25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COB-2019-0652
COMPARISON OF DIFFERENT FLIGHT CONTROL APPROACHES FOR A QUADCOPTER UAV

Fillipe C. Silva
Luiz S. Martins-Filho
Federal University of ABC (UFABC), Alameda da Universidade, s/nº - B. Anchieta - 09606-045 São Bernardo do Campo/SP, Brazil
e-mails fillipe.cerdan@gmail.com , luiz.martins@ufabc.edu.br

Abstract. *This article addresses the problem of the flight control of an unmanned aerial vehicle (UAV), considering a typical quadcopter configuration with four independently actuated rotors. The first step of this work concerns the setting of the dynamics model equations of the vehicle flight adequately with the application of different types of control methods. To design an appropriate control strategy for a reliable and quite simple system, the study comprises control approaches from the well-known PID to more advanced technics like optimal control and nonlinear control. The main purpose is pointing out the advantages and disadvantages of each approach for the specific problem of the controlled take-off/landing maneuvers and the displacements missions. An attractive design option can be defined by evaluating the compromise between the simplicity for an embedded processing system and the adequate command performance, considering the application in an open system experimental quadcopter platform.*

Keywords: *Quadcopter UAV; Flight control; PID control; LQR control; Optimal control.*

1. INTRODUCTION

This study aims to compare the performances resulting from the application of different types of control methods and pointing out the advantages and disadvantages of each for this specific problem. The first step of this work is to set the equations of the flight dynamics model of a quadcopter unmanned aerial vehicle. The conclusions of these analyses will drive future implementation in an experimental educational quadcopter with open and programmable systems. The fundamental characteristics of this study qualify it as a guide for educational activities of control systems. The main focus will be put in the altitude control that will later compose with an attitude control approach a complete quadcopter command tool (Hoffmann, 2007; Jirinec, 2011). The control technics here investigated are the PID with manual tuning, the PID using Ziegler-Nichols methods of tuning, the PID using Root Locus, the Phase Lead using root locus, the Linear Quadratic Regulator (LQR), the LQR with an integrator state augmentation and, at last, the Optimal Control. As the dynamic model of the vehicle is not linear, the linearization will be sought. After this step, the PID, Phase Lead and LQR controllers will be designed. With those controllers designed, simulations in Simulink will be carried out, reintroducing nonlinearities of the model. As optimal controllers do not necessarily require the system linearity, the original nonlinear model can be used for control design. Results of the simulations will be compared to each other as to choose the best controller given the system requirements.

Hoffman et al. (2007) applied the PID control strategy to control the attitude, the altitude, and the positioning of the quadcopter with satisfactory results. However, the proposed linearization prevented more aggressive maneuvers from being carried out and rejection of the final system disturbance also fell short of expectations. In another work, Bouabdallah et al. (2005) adopted the PID approach considering a linearized model around the glide equilibrium point. With this, the controller was able to stabilize the attitude of the vehicle up to 3 seconds. In this case, it was observed that the linearization restricts its use only in small disturbances around the gliding equilibrium point.

Considering the LQR control strategy, this approach was applied in Castillo et al. (2005) to obtain a satisfactory performance during numerical simulations tests. Due to the linearity features of this control, strong disturbances tended to destabilize the vehicle causing loss of command. In the actual model, this control can present critical problems to stabilize the vehicle. In another work using LQR, this strategy was implemented with several equilibrium points, but without the dynamics of the propulsion motors, which resulted in worse performance than those obtained by a PID controller (Bouabdallah, 2005).

Chen and Huzmezan (2003) used the $H\infty$ method to develop a two-degree-of-freedom controller coupled with a predictive model control (MPC). The numerical simulations results indicated that the controller generated robustness in monitoring the reference and good rejection of disturbance.

Frederico (2015) implemented a nonlinear backstepping approach to control the position and attitude of a quadcopter in a search and surveillance mission. This controller was tested in simulations obtaining satisfactory results.

This controller, however, considered a linearization, which can lead to instabilities and loss of command of the vehicle. In the same study, another strategy was tested to control the position and attitude of a quadcopter in the same type of mission: the sliding mode control. The obtained simulations tests results were satisfactory. In this case, the quadcopter tended to have faster responses than the previous one, but with a greater error concerning the trajectory taken. This controller, however, was not linearized.

In the following sections, the article presents a succinct description of the modeling of the flight dynamics, the main ideas of the considered control approaches, and the results of numerical simulations tests to comparatively analyze the controllers' performance.

2. FLIGHT DYNAMICS OF THE QUADCOPTER

We consider that the propulsion of the quadcopter is given by two motors, numbers 1 and 3, rotating counterclockwise and two motors, numbers 2 and 4, rotating clockwise (see Fig (1)). The reference frames considered in the modeling are the inertial (NED), denoted by (X, Y, Z), and the reference fixed in the body of the UAV, denoted by (x, y, z). The description of the attitude movement is parameterized by the roll, pitch and yaw angles, represented by (θ, ϕ, ψ) (Bouabdallah et alli, 2005; Castillo et alli, 2005; Frederico et alli, 2015).

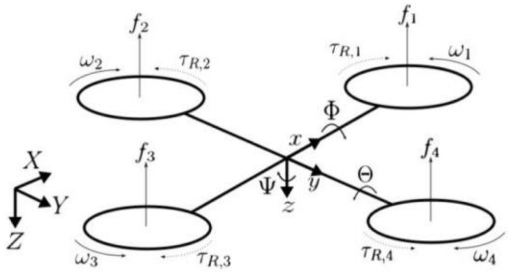

Figure 1. The schematic model of the quadcopter flight propulsion system, including the forces and torques, and the reference frames (Adigbli, 2007).

The model of the flight dynamics is obtained from the Euler-Lagrange approach, defining the Lagrangian by $L = E - V$. The state variables are the position of the center of gravity of the UAV and the attitude:

$$q = \{x \;\; y \;\; z \;\; \theta \;\; \phi \;\; \psi\}^T \tag{1}$$

The movement equations can be derived from the kinematics and potential energy, as shown by the following equations:

$$E = \frac{1}{2}[m(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) + I_{xx}(\dot{\phi} - \dot{\psi}\sin\theta)^2 + I_{yy}(\dot{\phi}\cos\phi - \dot{\psi}\cos\theta\sin\phi)^2 + I_{zz}(\dot{\phi} - \dot{\psi}\sin\theta)^2 \tag{2}$$

$$V = -mgZ \tag{3}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k, \quad k = 1, 2, \dots, 6 \tag{4}$$

An experimental platform was considered as the quadcopter for this study. This UAV is a vehicle of the GyroFly Company, model Gyro 200-ED (Gyrofly, 2014). The physical parameters of this model are shown in the Tab. (1). The flight dynamics model also includes the dynamics of the motors' aerodynamic drag aspects considering the propellers' simplified model.

Table 1. Physical parameters of the GyroFly 200ED experimental platform (Gyrofly, 2014).

Mass	$m = 1.03$ kg
Lever arm	$l = 0.26$ m
Torque constant	$d = 2.4086 \times 10^{-7}$ Nm/(rad/s)^2
Max angular velocity (motors)	$\omega_{Max} = 723$ rad/s
Inertial matrix	$I = \begin{bmatrix} 16.83 \times 10^{-3} & 0 & 5.72 \times 10^{-6} \\ 0 & 16.83 \times 10^{-3} & 0 \\ 5.72 \times 10^{-6} & 0 & 28.34 \times 10^{-3} \end{bmatrix}$ kgm^2

3. STUDIED CONTROL APPROACHES

The main purpose of the study is to compare different control strategies in the problem of quadcopter flight control and accumulate performance evaluations and analysis to understand the main problems that arise from linearization and simplification of the model and for posterior application in the experimental platform (Dorf, 2005; Skogestad and Postlethwaite, 2005).

The PID controller is the de-facto standard on the industry, for its simplicity and ease of tune. It consists of three main parts: Proportional, Derivative and Integral action and can be further incremented with anti-windup strategies, derivative action filtering and choosing the derivative action to be used only on the process variable. The well-known equations of PID are given by Eq. (5) and (6). In this paper, this controller was designed with gains defined through Ziegler-Nichols methods (and also with adjustments by manual tuning).

$$G(s) = K_P + K_D s + \frac{K_I}{s} \tag{5}$$

$$g(t) = K_P e(t) + K_D \frac{de(t)}{dt} + K_I \int_0^t e(t)dt \tag{6}$$

An alternative PID control design method is based on the gain determination using the Root Locus method, taking as project requirements the maximum overshoot and the settling time. This method consists of placing the system poles in such a way that the above-mentioned requirements are met. In this paper, the chosen controller was in a PD form, which transfer function can be seen in Eq. (7).

$$G(s) = (K_P + K_D s) = K_d(s + \frac{K_P}{K_D}) \tag{7}$$

After the design of the PID Controller using root locus, it was investigated a controller based in lead compensator to introduce a pole-zero pair into the open-loop transfer function. This controller transfer function is shown in (Eq (8)). The parameters of the control compensator can be defined using the Root Locus of the system.

$$G(s) = K\left(\frac{s+z_c}{s+z_c}\right) \tag{8}$$

The linear quadratic regulator (LQR) is based on optimal control. Its controller design imposes the linearization of the original dynamics model and application of a cost function, shown below Eq. (9). One of its drawbacks is that it assumes all the plant states to be known, which is rather impractical in the real world. To solve this problem, a Kalman Filter could be added to the dynamics, changing it to a Linear Quadratic Gaussian Regulator (LQG), not discussed in this paper.

$$J_{LQR}(t_0) = \frac{1}{2}\boldsymbol{x}^T(T)S(T)\boldsymbol{x}(T) + \frac{1}{2}\int_{t_0}^{T}[\boldsymbol{x}^T(t)Q(t)\boldsymbol{x}(t) + \boldsymbol{u}^T(t)R(t)\boldsymbol{u}(t)]dt \tag{9}$$

where the matrices Q(t) is positive semidefinite, R(t) is positive definite and S(T)≥ 0.

The linearized expression of the flight dynamics becomes:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ -0.04676 \end{bmatrix}\boldsymbol{u} \qquad \boldsymbol{y} = [1 \ \ 0]\boldsymbol{x} \tag{10}$$

To eliminate the steady-state error that appears in the application of the LQR control in the nonlinear flight dynamics, an integrative term is included in the mathematical modeling. This inclusion results in an augmented state space given by $\dot{\boldsymbol{g}} = \boldsymbol{y} - \boldsymbol{r} = C\boldsymbol{x} - \boldsymbol{r}$, the state equation becomes:

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{g}} \end{bmatrix} = \begin{bmatrix} A\boldsymbol{x} + B\boldsymbol{u} \\ C\boldsymbol{x} - \boldsymbol{r} \end{bmatrix} \qquad \boldsymbol{y} = C\boldsymbol{x} \tag{11}$$

The closed-loop expression, including the control action, is given by:

$$\dot{\boldsymbol{x}_a} = \begin{bmatrix} A - BG_c & -BK_i \\ C & \boldsymbol{0} \end{bmatrix}\boldsymbol{x}_a + \begin{bmatrix} BN \\ -1 \end{bmatrix}\boldsymbol{r} \qquad \boldsymbol{y} = [C \quad \boldsymbol{0}]\boldsymbol{x}_a \tag{12}$$

To analyze and evaluate the performance of all control methods above the nonlinearities were in the flight dynamics model. That way it was possible to accentuate the problems caused by the simplification of the model.

The next section presents the numerical simulations tests and the comparison of the response of the designed controllers with linear and nonlinear models.

4. SIMULATIONS TESTS RESULTS

The numerical simulations configuration considers the application of each control approach described in the precedent section in both linearized and nonlinear models for the quadcopter flight dynamics model. The considered input has the shape of a step applied at the initial simulation time.

The first analysis of the control strategies was made using a step input of 200 m of amplitude with the linearized model. The results of tests involving the problem of the quadcopter vertical flight are shown in Figures (2) to (11). It can be seen that the performance on the nonlinear model is not near that of the linearized model.

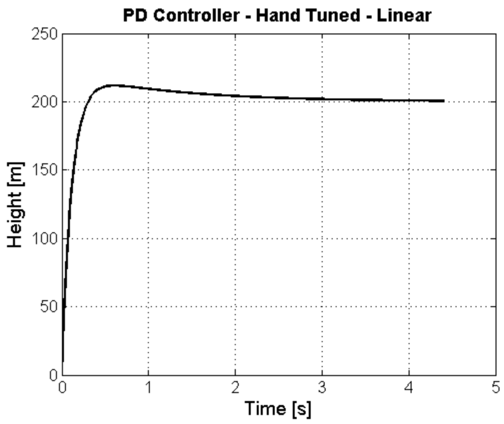

Figure 2. The response of the hand-tuned PD applied on linear model.

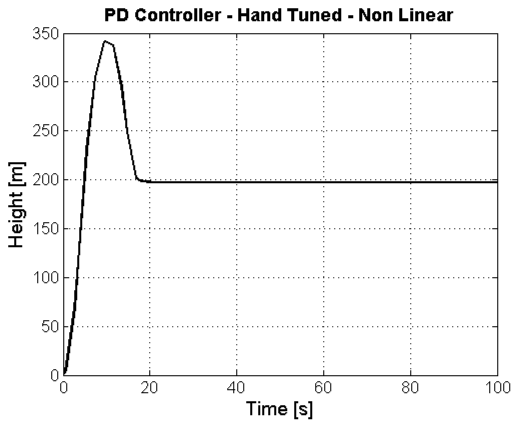

Figure 3. The response of the hand-tuned PD applied on nonlinear model

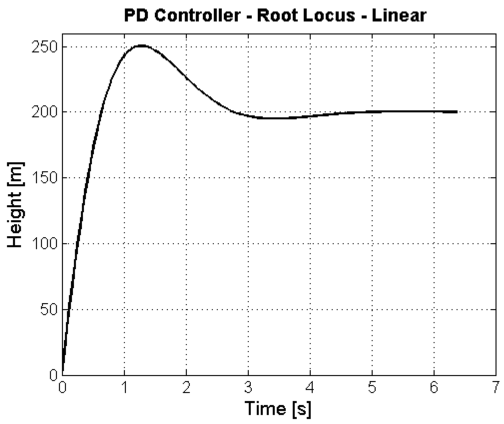

Figure 4. The response of the PD controller designed with Root Locus applied on linear model.

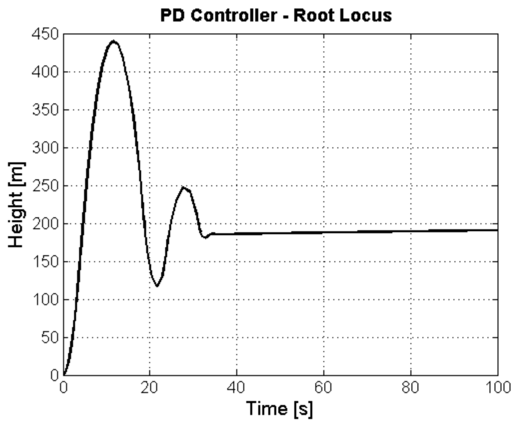

Figure 5. The response of the controller designed with Root Locus applied on nonlinear model.

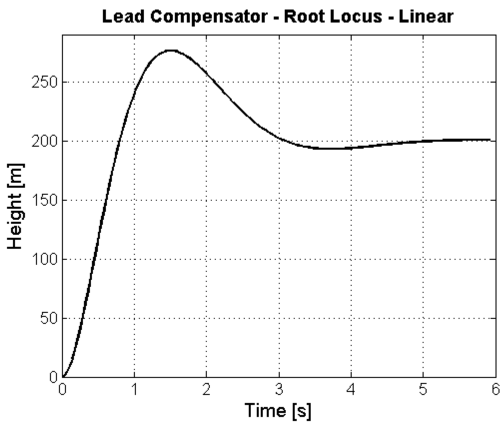

Figure 6. The response of the lead compensator applied on linear model.

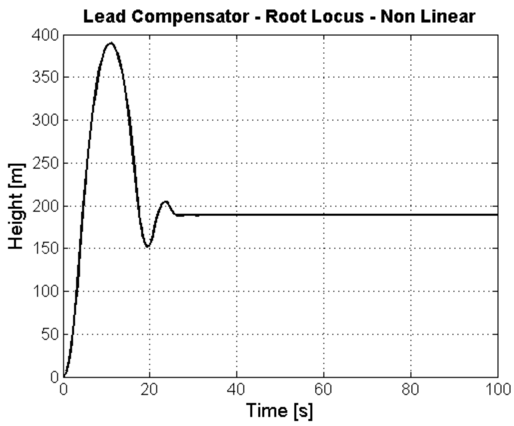

Figure 7. The response of the lead compensator applied on nonlinear model.

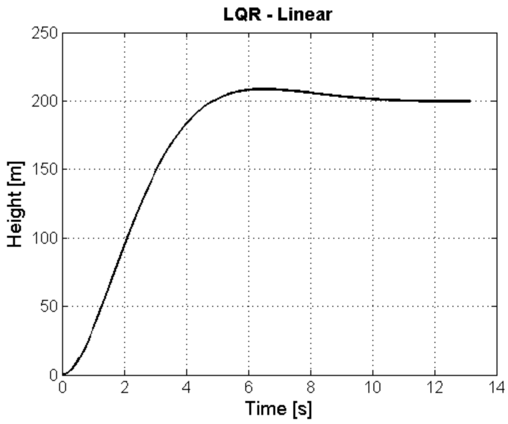

Figure 8. The response of the LQR controller applied on linear model.

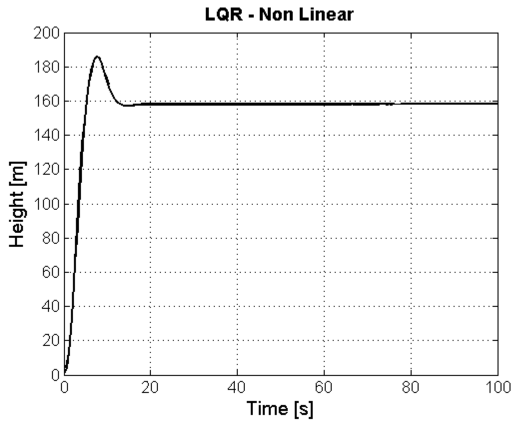

Figure 9. The response of the LQR controller applied on nonlinear model.

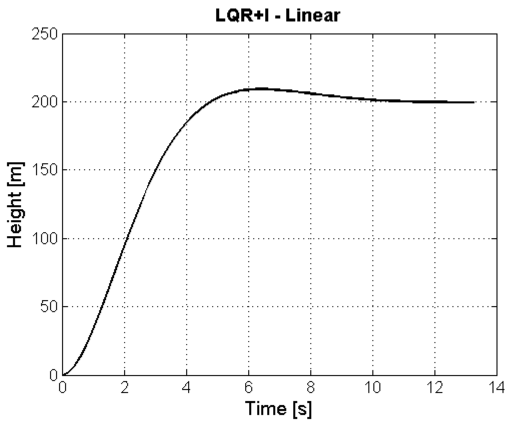

Figure 10. The response of the integral augmented LQR controller applied on linear model.

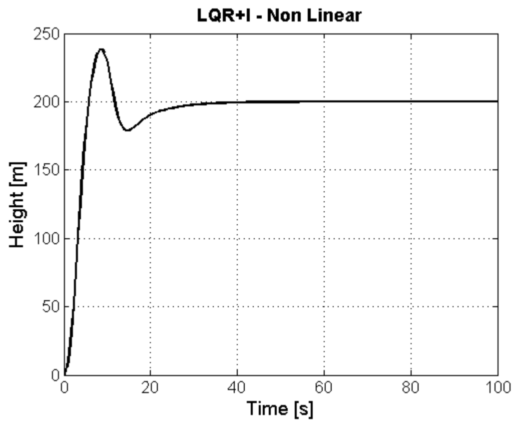

Figure 11. The response of the integral augmented LQR controller applied on nonlinear model.

The controller that had the worst performance is the PD designed with root locus. It had 250 m of overshoot, and a settling time of 35 s on the nonlinear case, as opposed to 51 m and 5 s using the linear model. Another drawback is the fact that it has a greater oscillation when compared to the other controllers. The augmented LQR controller, on the other side, had the best performance among all tested controllers. It had a 30 s settling time which is higher than the others, but its small overshoot and zero steady-state error makes it the best candidate so far. Table 2 shows all the acquired data.

Table 2. The results for the step input of 200 m of amplitude.

Controller	Settling time (s)	Max overshoot (m)	Steady-state error (m)
PID	15	141	0
PD + Root Locus	35	250	6
Phase Lead	25	190	10
LQR	20	27	42
LQR + Integrators	30	33	0

One of the main reasons why the performance on the nonlinear model suffers is controller saturation. The linear model assumes the controller can output infinite control effort, which is not feasible in real life, as every actuator has its limits. Figures (12) and (13) demonstrate the difference between the control effort from the linear and the nonlinear model, controlled by the PD Controller with Root Locus design.

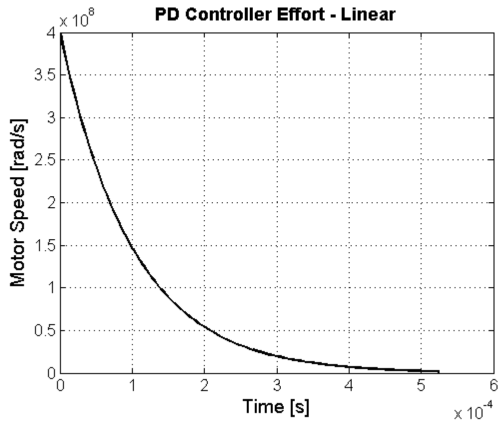

Figure 12. The control effort for the PD controller applied to the linear model.

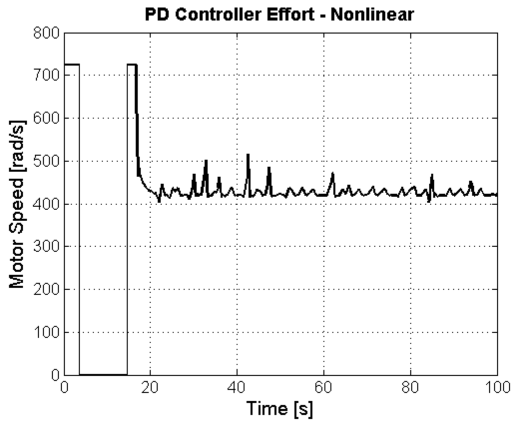

Figure 13. The control effort for the PD controller applied to the nonlinear model.

It can be seen that on the linear model, the output accelerates instantly to $4\cdot10^8$ rad/s, while on the nonlinear case, the output remains at 723 rad/s, the maximum speed the real motors can achieve. To mitigate the degrading effect of this saturation, instead of a large step as the reference for each controller, the rate at which the input could rise or fall was limited to 20 m/s, creating a ramp profile. The simulation results for the nonlinear case can be seen in Figures (14) through (18) and the numerical data about the performed simulations test are summarized in the Tab. (3).

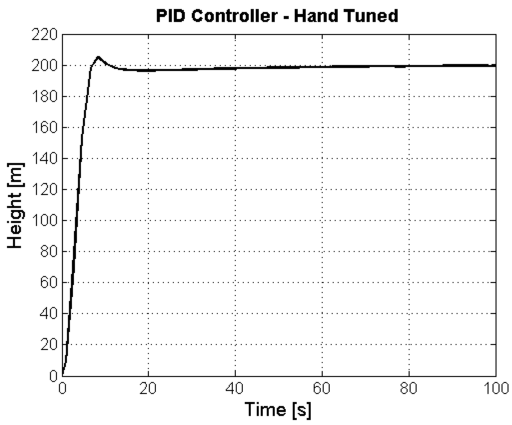

Figure 14. The results for the PD control.

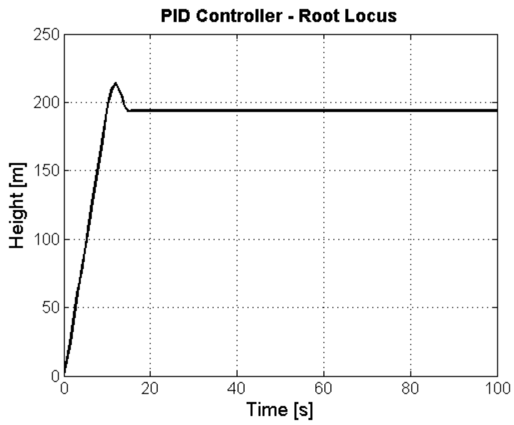

Figure 15. The results for the PD control with Root Locus.

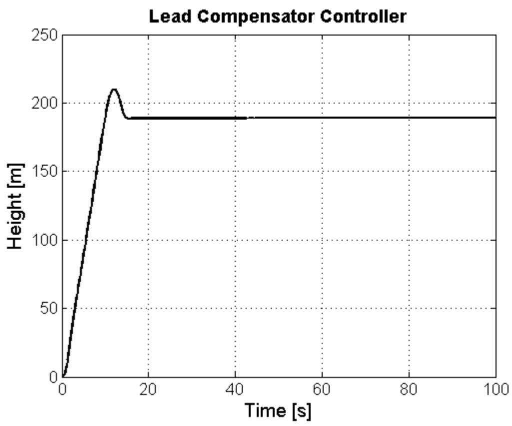

Figure 16. The results for the lead compensator controller.

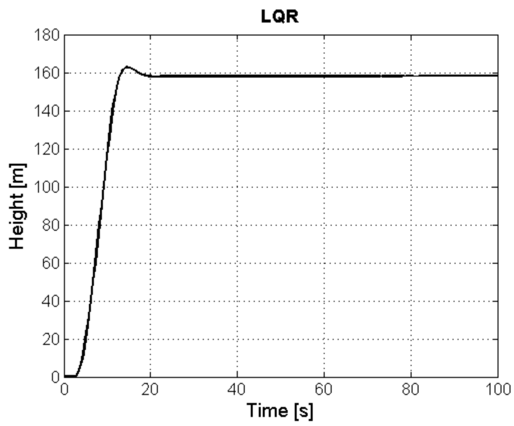

Figure 17. The results for the LQR control.

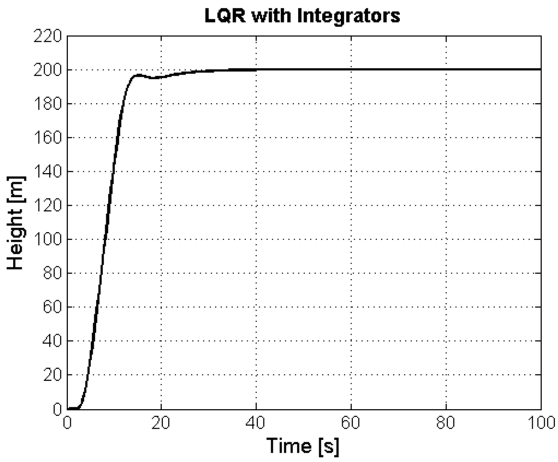

Figure 18. The results for the LQR control plus integrator.

Table 3. Results for ramp input with a rate of 20m/s and limit of 200m.

Controller	Settling time (s)	Max overshoot (m)	Steady-state error (m)
PD	90	1	0
PD + Root Locus	15	20	6
Phase Lead	15	21	11
LQR	20	5	42
LQR + Integrators	30	-	0

This reference change improved the settling time and maximum overshoot for all controllers, except for the hand-tuned PID. This controller had its settling time increased almost sevenfold. As the reference changes less abruptly, the controller does not need to have a high output, not saturating as much as before.

With all the above data, the best controller design, in terms of Settling Time and Maximum Overshoot was the integral augmented LQR. It presented a null overshoot, with no steady-state error and reasonable settling time of 30 s, for the case where the ramp reference was used. On the step reference, it also presented the best results, with the smallest overshoot, no steady-state error and the same settling time of the ramp case.

5. CONCLUSION

The first performance difference that the numerical simulations test results indicate is that the PID controller with manual tuning presented a better result than the PID designed using gains obtained through the Root Locus method. In both cases, the control is applied in linearized and nonlinear flight dynamics models. The phase gain controller obtained through the Root Locus method also presents worse performance than the PID with gains defined using manual tuning. Analyzing the case of the LQR controller, it can be seen that it presents a large steady-state error. If the LQR is boosted with an integrator, the controller can ensure null stationary error for a step input.

Considering the ramp input, the augmented LQR controller still had the best performance among the other four strategies. This kind of input improved both settling time and overshoot of the response for all controllers except for the PID with manual tuning.

The ongoing numerical simulations and controller design adjustments can allow more conclusive analysis and will point the best option, or options, for the embedded flight control system to be applied in the experimental quadcopter platform. The priority for the future steps of this study is the implementation of these improved controllers analyzed here in the available experimental quadcopter vehicle. The two options that first appear as candidates with more possibilities for success and adequate performance are the PID controller with manual gain tuning (taking into account the gains adjusted in the numerical simulations) and the LQR controller enhanced with the integrator action as they have no steady-state error.

6. ACKNOWLEDGMENTS

The authors acknowledge the financial support of FAPESP (Fundação de Apoio à Pesquisa do Estado de SP), of UFABC (Federal University of ABC), and CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior).

7. REFERENCES

Adigbli, P., 2007. “Nonlinear attitude and position control of a micro quadrotor using sliding mode and backstepping techniques.” In *Proceedings of the 3rd US-European Competition and Workshop on Micro Air Vehicle Systems and European Micro Air Vehicle*, Toulouse, France.

Bouabdallah, S., Murrieri, P., Siegwart, R., 2005. “Towards Autonomous Indoor Micro VTOL”. *Autonomous Robots*, Vol. 18, No. 2.

Castillo, P., Lozano, R., Dzul, A., 2005. “Stabilization of a mini rotorcraft with four rotors”. *Control Systems, IEEE*, Vol. 25, No. 6, p.45- 55..

Dorf, R. C., Bishop, R. H., 2005. “*Modern control systems*”. Upper Saddle River, USA: Pearson/Prentice Hall.

Frederico, L. A., Martins-Filho, L. S., Silva, A. L., 2015. “Navigation and control of a UAV quadrotor in search and surveillance missions”. *Journal of Physics: Conference Series*. 641 012031.

Gyrofly Innovations, 2014. “Gyro-200ED-X4 Manual do Usuário”. <http://www.gyrofly.com.br/>

Hoffmann, G.M. et al., 2007. “Quadrotor helicopter flight dynamics and control: Theory and experiment”. In *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, Hilton Head, South Carolina, USA.

Jirinec, T., 2011. “*Stabilization and control of unmanned quadcopter*”. Master Thesis, Luleå University of Technology, Sweden.

Skogestad, S., Postlethwaite, I., 2005. “*Multivariable feedback control: analysis and design*”. 2. ed. Chichester, UK: John Wiley & Sons.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.