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An Argument in Support of FE Discretizations of Unbounded Media

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Abstract. *This work investigates finite element (FE) discretizations of unbounded media in terms of their compliance to Sommerfeld's Radiation Condition (SRC). The SRC poses a problem for the modeling of such media with FE discretizations, since by definition they can only discretize finite domains. However, in practical applications in which the medium has some material damping, it is expected that the energy from the excitation source would naturally vanish at moderately large distances from it. SRC would be satisfied, since little energy would be left to reach the finite boundary of the medium and be reflected. In this work, FE models of hemispheres with different damping ratios were excited with impulse loads, and the resulting perturbation away from the excitation source was measured. The results establish which is the necessary size of the hemisphere in order for the perturbation of points of its boundary to be one percent or less than that of points immediately under the loading surface. The results show that the size of the finite hemisphere in which violation of SRC is negligible is much smaller than initially thought. This work is important in the context of wave propagation analyses of unbounded media, as the results show that overly complicated models may not be necessary in some cases.*

Keywords: *Unbounded media, Wave Propagation, Sommerfeld Radiation Condition*

1. INTRODUCTION

This work investigates finite element discretizations of unbounded media in terms of their compliance to Sommerfeld's Radiation Condition (SRC). The SRC concerns wave propagation within elastic and viscoelastic unbounded media, such as electromagnetic propagation from antennae through the air, and seismic wave propagation in the soil, and it states that the phenomenon must be such that energy propagates from the excitation source and dissipates at vanishingly large distances from it (Sommerfeld, 1949). Finite media cannot comply with SRC because of the reflection of waves at the finite boundaries. This poses a problem for the modeling of such media with finite element (FE) discretizations, since by definition they can only discretize finite domains (except for special techniques employing boundary-like elements that are outside the scope of this work; Wolf and Song, 1991; Basu and Chopra, 1996). Boundary element (BE) formulations present as an advantage the fact that they allow, for example, that only the antennae or the surface of the soil be discretized, while yielding exact solution anywhere within the domain, and adequately complying with SRC (Zienkiewicz, Kelly and Bettess, 1979). However, in practical applications in which media have some material damping (Gaul, 1999), and considering the geometric damping associated with unbounded media (Lamb, 1903), waves naturally dissipate away from the excitation source, in such a way that there is no energy left to cause significant violations of SRC when and if they reach the finite boundary of the medium.

This work investigates the conditions under which FE discretizations of unbounded media comply with the SRC. FE representations of the title problem in this work consist of finite hemispheres of radius R, on the surface of which an impulse vertical or horizontal load is applied. The results present guidelines of radii R that must be selected for given damping factors of the medium. The size of the finite hemisphere in which violation of SRC is negligible is much smaller than initially thought. This finding supports the use of FE discretizations to model unbound media under certain conditions.

2. ANALYSIS

The present analysis takes as a representative model of the problem a three-dimensional, viscoelastic, isotropic, homogeneous half-space under time-harmonic vertical and horizontal loads. The loads are uniformly-distributed over a circular area of radius s of its free surface. A semi-analytical solution for this problem that properly accounts for Sommerfeld's Radiation Condition has been given by Rajapakse and Wang (1993) in terms of axisymmetric Green's functions. An attempt to obtain an equivalent problem through a finite medium is proposed in this work through a finite hemisphere of outer radius R. The flat surface of the hemisphere is under impulse vertical and horizontal loads uniformly distributed on a circular surface of radius s. Wave propagation within the hemisphere is computed in commercial software Ansys 19.2, using the module Explicit Dynamics (Fig. 1b). Boundary conditions of zero velocity were applied throughout the spherical region of the hemisphere, and zero stress boundary conditions were defined on the unloaded flat surface. The coordinate system is aligned in such a way that the x-y plane is contained within the free surface, centered at the center of the loaded area, and the z axis points towards the middle of the hemisphere. The response of the medium is measured in terms of the velocity of displacement of three points within the hemisphere, as acquired by three probes placed as follows: a) probe 1 is immediately under the loaded area (x=y=0, z=s), b) probe 2 is buried deeply within the hemisphere (x=y=0, z=R−s) and c) probe 3 is at the outer edge of the free surface (x=0, y=R−s, z=s) (Fig. 2).

By measuring the attenuation from probe 1 to probes 2 or 3, one may understand how quickly energy dissipates from the vicinities of the excitation source (probe 1) to the edge of the hemisphere. In this work, we define as appropriate size of hemisphere that in which the velocity of displacement between probes 1 and probes 2 or 3 is reduced by at least 99%. It is expected that the necessary radius of hemisphere for this to occur should depend on the damping factor of the medium.

Although there is spherical symmetry in the direction of wave propagation in this problem, two probes are necessary due to different types of wave that irradiate in different locations within the medium. Near the free surface, the presence of Rayleigh waves (Graff, 2012) should cause the vibration of the medium to differ from that of deeply buried points, hence the need for two different probes.

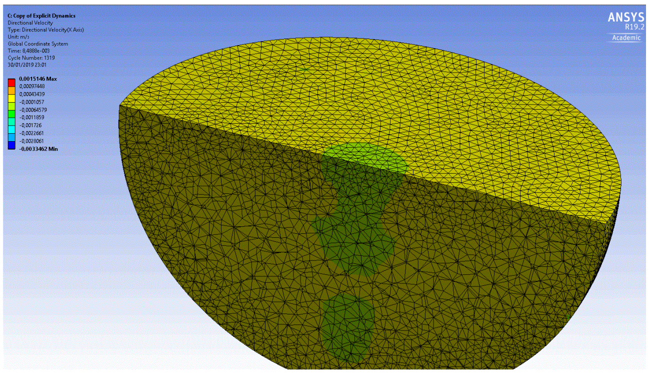

Figure 1. Unbounded half-space and finite element discretization of hemisphere.

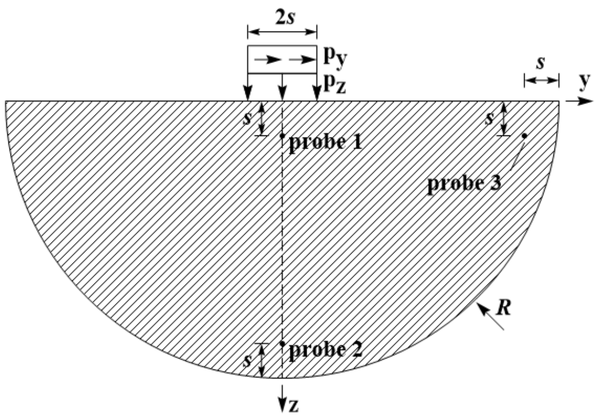

Figure 2. Position of probes within the hemisphere. The dimension 5s is out of scale for illustration purposes.

3. RESULTS

The results of this analysis are shown in terms of the necessary radius the hemisphere must have in order for the amplitude of vibration at its outer boundary to be reduced to less than one percent that of the vicinity of the excitation source. Since the measurements from probes 2 and 3 differ, the necessary radius to comply with that criterion varies from the point of view of each probe. The results are presented in terms of radius R_j, the necessary radius of the hemisphere to comply with the 99% vibration reduction criterion according to probe j (j=2,3). Since these radii depend on the radius of the loaded surface, results are normalized according to this quantity as well: $R'_j=R_j/s$.

Tables 1 and 2 and Fig. 3 show the normalized necessary radius of the hemisphere to comply with the 99% minimum vibration reduction criterion from the point of view of probes 2 and 3, for vertical and horizontal loads. These results consider different damping ratios ξ for the medium.

Table 1: Normalized necessary radius for vertical loads (in the z direction)

ξ	R'₃	R'₂
1e-5	26	24
1e-4	23	22
1e-3	19	19
1e-2	13.6	13
1e-1	8	8
2e-1	6.4	6

Table 2: Normalized necessary radius for horizontal loads (in the y direction)

ξ	R'₃	R'₂
1e-5	20	24
1e-4	19	20
1e-3	16	17.4
1e-2	12	13
1e-1	6	7
2e-1	5	5

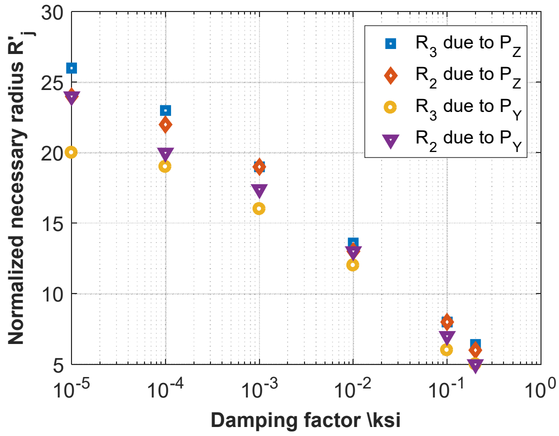

Figure 3 Normalized necessary radii for vertical and horizontal loads

As expected, the necessary radius decreases for increasing damping factors. Table 1 shows that a hemisphere with radius R'_2=24 is capable of reproducing SRC in the finite medium under vertical loads even for a relatively low value of damping ($\xi=10^{-5}$). For real soil media problems, in which the damping factor is much higher, these results show that a hemisphere of radius 10 times larger than that of the loaded area is capable of complying with SRC.

Note that this case, in which a homogeneous, isotropic hemisphere is considered, energy is allowed to propagate freely from the excitation source at the origin of the coordinate system to the outer boundary of the medium. This is a less energy-scattering medium than, for example, a multilayered soil, or one containing heterogeneities. Nevertheless, the results show that a finite hemisphere with moderate radii is capable of complying with SRC. These observations can reasonably be expected to hold for multilayered media, anisotropic media, porous media, etc.

4. CONCLUDING REMARKS

This paper investigated finite element approximations of the response of unbounded media, with respect to their ability to comply with Sommerfeld's Radiation Condition. A criterion that the amplitude of the velocity of displacements far from the excitation source had to be less than one percent of that of the vicinity of the excitation source was established. Results showed that this criterion is met with relatively small hemispheres. These analyses indicate that finite element discretizations provide acceptably accurate approximations for the propagation of waves in unbounded media, without the need to derive sophisticated Green's functions for specific soil media.

5. ACKNOWLEDGEMENTS

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