (1995), mode tracking is a technology used in eigenvalue problems to perform the correct association of these values when certain system parameters are changed. The classification of this problem involves two possible types: self-adjoint or nonself-adjoint. In the first case, energy is conserved in the system, the eigenvalue pairs are real and both values are identical. In the second case, the energy of the system is not conserved and usually comprises complex values for both eigenvalues and eigenvectors. In addition, there are differences between left and right values.

The mode tracking originates, in practice, an erroneous ordering of the results generated in the velocity simulations. It is a very common problem and is object of research (Van Zyl, 1993; Eldred *et al.*, 1995; Qiu and Sun, 2009; Hang *et al.*, 2018). The problem occasioned can be seen in Fig. 4.

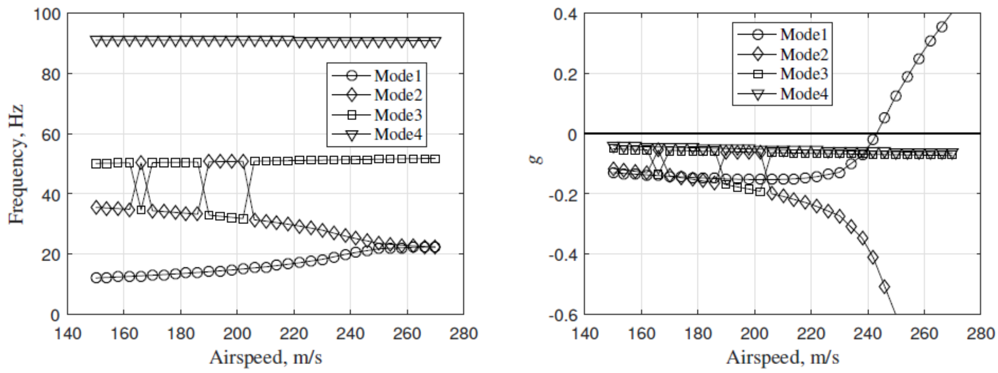

Figure 4. Effects originated by mode tracking.
Available from: Hang *et al.* (2018).

It is noticeable in Fig. 4 that undue changes occurred in the data and there is the presence of jumps in the plots. It was required to solve this problem to correct the order of the data and obtain, as final result, each line of the eigenvalues matrix and each column of eigenvectors matrix corresponding to the same aeroelastic mode.

The solution used in this paper to solve this problem is based on the calculus of the order of the values in a velocity that maximizes the smooth behavior of the curves for each aeroelastic mode. In other words, the relation calculates the distance between the values from the actual simulation velocity and the previous one and provides as an output the order that the values of the actual step need to be rearranged to guarantee the best smooth behavior between each couple of values (e.g., each aeroelastic mode). The relation used is presented in Eq. (1).

$$\boldsymbol{dist} = \left(1-\left|\boldsymbol{V_1}^T\cdot\boldsymbol{V_2}\right|\right)\cdot\sqrt{\left(distM\left(\boldsymbol{D_{1RE}},\boldsymbol{D_{2RE}}\right)^2\right)+\left(distM\left(\boldsymbol{D_{1IM}},\boldsymbol{D_{2IM}}\right)^2\right)} \quad (1)$$

Where:

- ***dist***: output matrix with the data that is used to generate a sorting sequence;
- *distM*: internal function that calculates the difference between the modules of the input arguments;
- $\boldsymbol{V_1}$: eigenvectors matrix of previous step;
- $\boldsymbol{V_2}$: eigenvectors matrix of current step;
- $\boldsymbol{D_{1RE}}$: real part of eigenvalues matrix of previous step;
- $\boldsymbol{D_{2RE}}$: real part of eigenvalues matrix of current step;
- $\boldsymbol{D_{1IM}}$: imaginary part of eigenvalues matrix of previous step;
- $\boldsymbol{D_{2IM}}$: imaginary part of eigenvalues matrix of current step.

Once the problem was solved by the data filters process, the calculation process can be done to achieve the results for the selected aeroelastic modes.

2.1.2 Calculation

Upon the selection of the aeroelastic modes, which is done in data filters, the code was updated to calculate the values for amplitude, phase, frequency and damping ratio. The first two quantities are extracted from the eigenvectors, while the two latter are from the eigenvalues. The frequency and damping ratio are associated with a specific mode and calculated by Eq. (2) and Eq. (3), respectively. The phase and amplitude are obtained for each element in each member of the aircraft by Eq. (4) and Eq. (5), respectively. These four quantities are used in Siqueira (2019) in order to exam the flutter physical mechanism of the aircraft model done in Sousa (2013).

$$Frequency = \frac{\sqrt{(Re_{va}{}^2 + Im_{va}{}^2)}}{2\pi} \quad (2)$$

$$Damping\ Ratio = \frac{-100Re_{va}}{\sqrt{(Re_{va}{}^2 + Im_{va}{}^2)}} \quad (3)$$

$$Phase = tan^{-1}\left(\frac{Im_{ve}}{Re_{ve}}\right) \quad (4)$$

$$Amplitude = \sqrt{(Re_{ve}{}^2 + Im_{ve}{}^2)} \quad (5)$$

Where:

- Re_{va}: eigenvalue real part;
- Im_{va}: eigenvalue imaginary part;
- Re_{ve}: eigenvector real part;
- Im_{ve}: eigenvector imaginary part.

The data obtained by Eq. (2), (3), (4) and (5) can be used to plot the variation of each variable along the velocity for a specific member of the aircraft (wing, horizontal tail or vertical tail). For example, if a member with five elements is chosen, there will be five curves of phase and amplitude, each one in one chart. For the frequency and damping ratio, there will be just one curve, since these two variables are related to the aeroelastic mode itself.

Two different groups of routines were created in order to generate the charts of interest:

- The first group is focused on providing a palpable perception of how an aeroelastic mode is acting upon the structure. This subroutine receives data from the eigenvectors (amplitudes) at a chosen velocity and converts the deformations of the aircraft into displacements. In order to determine the absolute amplitude of element 05, for example, the code automatically adds the value found for this element to the amplitudes of elements 01, 02, 03 and 04. In addition to this subroutine, there is a factor that multiplies and makes possible a better visualization of the aeroelastic mode. Since the eigenvectors express a relative behavior among themselves, this multiplier used maintains the proportionality between the values and guarantees a good illustration of the aeroelastic mode. The amplitudes provided by the eigenvectors represent the degree of oscillation of the structure in relation to the equilibrium (trimmed condition). It means that three-dimensional charts present qualitatively the aeroelastic mode format taking into account how certain part of the aircraft deforms in relation to the equilibrium condition. These charts do not consider the phase difference present between the elements. An important consideration about three- dimensional charts is that they always illustrate the right wing down and the left wing up when it is plotted an aeroelastic mode. It is caused because this subroutine receives the modulus of the eigenvectors and plots only positive values. According to the reference axis of NFNS_s methodology (Ribeiro, 2011; Sousa, 2013), positive bending causes right wing to move down and left wing to move up. This does not identify that it is an antisymmetric mode. It just represents how the aeroelastic is formed (mainly by bending and twist);
- The second group of subroutines created treats the results calculated from eigenvalues and eigenvectors. The graphs can be subdivided into two parts, which are: the first part is responsible to generate the frequency and damping ratio charts and the second contains the phases and amplitudes values for each element of each member. If is desired, the code can also calculate and average of the curves of the elements to provide an overview about the behavior of the member.

3. RESULTS

The results that can be obtained with the modified model in the AEROFLEX platform include three-dimensional charts and plots of frequency, damping ratio, amplitude and phase along the velocity. This is the basis of the results for future analysis about what is happening in each part of the structure. Some of the results obtained are presented in sequence.

Figure 5 presents an example of three-dimensional plot for the airplane model at 250 m/s and 10000 meters of altitude. This representation is intended to show the behavior of the aircraft under a given load. The nomenclatures of each member are exposed in the frontal view. It is possible to observe two wings, two horizontal tails and the vertical tail too. The fuselage is not illustrated because the numerical model considers it as a rigid unit (e.g., the internal node of element 01 is cantilevered in the fuselage). Although the model incorporates beams to represent the aircraft structure, the graphical representation shows surfaces on the members because these surfaces are considered in the calculation of the aerodynamic loads along the elastic beams (members).

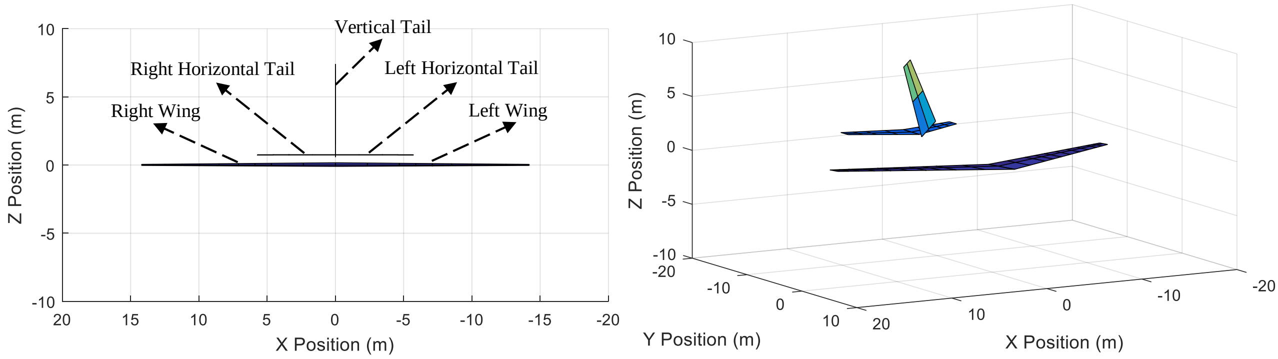

Figure 5. 3D representations of the aircraft model at 250 m/s: front view (left) and perspective view (right).

In order to exemplify the distinction of aircraft at a trimmed and stationary conditions, Fig. 6 is presented. According to Ribeiro (2011), the trimming is performed by calculating the structural and rigid body equilibrium for a straight and level flight condition at the specified input of altitude and speed (in this case, 10000 m and 250 m / s). With this, it is possible to obtain the equilibrium structural deformations, pitching angle, thrust and elevator deflection. More details can be found in Ribeiro (2011).

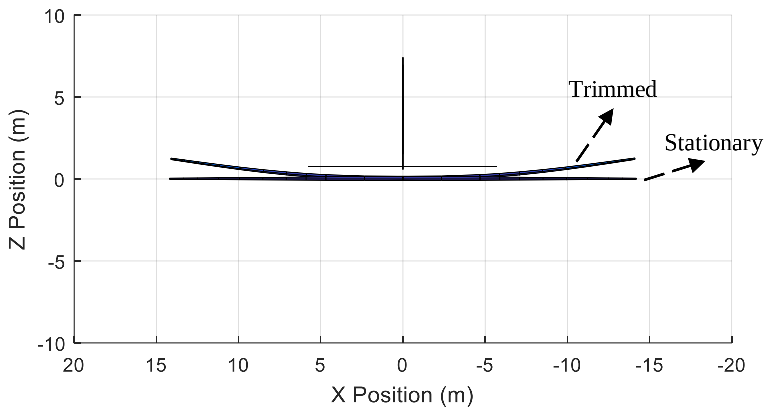

Figure 6. Comparison between the trimmed and stationary conditions.

Figure 7 shows the phase and amplitude differences between the movements of twist (*kx*) and bending (*ky*) for each element of the right wing (left one is analogous). This chart is important to evaluate if the movements are in or out of phase, which has a substantial effect in flutter mechanism, because one movement can energize the other.

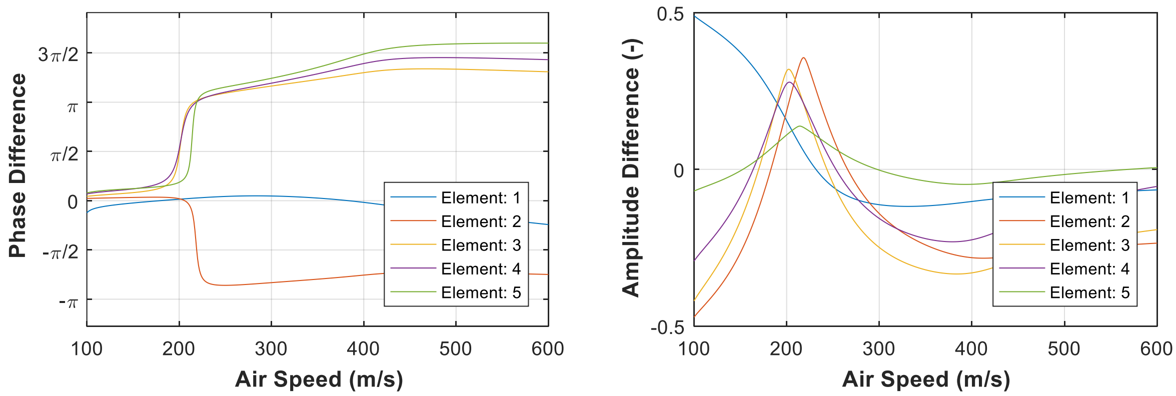

Figure 7. Differences calculated between the wing movements of twist (k_x) and bending (k_y).

Figure 8 illustrates how the frequency and damping ratio of three aeroelastic modes (selected ones) vary over the velocity for a very flexible aircraft. The second mode presented a negative damping ratio (flutter condition) around 425 m/s.

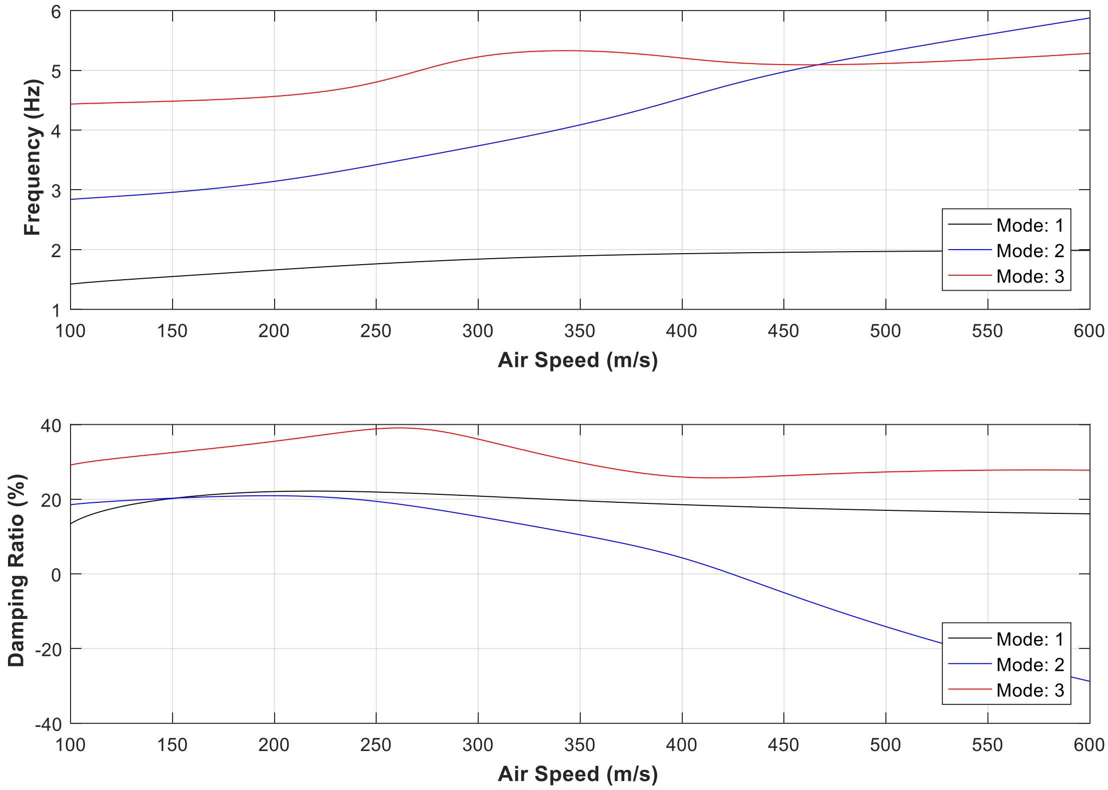

Figure 8. Frequency and damping ratio of three aeroelastic modes.

It can be noted that the numerical platform created can support a wide range of future analysis, which can be conducted in order to determine aeroelastic features of a numerical aircraft modeled in AEROFLEX and propose solutions that can be applied to avoid unwelcome behaviors.

4. CONCLUSIONS

A numerical model, created initially in Sousa (2013), was updated in order to add important tools to enable aeroelastic analysis. The improvement adapted the initial code to permit the acquisition of eigenvalues and eigenvectors matrices in a wide range of velocity simulation. The matrices are stored and used by new routines to calculate relevant data that can serve as a basis to support aeroelastic analysis, such as a deep comprehension in each part of the aircraft's structure. This deep understanding is achieved due to the methodology used, NFNS_s.

The routines incorporated a mode tracking process and amplified the capability to study the influence of one parameter (e.g., stiffness and/or mass distribution) in aircraft's aeroelastic performance. Data filters were used to select and analyze the aeroelastic modes of interest.

The platform and the tools created can help the physical understanding and contribute to generate applicable solutions to avoid unwelcome behaviors, since it is possible to evaluate each part of the structure separately and its correlation with the entire aircraft.

There is a great quantity of researches that can be developed in the future. For instance, the platform can be used to model different types of aircraft and analyze the physics involved in the flutter mechanism, whereas it is possible to evaluate the relation established between the twist and the bending among different aeroelastic modes or inside each of them.

5. ACKNOWLEDGEMENTS

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