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A MIXTURE THEORY APPROACH FOR MASS AND MOMENTUM TRANSPORT OF FLUID AND POLLUTANTS THROUGH POROUS MEDIA

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Abstract. *The motion of a fluid polluted by three pollutants though a rigid porous medium is modeled using a Mixture Theory approach. A very small quantity of the pollutants – when compared with the main fluid – is assumed and they react solely among themselves. This hypothesis allows the motion equation to consist of mass balance for the three pollutants constituents and the fluid constituent and momentum balance solely for the fluid constituent. Combining with a constitutive assumption to represent pollutants mass generation, the model gives rise to a nonlinear nonhomogeneous hyperbolic problem. The simulation combines Glimm scheme, that marches in time with the solution of a chosen number of Riemann problems, with an operator splitting technique. Some results show the success of this numerical strategy.*

Keywords: *Mixture Theory, mass transfer, flow through porous medium, Glimm scheme, operator splitting.*

1. INTRODUCTION

This work employs a preliminary hyperbolic model to describe the transport of a small amount of pollutants contaminating a fluid, a carrier fluid, which is flowing through a porous matrix. A mixture theory approach is used to build the mechanical model. The porous matrix is assumed rigid, so it suffices to solve mass and momentum balances for all the fluid constituents of the mixture, which is assumed isothermal. Assuming the pollutants constituents mass density with the same order of magnitude of the fluid constituent, representing the carrier fluid, a simplifying assumption may be used: while the mass balance must be postulated for all the pollutant constituents and for the fluid constituent – representing the carrier fluid, the momentum balance is postulated solely for the fluid constituent. A simple constitutive assumption for pollutant generation is proposed, supposing that the pollutants react among themselves, but do not react either with the fluid constituent or with the solid constituent – representing the porous matrix.

The mathematical representation gives rise to a non-linear system of hyperbolic equations presenting a contact shock. At this spatial point all the pollutants concentration jump, while their mass density and velocity suffer no variation. This problem is simulated by using an operator splitting technique which separates the hyperbolic part of the operator from the time evolutionary portion. The former is simulated by Glimm's scheme, that employs the solution of a Riemann problem for each two consecutive time steps. A complete solution of the associate Riemann problem is presented.

Glimm's method ensures not only that the shock identity, shock magnitude and position, is preserved but also that the problem's approximation tends to its solution in the weak formulation when the width of the steps teds to zero.

Although the convergence of the operator splitting technique is not mathematically ensured as Glimm's method, it has been successfully employed to treat many nonlinear hyperbolic problems, such as gas dynamics problems (Sod, 1977), wave propagation in pipelines (Marchesin and Paes-Leme, 1983), dynamical behavior of nonlinear elastic rods (Saldanha da Gama, 1990), wave propagation in damageable elasto-viscoplastic pipelines (Freitas Rachid et al., 1994), pollutants flow in an atmosphere (Martins-Costa and Saldanha da Gama, 2003; 2006; Porto et al., 2011) and fluid flows through unsaturated porous media (Saldanha da Gama and Sampaio, 1987; Saldanha da Gama and Martins-Costa, 1997; Martins-Costa and Saldanha da Gama, 2001; Martins-Costa and Saldanha da Gama, 2005). Essentially the methodology separates the hyperbolic part from the time evolutionary one, treating a simultaneous problem sequentially. The purely hyperbolic problem is simulated by Glimm scheme, that employs the solution of a certain, previously chosen, number of Riemann problems for each consecutive time steps, giving rise to a "prediction" step, subsequently, the solution of the time evolutionary problem gives rise to the "correction" step.

2. MECHANICAL MODEL

Since chemical reactions are allowed, each constituent´s mass is not necessarily conserved. The mass balance must be satisfied by four constituents: the carrier fluid – denoted as fluid constituent and the pollutants 1, 2 and 3. There are two other constituents coexisting in the mixture, an inert gas to account for the compressibility of the mixture as a whole and a solid constituent, assumed rigid, homogeneous and isotropic.

At this point an important hypothesis must be stated – the pollutants 1, 2 and 3 may react among themselves, but they do not react either with the carrier fluid, the fluid constituent, or with the porous matrix, the solid constituent, or with the gas – assumed with very low inertia. So, the mass balance for the constituents may be stated as:

$$\begin{aligned} &\frac{\partial \rho_F}{\partial t} + \nabla \cdot \left(\rho_F \mathbf{v}_F \right) = 0 \\ &\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot \left(\rho_\alpha \mathbf{v}_\alpha \right) = m_\alpha \qquad \alpha = 1,3 \end{aligned} \tag{1}$$

where ρ_α is the α^{th} pollutant mass density, $\mathbf{v}_\alpha$ its velocity, and m_α is its mass production or destruction. Also, ρ_F is the fluid constituent mass density and $\mathbf{v}_F$ its velocity. For the mixture as a whole the mass is preserved.

The momentum balance should be satisfied by the three pollutant constituents as well as by the so-called fluid constituent, since the gas constituent is assumed with very little inertia and the solid constituent is assumed rigid. So, it could be represented by

$$\rho_\alpha \frac{D_\alpha \mathbf{v}_\alpha}{Dt} = \rho_\alpha \left[\frac{\partial \mathbf{v}_\alpha}{\partial t} + \nabla \left(\mathbf{v}_\alpha \right) \mathbf{v}_\alpha \right] = \nabla \cdot \boldsymbol{\sigma}_\alpha + \mathbf{p}_\alpha - m_\alpha \left(\mathbf{v}_\alpha - \mathbf{J}_\alpha \right) + \rho_\alpha \mathbf{F}_\alpha \quad ; \ \alpha = 1,4 \tag{2}$$

where $\boldsymbol{\sigma}_\alpha$ represents the partial stress tensor acting on the three pollutant constituents and on the fluid constituent, which is analogous to Cauchy stress tensor $\boldsymbol{\sigma}$. Assuming the partial stress tensor symmetrical, the angular momentum balance is automatically satisfied. Also, $\mathbf{F}_\alpha$ represents the external body force acting on each of the four constituents and $m_\alpha \mathbf{J}_\alpha$ represents a momentum source resulting from chemical reactions among the constituents (where $\mathbf{J}_\alpha$ has dimensions of velocity). An interactive force caused by other interaction effects, such as the relative motion of the constituents is given by $\mathbf{p}_\alpha$, a force applied on the α-th constituent by all the remaining constituents, due to their interaction, actually a momentum generation term. For instance, in the special case of a fluid flowing through an unsaturated porous matrix this interaction force would represent the viscous drag of the matrix on the fluid and the capillary effects arising from a nonuniform distribution of fluid in the porous matrix.

At this point some additional hypotheses are made. The carrier fluid, represented by the fluid constituent, does not react chemically, so that $m_F = 0$. Also, there is a sufficiently small quantity of the three constituent pollutants in the flow – at any time instant, along with all mass densities with the same order of magnitude, so these pollutant constituents do not to affect the linear momentum balance. In other words, the pseudo-mixture, containing the fluid constituent and the three constituent pollutants, velocity is approximated by the fluid constituent velocity. Another important assumption is to suppose an isothermal problem, it suffices to solve mass and momentum equations. These equations are given by the following system, that characterize the mechanical model

$$\begin{aligned} &\frac{\partial \rho_F}{\partial t} + \nabla \cdot \left(\rho_F \mathbf{v}_F \right) = 0 \\ &\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot \left(\rho_\alpha \mathbf{v}_F \right) = m_\alpha , \quad \alpha = 1,3 \\ &\rho_F \left[\frac{\partial \mathbf{v}_F}{\partial t} + \nabla \left(\mathbf{v}_F \right) \mathbf{v}_F \right] = \nabla \cdot \boldsymbol{\sigma}_F + \mathbf{p}_F + \rho_F \mathbf{F}_F \end{aligned} \tag{3}$$

The partial stress tensor constitutive equation assumes that the normal fluid stresses are dominant over shear stresses and interphase tractions. Neglecting the viscous drag of the matrix on the Newtonian fluid, usually called Darcian term, only the term related to the fluid fraction gradient is present in the momentum source $\mathbf{p}_\alpha$ (see Martins-Costa and Saldanha da Gama (2001) for details), so the following constitutive hypotheses are assumed

$$\boldsymbol{\sigma}_F = -\varphi\bar{p}\,\mathbf{I} \qquad\qquad \mathbf{p}_F = -\frac{\mu_f \mathcal{D}}{K}\varphi\nabla\varphi \tag{4}$$

where $\bar{p}$ is a pressure assumed constant while the flow is unsaturated, φ represents the fluid fraction ($\rho_F = \varphi\rho_f$ with ρ_f representing the fluid density from a Continuum Mechanics viewpoint), μ_f is the fluid viscosity, measured from a Continuum Mechanics viewpoint, $\mathcal{D}$ is a diffusion coefficient, analogous to the mass diffusion coefficient, and K is the porous matrix specific permeability.

The following constitutive relation is assumed for the three pollutants' mass generation; which is such that the α -th pollutants mass production or destruction is given by assuming that the reaction between the pollutants 1 and 2 gives rise to the pollutant 3, may be stated as

$$m_1 = -\beta_1\omega_1 \quad ; \qquad m_2 = -\beta_2\omega_2 \quad ; \qquad m_3 = -m_1 - m_2 \tag{5}$$

At this point the definition of the α -th constituent mass fraction, along with some other definitions are given

$$\omega_\alpha = \frac{\rho_\alpha}{\rho_F} \quad \alpha = 1,3\,; \qquad \gamma_1 = \frac{\beta_1}{\rho_f} \qquad , \qquad \gamma_2 = \frac{\beta_2}{\rho_f} \quad ; \qquad p = c^2\varphi, \quad c > 0 \quad ; \qquad \mathbf{v}_F = \mathbf{v} \tag{6}$$

where ω_α is the mass fraction of the α^{th} constituent, the ratio of its mass density and the fluid constituent mass density ρ_F , ρ_f is the main fluid mass density, in a Continuum Mechanics viewpoint, β_1 and β_2 are positive constants, p is the pressure, which depends on the fluid fraction ($\varphi = \rho_F / \rho_f$) with *c* being a constant and $\mathbf{v}_F$ is the fluid constituent velocity.

Considering a mixture theory approach (Atkin and Craine, 1976), neglecting gravitational effects and all the previously stated hypotheses and a one-dimensional flow, so that all variables depend only on time *t* and position *x*, and, considering Eq. (4), it comes that

$$p = \frac{1}{\rho_f}\left[\varphi\bar{p} + \frac{\mu_f\mathcal{D}}{2K}\varphi^2\right] \qquad \frac{\partial p}{\partial x} = \frac{\partial}{\partial x}\left(\varphi\bar{p}\right) + \frac{\mu_f\mathcal{D}}{K}\varphi\frac{\partial\varphi}{\partial x} \tag{7}$$

Combining the balance equations (Eq. (3)) with the constitutive assumptions and definitions (Eqs. (4)-(7)), the mechanical model is represented by

$$\begin{aligned}
&\frac{\partial\varphi}{\partial t} + \frac{\partial(\varphi v)}{\partial x} = 0 \\
&\frac{\partial}{\partial t}(\varphi\omega_1) + \frac{\partial}{\partial x}(\varphi\omega_1 v) = -\gamma_1\omega_1 \\
&\frac{\partial}{\partial t}(\varphi\omega_2) + \frac{\partial}{\partial x}(\varphi\omega_2 v) = -\gamma_2\omega_2 \\
&\frac{\partial}{\partial t}(\varphi\omega_3) + \frac{\partial}{\partial x}(\varphi\omega_3 v) = \gamma_1\omega_1 + \gamma_2\omega_2 \\
&\frac{\partial}{\partial t}\left(\varphi v\right) + \frac{\partial}{\partial x}\left(\varphi v^2 + c^2\varphi\right) = 0
\end{aligned} \tag{8}$$

3. NUMERICAL MODEL

Equation (8) is a nonlinear nonhomogeneous hyperbolic system. The associated Riemann problem is obtained by combining the homogeneous portion of Eq. (8) with the following condition:

$$\begin{cases} (\varphi, \varphi v, \varphi\omega_\alpha) = \left((\varphi)_{n_j}, (\varphi v)_{n_j}, (\varphi\omega_\alpha)_{n_j}\right) & \mathrm{t} = t_n\ , -\infty < x < x_j + \dfrac{\Delta x}{2} \\ (\varphi, \varphi v, \varphi\omega_\alpha) = \left((\varphi)_{n_{j+1}}, (\varphi v)_{n_{j+1}}, (\varphi\omega_\alpha)_{n_{j+1}}\right) & \mathrm{t} = t_n\ ,\ x_{j+1} - \dfrac{\Delta x}{2} < x < \infty \end{cases} \tag{9}$$

giving rise to the following Riemann problem:

$$\begin{aligned} &\frac{\partial\varphi}{\partial t} + \frac{\partial(\varphi v)}{\partial x} = 0 \\ &\frac{\partial}{\partial t}(\varphi v) + \frac{\partial}{\partial x}\left(\varphi v^2 + p\right) = 0 \\ &\begin{cases} (\varphi, \varphi v) = \left((\varphi)_{n_j}, (\varphi v)_{n_j}\right) & \mathrm{t} = t_n\ , -\infty < x < x_j + \dfrac{\Delta x}{2} \\ (\varphi, \varphi v) = \left((\varphi)_{n_{j+1}}, (\varphi v)_{n_{j+1}}\right) & \mathrm{t} = t_n\ ,\ x_{j+1} - \dfrac{\Delta x}{2} < x < \infty \end{cases} \end{aligned} \tag{10}$$

where $(\varphi)_{n_j}$, $(v)_{n_j}$, $(\varphi)_{n_{j+1}}$ and $(v)_{n_{j+1}}$ are constants and can be renamed as $\varphi_L, \varphi_R, v_L$ and v_R .

The generalized solution of the Riemann problem is obtained by linking the left (L) $L \to *1$ and right (R) $*3 \to R$ states either by rarefactions or shocks, but the connection between the intermediate states $*1 \to *2$ and $*2 \to *3$ is always a contact shock, with a propagation speed v, in which only the variables $(\varphi\omega)_\alpha$ can jump, so that $(\varphi)_{*1} = (\varphi)_{*2} = (\varphi)_{*3}$ and $(\varphi v)_{*1} = (\varphi v)_{*2} = (\varphi v)_{*3}$, because the variables (φ) and (φv) never jump across these contact shocks.

It is important to note that the same time step must be employed both in Glimm scheme and in the approximation of the time evolutionary problem. The "prediction" step, or Glimm method, consists in reaching the approximations $(\hat{\varphi})_{n+1}$ and $(\hat{v})_{n+1}$ from the solution of some previously determined number of Riemann problems. The pollutants $(\hat{\omega}_1)_{n+1}$, $(\hat{\omega}_2)_{n+1}$ and $(\hat{\omega}_2)_{n+1}$, travel with a speed v through the contact shock. In order to employ the Riemann method, between each two consecutive spatial steps, the arbitrary initial condition given as a function of the position *x*, is approximated by a piecewise constant function, since Riemann method requires a step function as initial condition.

At a given time, the approximation, is conveniently chosen with equal width steps, so that for any function $(\hat{\varphi})_n$, $(\hat{v})_n$ and $(\hat{\omega}_i)_n$ are obtained at $x_j + \theta_n \Delta x$, with $\Delta x = x_{j+1} - x_j$ being the width of the interval and θ_n a number randomly chosen in the interval $(-1/2, +1/2)$. After each advance in time the choice of θ_n must be repeated, assuring that the Riemann problem initial condition is a step function.

To ensure uniqueness, there can be no shock interaction among shocks from adjacent Riemann problems, the Courant-Friedrich-Lewy condition, given by $\Delta t = t_{n+1} - t_n \le \Delta x / (2|\lambda|_{\max})$ must be satisfied, with $|\lambda|_{\max}$ being the maximum absolute value of the shock propagation speeds.

Table 1 states the conditions for the four possible solutions for the Riemann problem, when the intermediate state is different from the left and from the right ones.

Table 1. Conditions for each of the four possible solutions for the Riemann problem.

if	then the solution is
$v_R - v_L > c\left\|\ln\dfrac{\varphi_L}{\varphi_R}\right\|$	1-rarefaction/5-rarefaction
$-c\left\|\sqrt{\dfrac{\varphi_R}{\varphi_L}} - \sqrt{\dfrac{\varphi_L}{\varphi_R}}\right\| > v_R - v_L$	1-shock/5-shock
$c\left[\sqrt{\dfrac{\varphi_R}{\varphi_L}} - \sqrt{\dfrac{\varphi_L}{\varphi_R}}\right] < v_R - v_L < c\ln\dfrac{\varphi_L}{\varphi_R}$	1-rarefaction/5-shock
$-c\left[\sqrt{\dfrac{\varphi_R}{\varphi_L}} - \sqrt{\dfrac{\varphi_L}{\varphi_R}}\right] < v_R - v_L < -c\ln\dfrac{\varphi_L}{\varphi_R}$	1-shock/5-rarefaction

Once known the intermediate state (φ_*, v_*), the solution (φ, v) is given by

a) 1-rarefaction/5-rarefaction

$$(\varphi,v)=\begin{cases}(\varphi_L,v_L) & \text{if} \quad -\infty < x/t < v_L - c\\ (f_1,g_1) & \text{if} \quad v_L - c \le x/t \le v_* - c\\ (\varphi_*,v_*) & \text{if} \quad v_* - c < x/t < v_* + c\\ (f_5,g_5) & \text{if} \quad v_* + c \le x/t \le v_R + c\\ (\varphi_R,v_R) & \text{if} \quad v_R + c < x/t < \infty\end{cases} \tag{11}$$

b) 1-shock/5-shock

$$(\varphi,v)=\begin{cases}(\varphi_L,v_L) & \text{if} \quad -\infty < x/t < s_1\\ (\varphi_*,v_*) & \text{if} \quad s_1 < x/t < s_2\\ (\varphi_R,v_R) & \text{if} \quad s_2 < x/t < \infty\end{cases} \tag{12}$$

c) 1-rarefaction/5-shock

$$(\varphi,v)=\begin{cases}(\varphi_L,v_L) & \text{if} \quad -\infty < x/t < v_L - c\\ (f_1,g_1) & \text{if} \quad v_L - c \le x/t \le v_* - c\\ (\varphi_*,v_*) & \text{if} \quad v_* - c < x/t < s_2\\ (\varphi_R,v_R) & \text{if} \quad s_2 < x/t < \infty\end{cases} \tag{13}$$

d) 1-shock/5-rarefaction

$$(\varphi,v)=\begin{cases}(\varphi_L,v_L) & \text{if} \quad -\infty < x/t < s_1\\ (\varphi_*,v_*) & \text{if} \quad s_1 < x/t < v_* + c\\ (f_5,g_5) & \text{if} \quad v_* + c \le x/t \le v_R + c\\ (\varphi_R,v_R) & \text{if} \quad v_R + c < x/t < \infty\end{cases} \tag{14}$$

in which f_1, g_1, f_5 and g_5 depend on the ratio x/t and are obtained from the Riemann invariants, being given by

$$g_1 = \frac{x}{t} + c, \quad g_5 = \frac{x}{t} - c, \quad f_1 = \varphi_L \exp\left[-\frac{x}{ct} + \frac{v_L}{c} - 1\right] \quad \text{and} \quad f_5 = \varphi_R \exp\left[\frac{x}{ct} - \frac{v_R}{c} - 1\right] \tag{15}$$

Since φ_L and φ_R are positive, then f_1, f_5 and φ_* are positive too. Therefore, the positiveness of the fluid fraction is ensured for all x and t.

At this point it is important to remark that, since contact shocks are present, $(\varphi)_{*1} = (\varphi)_{*2} = (\varphi)_{*3}$ and $(\varphi v)_{*1} = (\varphi v)_{*2} = (\varphi v)_{*3}$, so that the shock is propagated with a speed v, and only the pollutants mass fraction ω_i is to be determined.

After obtaining the approximations $(\hat{\varphi}\hat{\omega}_1)_{n+1}$, $(\hat{\varphi}\hat{\omega}_2)_{n+1}$ and $(\hat{\varphi}\hat{\omega}_3)_{n+1}$ by Glimm method they are "corrected", by marching the same time interval Δt with the solution of the ordinary system obtained by the solution of following the time evolutionary system,

$$\begin{cases}\dfrac{\partial(\varphi\omega_1)}{\partial t} = -\gamma_1\omega_1\\ \dfrac{\partial(\varphi\omega_2)}{\partial t} = -\gamma_2\omega_2\\ \dfrac{\partial(\varphi\omega_3)}{\partial t} = \gamma_1\omega_1 + \gamma_2\omega_2\end{cases} \qquad \left.\begin{aligned}(\varphi\omega_1) &= (\hat{\varphi}\hat{\omega}_1)_{n+1}(x)\\ (\varphi\omega_2) &= (\hat{\varphi}\hat{\omega}_2)_{n+1}(x)\\ (\varphi\omega_3) &= (\hat{\varphi}\hat{\omega}_3)_{n+1}(x)\end{aligned}\right\}\text{at } t = t_n \tag{16}$$

System (16) is approximated by an Euler scheme as

$$
\begin{aligned}
(\varphi\omega_1) &= (\hat{\varphi}\hat{\omega}_1)_{n+1}(x) \approx (\tilde{\varphi}\tilde{\omega}_1)_{n+1}(x) - \{\gamma_1\omega_1\}\,\Delta t \\
(\varphi\omega_2) &= (\hat{\varphi}\hat{\omega}_2)_{n+1}(x) \approx (\tilde{\varphi}\tilde{\omega}_2)_{n+1}(x) - \{\gamma_2\omega_2\}\,\Delta t \\
(\varphi\omega_3) &= (\hat{\varphi}\hat{\omega}_3)_{n+1}(x) \approx (\tilde{\varphi}\tilde{\omega}_3)_{n+1}(x) + \{\gamma_1\omega_1 + \gamma_2\omega_2\}\,\Delta t
\end{aligned}
\tag{17}
$$

4. NUMERICAL RESULTS

Some representative results are depicted in Figures 1 to 5. In the Figs. 1 and 2, for all the graphs the initial data for the fluid fraction φ, the fluid constituent velocity v and the pollutants' concentrations ω_α are presented in the first line while the other five lines depict five selected time instants. It is interesting to note that in all cases ω_3 initial value was supposed zero throughout the whole domain, while ω_1 and ω_2 present rectangular waves or step functions as initial values.

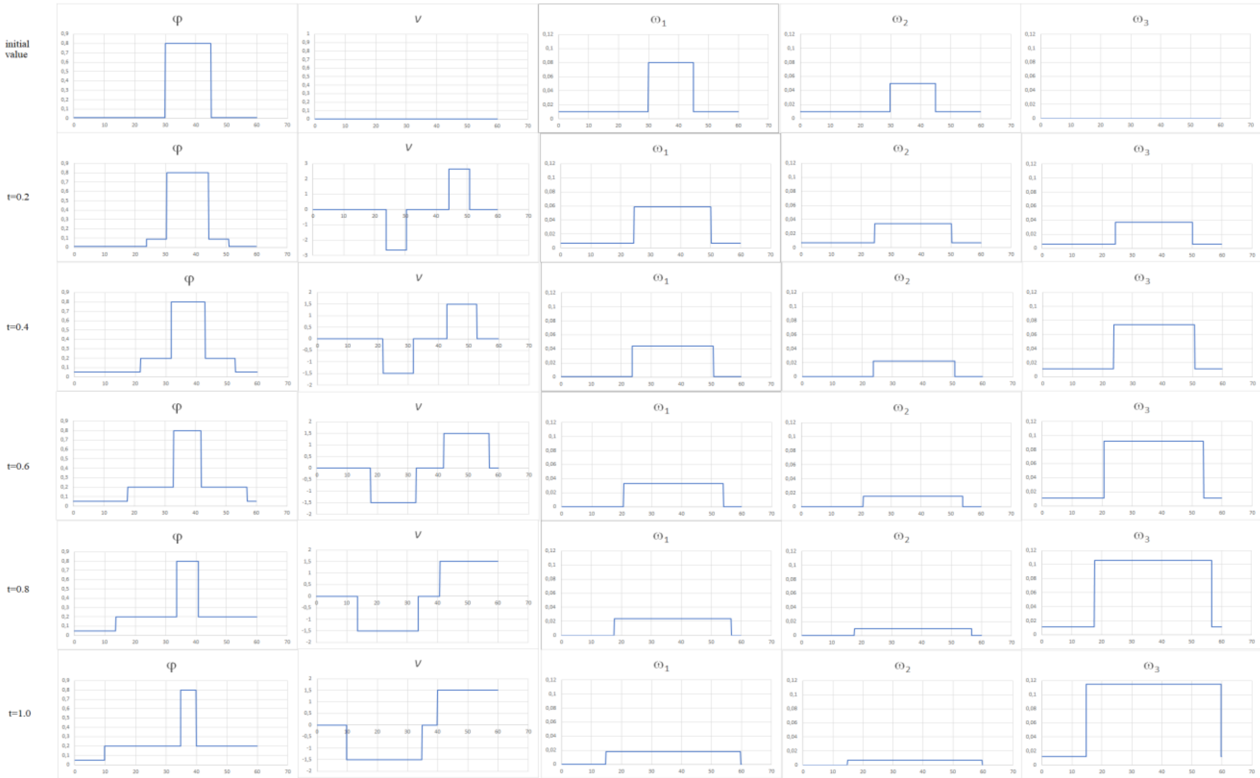

Figure 1. Behavior of fluid fraction φ, fluid constituent velocity v and the pollutants' concentration ω_α along the time for zero velocity and zero ω_3, rectangular waves for φ, ω_1 and ω_2.

Regarding the initial data, in Fig. 1 the fluid constituent velocity and the third constituent concentration ω_3 are zero along the whole domain, while rectangular waves are imposed from the middle of the domain until its last quarter for φ, ω_1 and ω_2. The fluid fraction is assumed almost zero out of this region and instantly assumes a large value ($\varphi = 0.8$) when it reaches half the domain, maintaining this value until reaching the third quarter of the domain, when it abruptly decreases to almost zero. An analogous rectangular wave is imposed on the first and second constituents concentrations ω_1 and ω_2 ($\omega_1 = 0.08$ and $\omega_2 = 0.05$). The fluid constituent velocity shows some shocks travelling along the domain as time goes by. The fluid fraction "spreads" along the domain as the time evolves. As expected, the contact shock for the pollutants' concentration is always at the same position $\forall \omega_\alpha$.

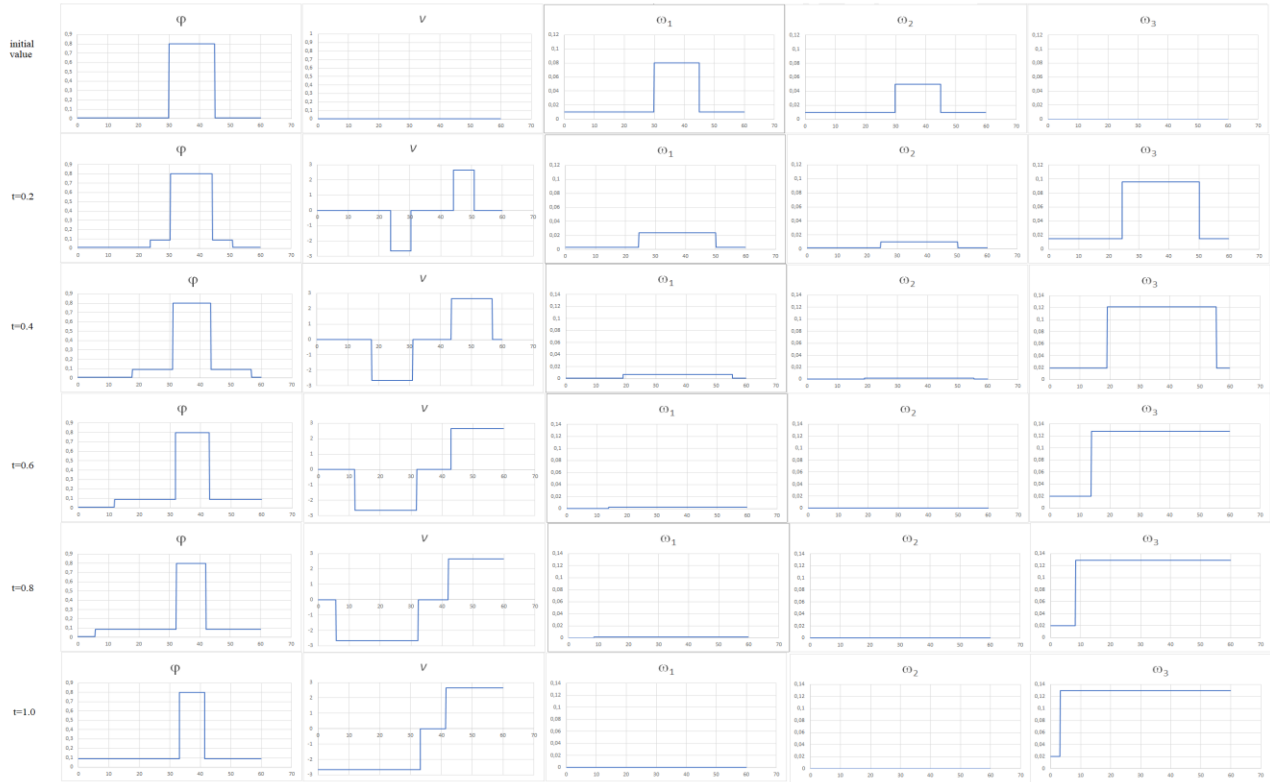

Figure 2. Behavior of fluid fraction φ, fluid constituent velocity v and the pollutants' concentration ω_α along the time for zero velocity and zero ω_3, rectangular waves for φ, ω_1 and ω_2 ($\gamma_1 = 4\gamma_1$ and $\gamma_2 = 4\gamma_2$, comparing with Fig.1).

It can be noticed that as predicted by equation (5), that ω_3, initially zero increases, while ω_1 and ω_2 decrease. In Fig. 2 all the initial data is the same considered in Fig. 1, except for the parameters β_1 and β_2 (and, consequently, γ_1 and γ_2), which are made four times bigger, impacting the behavior of all the pollutants' concentrations ω_α.

Distinct initial data is considered in Fig. 3: while the fluid fraction has the same initial behavior prescribed in Fig. 1, including the parameters β_1 and β_2, the fluid constituent initial velocity is a rectangular wave between the middle of the domain and its third quarter with a negative value, while step functions are prescribed for both ω_1 and ω_2.

In Fig. 4 the initial condition is a decreasing linear function for the fluid fraction φ, a rectangular wave between the middle of the domain and its third quarter with a positive value is prescribed for the initial velocity v and step functions for have been considered for both ω_1 and ω_2.

It is remarkable that the quantity $\varphi\rho_f\omega_\alpha$ has a physical meaning more relevant than the quantity $\rho_f\omega_\alpha$, once that the integral of $\varphi\rho_f\omega_\alpha$ over a given region of the mixture represents the amount of mass of the α^{th} constituent inside this region while $\rho_f\omega_\alpha$ does not play an important role.

So, the Figs. 5 and 6, for all the graphs the initial data for the fluid fraction φ, the fluid constituent velocity v and the pollutants' densities $\varphi\omega_\alpha$ are presented in the first line while the other five lines depict five selected time instants. As in the previous figures, Figs. 1 and 2, the initial value of ω_3 was supposed zero throughout the whole domain, while ω_1 and ω_2 present rectangular waves as initial values.

Figure 5 is actually the same case depicted in Fig. 2 but showing the time evolution of the three pollutants' densities.

Finally, a triangular wave is prescribed for the fluid fraction φ in Fig. 6, while the same rectangular waves for ω_1 and ω_2 employed in Fig. 1 are considered, along with and zero v and ω_3, as initial values.

Since the initial data for ω_α is always a piecewise constant function, the shape for ω_α remains piecewise constant. On the other hand, when $\varphi\omega_\alpha$ is considered, different types of behavior are found. The amount of the third pollutant is strongly dependent on the velocity distribution and on the rate of mass generation for this constituent.

For phenomena beginning from the rest, Figs. 1, 2, 5, 6, the initial fluid fraction distribution plays the main role in the pollutant distribution.

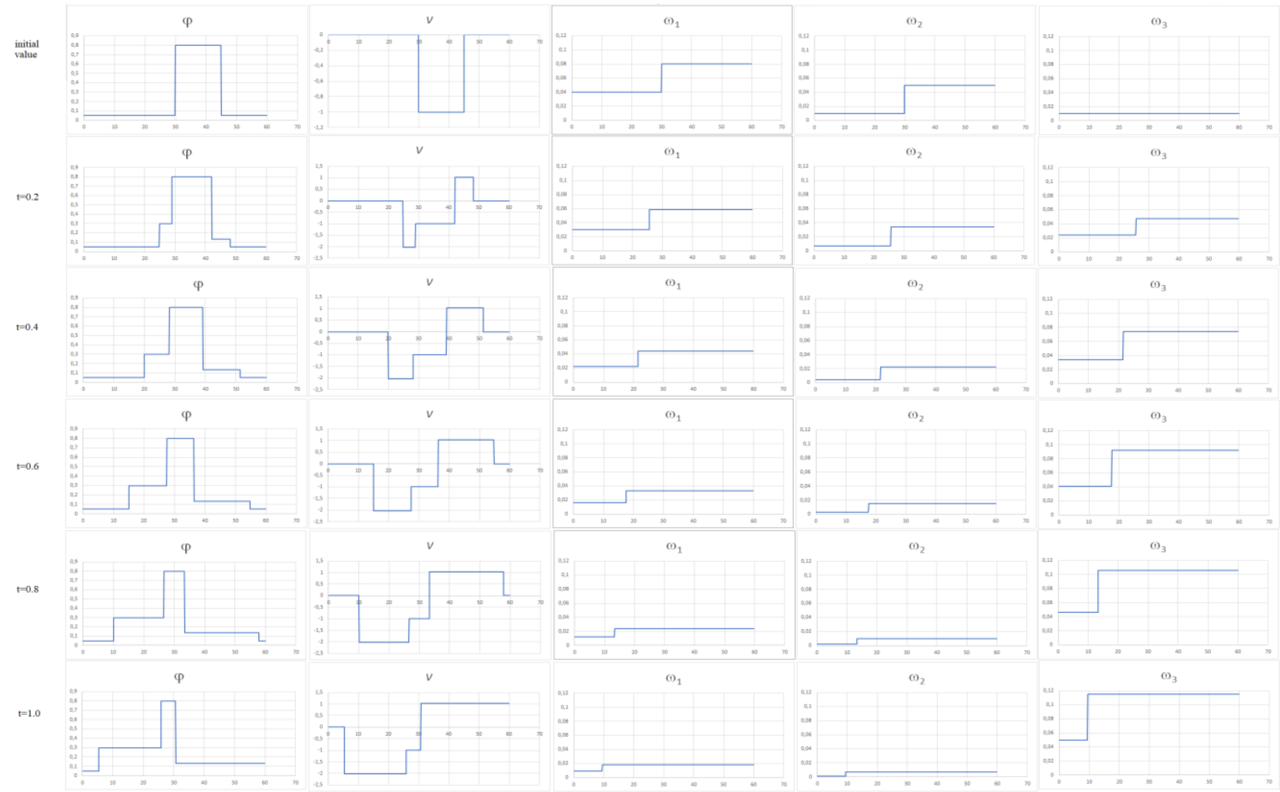

Figure 3. Behavior of fluid fraction φ, fluid constituent velocity v and the pollutants' concentration ω_α along the time for zero and zero ω_3, rectangular waves for fluid fraction and velocity and step functions for ω_1 and ω_2.

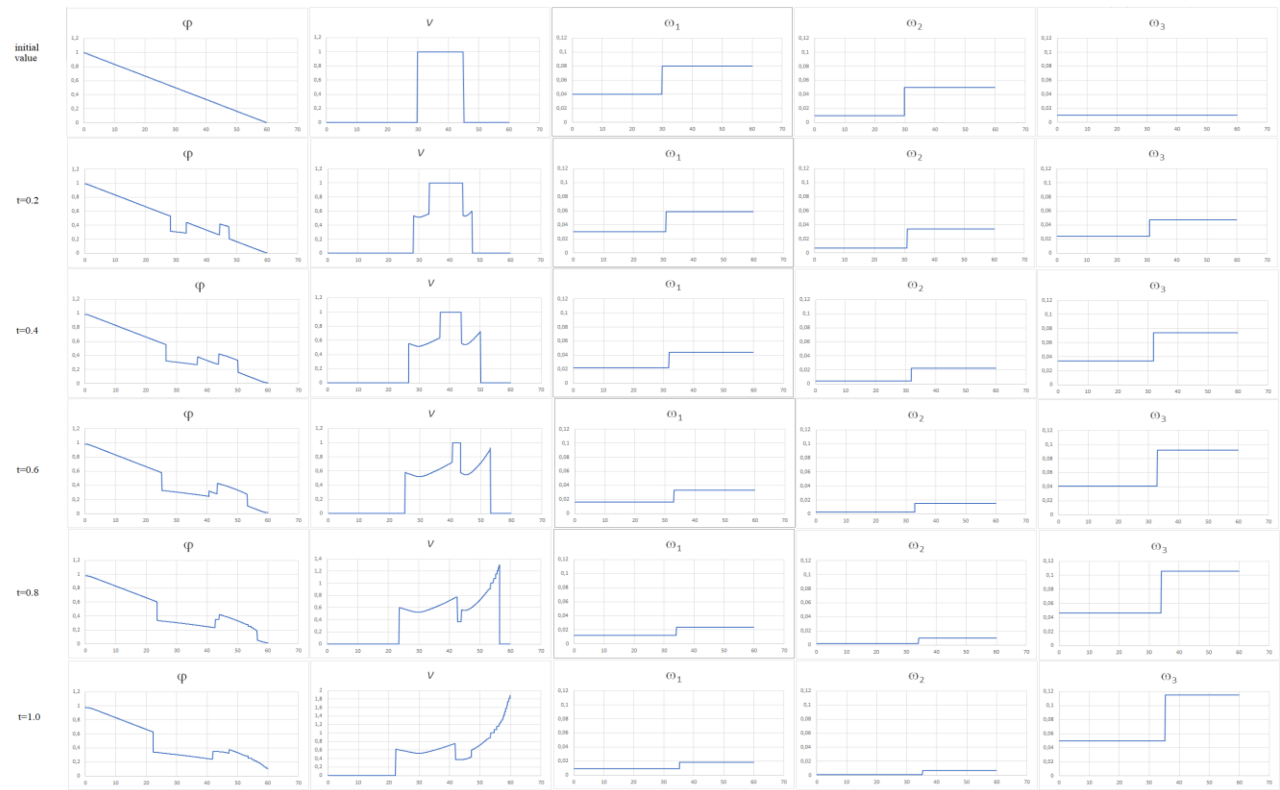

Figure 4. Behavior of fluid fraction φ, fluid constituent velocity v and the pollutants' concentration ω_α along the time for and zero ω_3, decreasing linear function for φ, rectangular waves for v and step functions for ω_1 and ω_2.

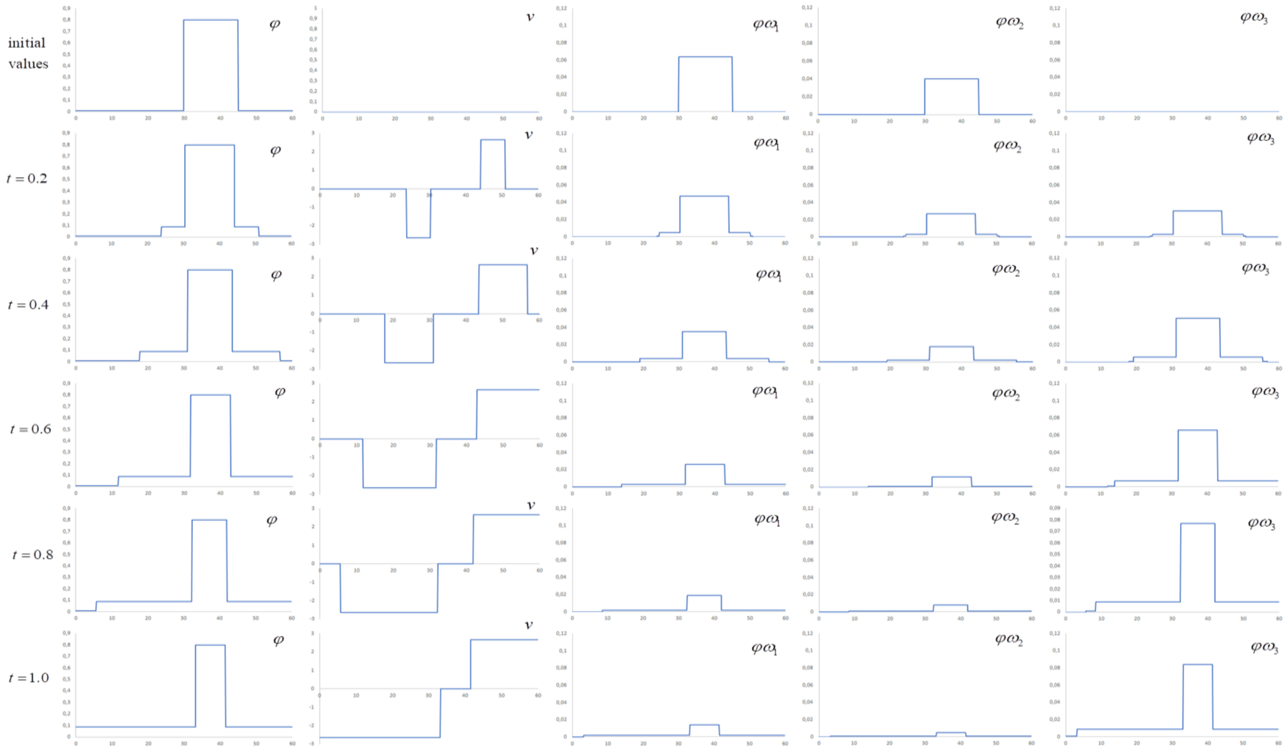

Figure 5. Behavior of fluid fraction φ, fluid constituent velocity v and the pollutants' densities $\varphi\omega_\alpha$ along the time for zero velocity and zero ω_3, rectangular waves for φ, ω_1 and ω_2 – γ-parameters 4 times bigger than Fig. 1.

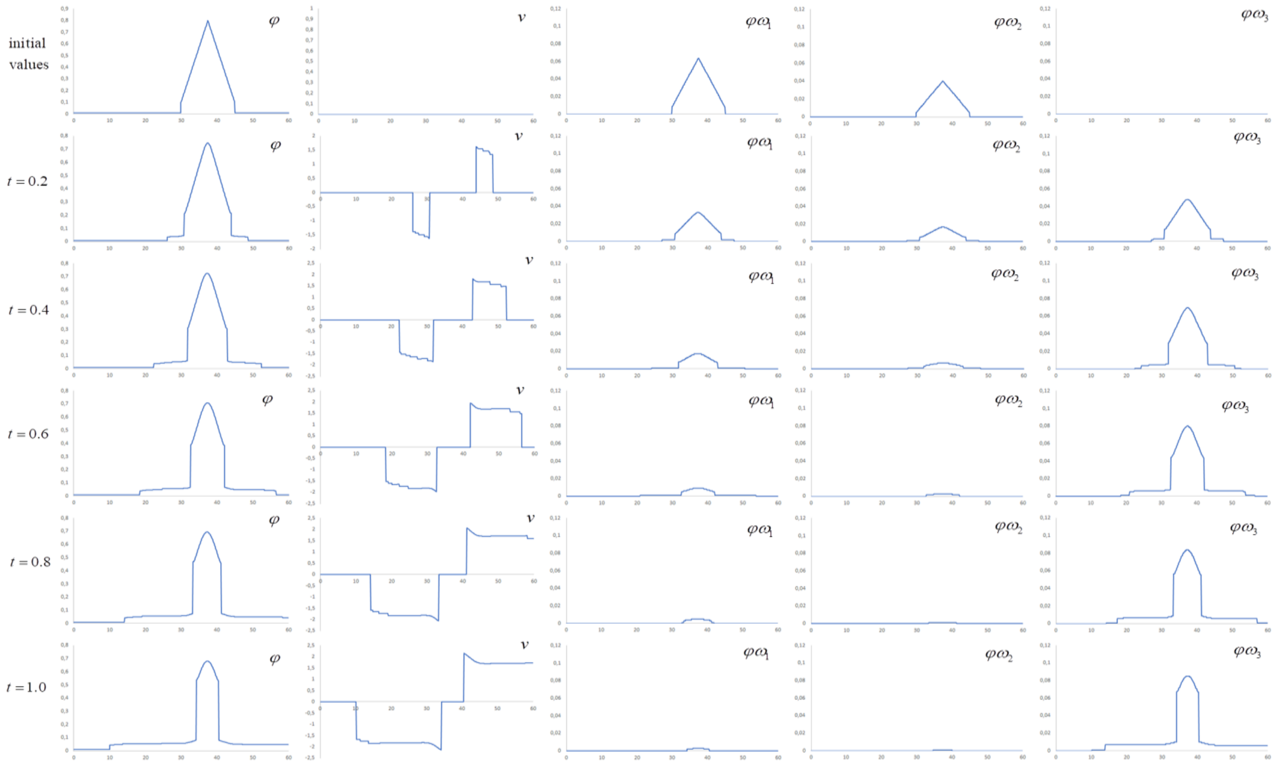

Figure 6. Behavior of fluid fraction φ, fluid constituent velocity v and the pollutants' densities $\varphi\omega_\alpha$ along the time for zero velocity and zero $\varphi\omega_3$, triangular waves for φ and rectangular waves ω_1 and ω_2 – $\gamma_1 = 4\gamma_1$ and $\gamma_2 = 4\gamma_2$, comparing with Fig.1.

5. FINAL REMARKS

A mixture theory approach studies the flow of a fluid containing pollutants through a porous matrix. Since the solid constituent is assumed rigid, there is a very small quantity of pollutants constituents in the fluid constituent and their mass densities are of the same order of magnitude, and the gas pollutant with negligible inertia is included just to account for the compressibility of the mixture, it suffices to solve the mass balance for the three constituents pollutants and the fluid constituent and the fluid constituent momentum balance. Constitutive relations for the partial stress tensor and the interaction force among the constituents were considered and a constitutive relation for the mass generation was proposed, giving rise to the mechanical model: a nonlinear nonhomogeneous hyperbolic system. An operator splitting technique, which treats a simultaneous problem as a sequential one, was employed to deal with the non-homogenous portion of the operator, while Glimm scheme, that marches in time employing the solution of a previously chosen number of Riemann problems, preserving the shock identity, approximated the homogeneous portion of the problem.

6. ACKNOWLEDGEMENTS

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