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PERFORMANCE OF FINITE VOLUME DISCRETIZATION SCHEMES IN THE BACKWARD-FACING STEP PROBLEM. PART I : STABILITY OF SCHEMES

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Abstract. *The objective of this paper is to evaluate the numerical performance related to the stability of six finite-volume discretization schemes, in the classic benchmarking problem of the two-dimensional backward-facing incompressible step flow. The central differencing (CDS), simple exponential (EXP), first order upwind (FOU), second order upwind (SOU), QUICK and UNIFAES schemes were employed in the discretization of the advective terms of the Navier-Stokes equations in primitive variables using the semi-staggered mesh. The momentum equations are integrated explicitly after the solution of a Poisson pressure equation that ensures mass conservation. The reattachment lengths, as well as the velocity profiles, were obtained as functions of Reynolds number in eight levels of mesh refinement adopted. For low Reynolds numbers all discretization schemes in intermediary or refined meshes produce very similar results. In the case of flows with high Reynolds numbers, EXP scheme and FOU proved to be excessively diffusive, providing unphysical results. The CDS, FOU, and QUICK schemes fail for most of the intermediate meshes, obtaining results only in the most refined mesh for moderate Reynolds numbers. The stability of the SOU and UNIFAES schemes is similar, these two are convergent for any Reynolds number and present better results than the other schemes.*

Keywords: *Backward-Facing Step, Laminar Flow, Discretization Schemes, Finite Volumes, CFD.*

1. INTRODUCTION

The Backward-Facing Step (BFS) flow is one of the most fundamental geometries where laminar separation occurs caused by the sudden change in the geometry. Flows that exhibit separation and recirculation phenomena, caused by obstacles or abrupt changes in channel geometry, play an important role in many engineering applications. First, because the separation of the flow produced by an abrupt change in geometry is of great importance in many engineering applications. Second, from a fundamental perspective, there is a strong interest in understanding instability and the transition to turbulence in open, non-parallel flows. Finally, from a strictly computational perspective, the two-dimensional flow on a step is a very well-founded reference in CFD.

Flows with these characteristics are commonly found in airfoils, piping systems, combustion chambers, heat exchangers among many other applications of daily life. The transport and mixing properties of such flows are of great interest. The BFS flow has become a benchmark geometry for tests in numerical studies of flows that present separation and recirculation, in the last 50 years, several experimental and numerical studies have been carried out in an attempt to investigate this phenomenon.

The main experimental investigations in this geometry were made by Armaly *et al.* (1983) and Lee and Mateescu (1998). Armaly *et al.* (1983) perform the first large study in the geometry of the step obtaining experimental data in the laminar, transition and turbulent regions for an extensive Reynolds number range of 70 to 8000, considering an expansion rate of 1:1.94. The authors observed that the length of the primary recirculation zone varies almost linearly with the Reynolds number until the transition from the laminar to the turbulent regime occurs, it was observed that the flow is stable to Reynolds less than 1200, flow is in the transition region characterized by instabilities. The transition region occurs to Reynolds numbers smaller than 6600, for Reynolds numbers larger than this the flow becomes considered turbulent. Lee and Mateescu (1998) repeated Armaly *et al.* (1983) research into BFS flow for the 1:2 expansion rate, which is an easily reproduced geometry and considered more reliable by Santos and Figueiredo (2011).

Armaly *et al.* (1983) also performed a numerical study and did not obtain good agreement with the experimental data for high Reynolds numbers, the authors make it clear in their findings that the flow can only be considered totally two-dimensional only for Reynolds numbers below 400, above this limit the flow had three-dimensional effects. Many authors consider that the three-dimensional effects are the cause of the discrepancy between the experimental and numerical data for moderate to high Reynolds numbers.

This classical benchmarking problem is governed by a set of non-linear equations and it is necessary to use some discretizing scheme in the solution of these advective-diffusive transport equations, considering the context of finite-volume method. The choice of a discretization scheme has been a dominant subject in the literature when it comes to the numerical solution of problems involving transport phenomena and fluid mechanics. The most used first order scheme is the First Order Upwind (FOU) proposed by Courant *et al.* (1952). The FOU scheme is simple, stable and provides a smooth solution, however it is excessively diffusive for medium to high Reynolds, Peclet or Rayleigh number problems, which results in inaccurate solutions.

The Central Differencing Scheme (CDS) is the simplest second order scheme, but presents numerical instability and spatially oscillatory solutions as the Reynolds number increases. Despite the increasing computing power, allowing perform calculations on meshes increasingly refined, have expanded the spectrum in which the classic second-order schemes are stable, its limitations are still present. Thus, in the last decades, there has been an extensive use of stabilization methodologies, such as flow limiters, insertion of artificial viscosity and the intensive search for alternative schemes.

To overcome these limitations new second order schemes have been developed, as the Second Order Upwind scheme (SOU) presented by Warming and Beam (1976) and the Quadratic Upstream Interpolation for Convective Kinematics (QUICK) developed by Leonard (1979). These methods do not present the stability problems like the central scheme, however in some cases QUICK exhibits non-monotonic convergence as observed by Rodrigues *et al.* (2018) and Nascimento *et al.* (2018).

Among these options there are also the exponential-type schemes, where the interpolation functions are obtained from the exact solution of a one-dimensional homogeneous or non-homogeneous linear equation that simultaneously approximates the advective and diffusive terms of the transport equation. The exponential-type schemes are so called because the exponential function always appears in their interpolating curves, based on a linearized generative equation. All exponential-type schemes are asymptotically second-order. The first FVM exponential-type scheme is the Simple Exponential of Spalding (1972) and Raithby and Torrance (1974), between the most sophisticated scheme of this class is the Unified Finite Approaches Exponential-type Scheme (UNIFAES) proposed by Figueiredo (1997).

The main objective of this paper is to investigate the effects of increasing the number of Reynolds on the stability of these discretization schemes through the reattachment lengths and pressure and velocity fields. This test does not have an exact analytical solution for comparison with the numerical results, thus using the experimental data of Armaly *et al.* (1983) and Lee and Mateescu (1998) as an experimental reference in the investigation of the performance of the numerical schemes in approaching the reattachment lengths, but such tests have intrinsically three-dimensional characteristics for Reynolds numbers greater than 400. For these reasons, the comparison of the accuracy of these schemes requires the estimation of the exact solutions through Richardson's extrapolation, in a methodical and comparative way that is carried out in the second part of this investigation in the companion paper of Rodrigues *et al.* (2019).

2. METHODOLOGY

This paper together with its companion paper of Rodrigues *et al.* (2019) compose an investigation of the performance of referred numerical schemes in the case problem of BFS flow. The non-linear equations that govern this problem are solved using the formulation of primitive variables and the momentum equations are integrated explicitly after the solution of a Poisson pressure equation that ensures mass conservation. The referred numerical schemes were employed in the discretization of the advective-diffusive terms of the Navier-Stokes equations using the semi-staggered mesh.

2.1 Semi-staggered mesh

The present investigation uses the semi-staggered mesh structure, as indicated in Fig. 1, which presents the union of the advantages of co-located and staggered mesh structures, the so-called semi-staggered mesh stores the velocity components co-located at the vertices and the pressure at the center of the continuity cell, leaving the pressure components internal to the components of velocity. This mesh structure, developed by Kuznetsov (1968), presents simplicity in determining boundary velocities, such as the co-located mesh, and also enables the use of the MAC approach, as in the staggered mesh. According to Peyret and Taylor (1983) this configuration is also prone to have decoupled pressure fields as in the co-localized mesh, however, the pressure gradient of this field itself is not oscillatory and thus it is possible to correct the pressure field by techniques of post-processing that aims to smooth the pressure field without loss of the conservation of velocity fields.

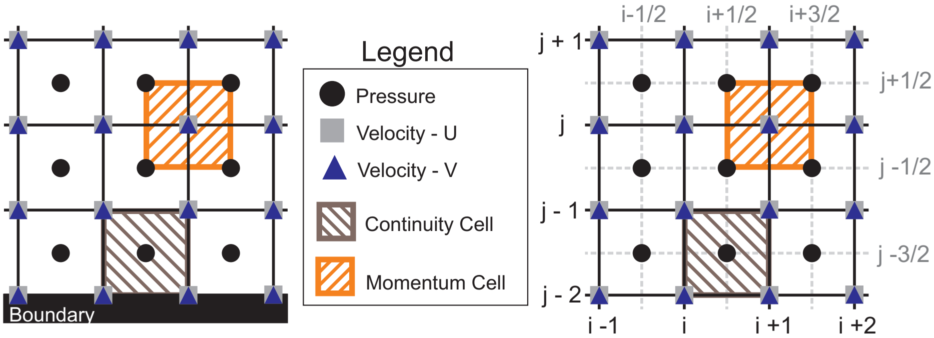


Figure 1. Semi-staggered mesh.

Among the co-located, staggered and semi-staggered mesh structures, the semi-staggered mesh is less widespread in the literature. One of the reasons for this can be attributed to the various existing nomenclatures for that configuration which is also known as half-staggered mesh, hybrid-staggered mesh or even as a type of co-located mesh.

2.2 Solution of governing equations

For an incompressible, isothermal flow with constant viscosity the Navier-Stokes equations together with the continuity equation can be written in two-dimensional Cartesian coordinates in the conservative form given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} = -A_u - \frac{\partial P}{\partial x} \tag{2}$$

$$\frac{\partial v}{\partial t} = -A_v - \frac{\partial P}{\partial y} \tag{3}$$

Where A_u e A_v represents the combined advective and diffusive net flux, given in conservative form as:

$$A_\phi = Re\frac{\partial(u\phi)}{\partial x} + Re\frac{\partial(v\phi)}{\partial y} - \left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) \tag{4}$$

The spatial coordinates were made nondimensional by a characteristic length L_c, the velocity components u and v by a characteristic velocity V_c, and time t by ReL_c/V_c. The Reynolds number is $Re = \rho V_c L_c/\mu$. The pressure P is made nondimensional by the sum of the physical pressure and the hydrostatic charge.

Using the Euler-explicit temporal discretization scheme for time derivatives and finite second order formulas for spatial derivatives, the system of governing equations (1) to (3) can be discretized by applying the methodology developed by Harlow and Welch (1965) in the semi-staggered mesh structure. Thus, considering the control volume indicated in Fig. 1 the discretized equations are of the form:

$$D^n_{i-1/2,j-1/2} = \frac{u^n_{i,j} + u^n_{i,j-1} - u^n_{i-1,j} - u^n_{i-1,j-1}}{2\Delta x} + \frac{v^n_{i,j} + v^n_{i-1,j} - v^n_{i,j-1} - v^n_{i-1,j-1}}{2\Delta y} \tag{5}$$

$$\frac{u^{n+1}_{i,j} - u^n_{i,j}}{\Delta t} = -A^n_{u_{i,j}} - \frac{P^n_{i+1/2,j-1/2} + P^n_{i+1/2,j+1/2} - P^n_{i-1/2,j-1/2} + P^n_{i-1/2,j+1/2}}{2\Delta x} \tag{6}$$

$$\frac{v^{n+1}_{i,j} - v^n_{i,j}}{\Delta t} = -A^n_{v_{i,j}} - \frac{P^n_{i+1/2,j+1/2} + P^n_{i-1/2,j+1/2} - P^n_{i+1/2,j-1/2} + P^n_{i-1/2,j-1/2}}{2\Delta y} \tag{7}$$

A Poisson-type equation for the pressure is obtained numerically by taking the divergent of the momentum equation and rearranging to yield:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = -\left(\frac{\partial A_u}{\partial x} + \frac{\partial A_v}{\partial y}\right) - \frac{\partial D}{\partial t} \tag{8}$$

Where D is the dilation term, which must be null by the incompressibility condition, which is evaluated by:

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{9}$$

Replacing the equations (5) to (7) in (9) we obtain the Poisson equation for discretized pressure for the volumes of internal controls, given by:

$$\begin{aligned}
&\frac{1}{4}\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)\left(P^n_{i-3/2,j+1/2}+P^n_{i+1/2,j-3/2}+P^n_{i+1/2,j+1/2}+P^n_{i-3/2,j-3/2}\right)+\\
&\frac{1}{2}\left(\frac{1}{\Delta x^2}-\frac{1}{\Delta y^2}\right)\left(P^n_{i-1/2,j-1/2}+P^n_{i-3/2,j-1/2}-P^n_{i-1/2,j+1/2}-P^n_{i-1/2,j-3/2}\right)\\
&-\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)P^n_{i-1/2,j-1/2} = -\frac{A^n_{u_{i,j}}+A^n_{u_{i,j-1}}-A^n_{u_{i-1,j}}-A^n_{u_{i-1,j-1}}}{2\Delta x}\\
&-\frac{A^n_{v_{i,j}}+A^n_{v_{i-1,j}}-A^n_{v_{i,j-1}}-A^n_{v_{i-1,j-1}}}{2\Delta y}+\frac{D^n_{i-1/2,j-1/2}}{\Delta t}
\end{aligned} \tag{10}$$

The terms A_u and A_v correspond to the two-dimensional steady-state transport equation and are evaluated from the equation (4) by the referred discretizing schemes. The details of the discretization of the equation (4) are properly presented in the second part of this study.

2.3 Numerical setup

The schematic diagram of the flow on a Backward-facing step is presented in detail in figure (2). The domain of study adopted was the same of Lee and Mateescu (1998) and consists of a channel of total length $L = 50h$ and total height $H = 2h$, where h is the height of the upstream step. For an expansion ratio of 1:2 the height of the step is $h = 1/2$. The step is located near the fluid inlet region, the inlet length is $10h$ with height $H - h$. The investigation of the symmetrical sudden expansion in this geometry is presented in the companion paper of Nascimento *et al.* (2019).

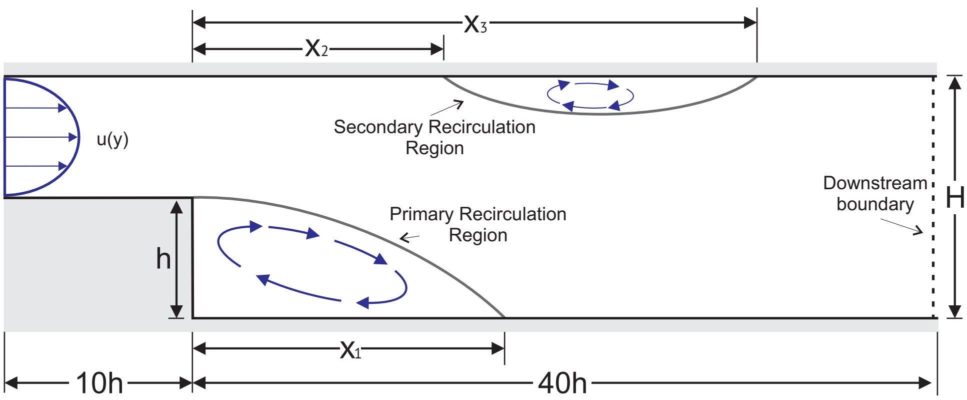


Figure 2. Backward-facing step geometry.

The input length of the channel was chosen so that it did not influence the primary recirculation region, as verified by Erturk (2008) and Santos *et al.* (2010) an input channel with length $10h$ is already sufficient to not affect the results in the primary recirculation region. The values of the reattachment and detachment lengths X_1, X_2 and X_3 were determined from interpolations in the velocity fields near the regions of separation and relocation, where the values found were extrapolated to the channel wall increasing the accuracy of the lengths found.

The sudden expansion caused by the presence of the step generates recirculation zones in the lower and upper walls of the channel according to Reynolds number. The main recirculation region is formed in the bottom wall, whose length is normalized by X_1/h. The secondary recirculation region occurs in the upper wall and the normalized length of this region is evaluated by $(X_3 - X_2)/h$.

The Poisson equation of pressure (10) is iteratively solved with 200 Gauss-Siedel scans by iteration, that number of scans was chosen because it is sufficient to ensure a satisfactory tolerance of the maximum rate of variation of the pressure field, thus dispensing with a computationally costly verification procedure of this rate of change. During the sweeping process, a sub-relaxation factor was adopted, which compensates for non-diagonality of the matrix.

The velocity at the channel entrance was used as the Dirichlet boundary condition, where the velocity profile is assumed to be fully developed. The fully developed and dimensionless velocity profile adopted is:

$$u = -24\left(y^2 - \frac{3}{2}y + \frac{1}{2}\right)$$
$$v = 0 \tag{11}$$

The fully developed velocity profile (11) is extended into the domain over the entire length of the input length. For other regions the dimensionless velocities are initialized as null. The null derivative is adopted as a boundary condition for the velocities at the exit of the channel. The pressure field is initialized as null for the entire computational domain. All walls of this geometry are impermeable. The region below the input channel is admitted as a solid wall and no computation is performed in this region of the computational domain.

The Reynolds number is determined by the expression (12), where V is the average speed at the entrance, equal to two thirds of the maximum input speed, D is the hydraulic diameter of the input $(2h)$, and ν is the kinematic viscosity.

$$Re = \frac{VD}{\nu} \tag{12}$$

According to Armaly *et al.* (1983) and Lee and Mateescu (1998) for this geometry the laminar flow is no longer stable around $Re = 1200$, so the tests of the present investigation will be carried out in the Reynolds range of $100 \leq Re \leq 1200$.

The stopping criterion was that the root mean square (RMS) of the momentum equation should be less than 10^{-5}, reducing that tolerance to 10^{-6} the results will only show differences in the eighth decimal place, indicating satisfactory accuracy for the purposes of this study.

3. RESULTS AND DISCUSSION

The test methodology used in this study consists of comparing the performance of the FOU, CDS, SOU, simple exponential and UNIFAES schemes in the eight refinement levels presented in the table 1, for the Reynolds range where the flow is considered laminar and stable, for practicality these meshes were named as set out in the table 1.

Table 1. Nomenclature of the levels of refinements used.

Mesh	Nomenclature
50x30	Mesh A
60x40	Mesh B
80x40	Mesh C
100x60	Mesh D
150x90	Mesh E
180x100	Mesh F
200x120	Mesh G
250x150	Mesh H

3.1 Preliminary results

Table 2 shows the results of the primary region recirculation length, at the eight refinement levels used, for $Re = 100$. From the results obtained, it is observed that for this Reynolds number all the schemes present similar solutions at almost all levels of refinement, where the results can be considered "independent" of the refinement from the F-mesh.

Table 2. Recirculation length X_1/h for $Re = 100$.

Mesh	FOU	CDS	EXP	SOU	QUICK	UNIFAES
A	2.69	1.83	3.07	3.42	2.62	3.47
B	2.69	2.15	2.98	3.35	2.72	3.34
C	2.70	2.55	2.93	3.30	2.96	3.27
D	2.75	2.93	2.93	3.23	3.04	3.21
E	2.81	3.00	2.95	3.14	3.05	3.11
F	2.83	3.01	2.95	3.10	3.04	3.08
G	2.84	3.00	2.95	3.09	3.03	3.07
H	2.86	2.99	2.95	3.06	3.01	3.03

The work of Lee and Mateescu (1998) only provides experimental data from $Re = 400$ for the 1:2 expansion rate. However, the experimental data of Armaly *et al.* (1983) for the expansion ratio 1:1.94 can be used as an estimate of the experimentally found length for the 1:2 expansion rate. For $Re = 100$, Armaly *et al.* (1983) found experimentally that $X_1/h = 3.11$. UNIFAES, QUICK and SOU schemes present almost identical results, obtaining the closest results of experimental data for X_1/h. In addition, SOU and UNIFAES provide a large reattachment length for coarse meshes and converge to smaller values as the mesh is refined. The EXP scheme also obtains good results, but it was not a highlight like the previous schemes. As expected the FOU scheme presents the least satisfactory results for $Re = 100$ given its first-order accuracy.

Table 3 shows the primary region recirculation lengths obtained for the eight refinement levels used in $Re = 300$. From the results obtained, it is observed that for this Reynolds number all schemes again present similar solutions at almost all refinement levels where for $Re = 300$ Armaly *et al.* (1983) found experimentally that $X_1/h = 6.76$.

Table 3. Recirculation length X_1/h for $Re = 300$.

Mesh	FOU	CDS	EXP	SOU	QUICK	UNIFAES
A	5.75	-	6.39	7.39	-	-
B	5.73	-	6.40	7.27	-	7.24
C	5.84	-	6.42	7.16	-	7.11
D	6.00	-	6.47	7.06	-	7.01
E	6.25	-	6.56	6.95	-	6.91
F	6.32	-	6.59	6.92	6.88	6.88
G	6.38	-	6.61	6.90	6.87	6.87
H	6.45	6.83	6.64	6.87	6.84	6.84

It is possible to observe that the CDS and QUICK schemes fail for intermediate meshes, this behavior was already expected for the CDS scheme, since the stability condition is no longer satisfied in $Re = 300$, where the CDS converges only for H-mesh. The QUICK presenting stability problems it is strange at first moment, because when comparing the performance presented by the same scheme in the investigation made by Rodrigues *et al.* (2018) and Nascimento *et al.* (2018) in a linear test problem, QUICK in all of the tests did not present any signal of instability. However, with a brief bibliographic search it is possible to verify that this is a common behavior for this scheme, as verified by Sharif and Busnaina (1988) and Shyy (1985). Instability issues like this have given rise to the various versions of QUICK. From the Tab. 3 it is observed that for $Re = 300$ the QUICK converges only to G and H meshes.

Table 4 displays the characteristic lengths of the two recirculation regions for $Re = 500$. For Reynolds number flows above 400 a second recirculation zone is formed in the upper wall of the channel, this second recirculation zone is determined by the difference between the lengths X_2/h e X_3/h.

Table 4. Recirculation Lengths X_1/h, X_2/h e X_3/h for $Re = 500$.

Length	Mesh	FOU	CDS	EXP	SOU	QUICK	UNIFAES	Experimental
X_1/h	A	-	-	7.97	9.95	-	-	9.21
	B	-	-	8.05	9.96	-	-	
	C	-	-	8.07	9.88	-	9.73	
	D	7.98	-	8.12	9.80	-	9.64	
	E	8.26	-	8.49	9.65	-	9.52	
	F	8.41	-	8.63	9.61	-	9.48	
	G	8.51	-	8.73	9.58	9.52	9.47	
	H	8.68	9.46	8.88	9.54	9.49	9.45	
X_2/h	A	-	-	0.00	7.49	-	-	7.6
	B	-	-	6.11	8.42	-	-	
	C	-	-	6.23	8.14	-	7.96	
	D	6.48	-	6.39	8.12	-	7.98	
	E	6.70	-	6.75	8.03	-	7.88	
	F	6.82	-	6.97	7.98	-	7.84	
	G	6.90	-	7.01	7.96	7.90	7.83	
	H	7.06	7.17	7.17	7.92	7.87	7.83	
X_3/h	A	-	-	0.00	14.14	-	-	14.25
	B	-	-	10.02	13.93	-	-	
	C	-	-	10.60	13.96	-	13.74	
	D	10.10	-	11.19	13.81	-	13.63	
	E	11.16	-	12.03	13.66	-	13.54	
	F	11.49	-	12.30	13.62	-	13.51	
	G	11.72	-	12.44	13.59	13.55	13.49	
	H	12.06	12.67	12.67	13.54	13.50	13.45	

SOU, QUICK, and UNIFAES schemes again provide very similar results, and when compared to the experimental measurements obtained by Lee and Mateescu (1998) these schemes are very accurate in approaching the three characteristic lengths of the recirculation zones. QUICK maintained the same unstable behavior, presented in the previous cases, in this case converging only for the G and H meshes.

As expected, for high Reynolds numbers, the EXP scheme presents similar results to FOU. The FOU scheme also proved to be non-convergent like QUICK, where it did not obtain results for the A, B and C meshes. The CDS scheme converged only in the H mesh, failing all other refinements investigated.

The SOU scheme has been surprising so far, presenting results that are closer than QUICK and UNIFAES of the experimental measurements, and this scheme has also demonstrated the greater stability for coarse meshes for this problem. UNIFAES did not converge for the A and B meshes, however it presented very accurate results, being paired with the SOU in the medium and refined meshes.

According to Shyy *et al.* (1985) the SOU scheme presents greater numerical stability than the QUICK because it uses only the upwind terms in its interpolating curve that evaluates the property ϕ in the face of the control volume, unlike the QUICK that employs a quadratic curve.

The input length in the channel geometry may be a possible cause for the instability presented by the FOU, CDS and QUICK schemes, since it is known that this input length harms the diagonal dominance of the system of equations. Many authors, such as Pollard and Siu (1982), Patel and Markatos (1986) and Barton (1995) do not consider the input length in the computational domain, favoring to a certain extent the performance of the schemes in this problem. These three works used alternative versions of QUICK, which do not present stability problems.

It can be seen from the results presented in the tables 2, 3 and 4 that the recirculation lengths of the CDS, SOU, QUICK and UNIFAES schemes decrease as the refinement level increases, with FOU and EXP schemes the recirculation length increases with increasing mesh size. Also it is observed that the results with the greater agreement with experimental data were obtained in intermediate meshes and not in the most refined meshes, which by definition should be the most accurate because of the more refined the mesh the lower the error numeric. Ferziger and Peric (2002) attribute this behavior to the cancellation of various forms of errors between-itself in intermediate meshes caused that results closer to the experimental in those levels of refinement.

3.2 Validation of method

The figure 3 and 4 show the lengths of the recirculation regions obtained for the H-mesh in the laminar Reynolds range of that flow. In figure 3 it is possible to observe the recirculation lengths of the primary region, where for Reynolds numbers below 400, all schemes predict similar results with each other. From Reynolds 400 a separation of the lengths obtained by the schemes takes place, where the FOU and the EXP follow parallelly presenting similar and non-accurate results among themselves. The CDS will fail for any mesh from Reynolds 600.

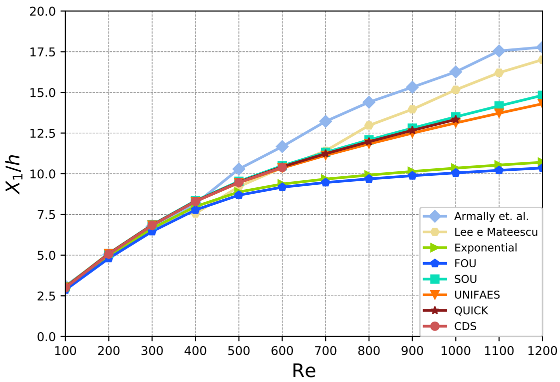
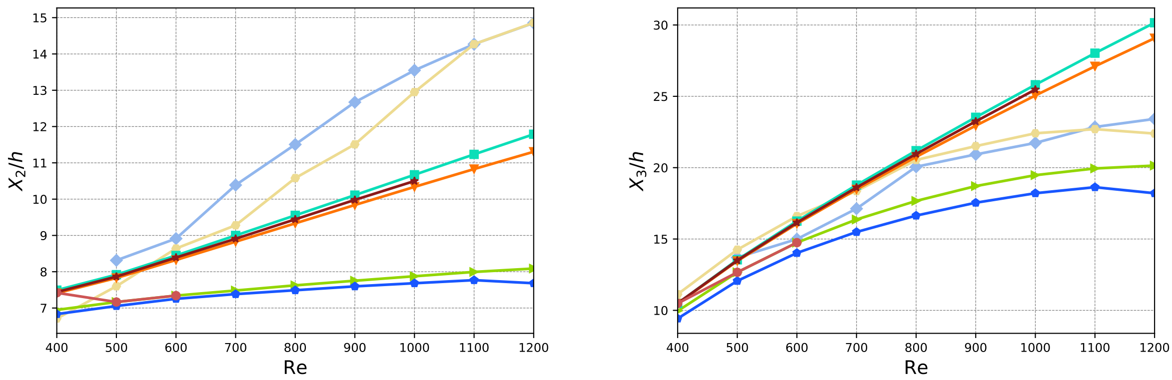


Figure 3. Reattachment length for bottom wall.

SOU, QUICK and UNIFAES present the closest results of the experimental data of Armaly *et al.* (1983) and Lee and Mateescu (1998) for the H-mesh. However QUICK stops converging from Reynolds 1000 and the SOU overcomes the UNIFAES presenting lengths slightly closer to the experimental one in the stable region of the laminar flow.

The lower reattachment length X_1/h increases in size with the elevation of the Reynolds number. The numerical predictions obtained by the SOU, QUICK and UNIFAES schemes satisfactorily agree with the experimental results of Lee and Mateescu (1998) up to $Re = 700$. Ferziger and Peric (2002) attribute these discrepancies between experimental and numerical data to modeling errors, where according to authors in turbulent or transient flows modeling errors become more expressive generating qualitatively and quantitatively wrong results. This hypothesis is consistent for the BFS flow given the present three-dimensional effects, in moderate to high Reynolds numbers.

Figure 4(a) displays the characteristic length X_2/h of the secondary recirculation region, the same observations made for the Fig. 3 are valid for the characteristic length X_2/h. Now for the characteristic length X_3/h, shown in Fig. 4(b), SOU, QUICK and UNIFAES schemes provide longer lengths than the experimental ones. FOU and EXP schemes provide for smaller lengths than the experimental data of Armaly *et al.* (1983) and Lee and Mateescu (1998).



(a) X_2/h (b) X_3/h

Figure 4. Reattachment lengths for the upper wall.

It should be noted that all analyzes were made based on the results of the H-mesh, which is considered the most accurate of the present investigation. In addition, some schemes failed in intermediate meshes for high Reynolds, making a more complete analysis impossible in intermediate meshes.

QUICK and CDS were very unstable, converging only to the most refined meshes. The other FOU, SOU, EXP and UNIFAES schemes presented instability problems only in coarse meshes A and B, but this is something already expected for high Reynolds numbers.

Table 5 presents another way of validating the results obtained by comparing the results of the present work with the numerical results of previous investigations. It can be observed that the results obtained are very close to the data previously published in the literature. However, all published numerical results, including the current one, underestimate the experimental lengths of the recirculation zones. In addition, the authors who consider the input length in the channel geometry have further underestimated in relation to those that do not consider, as can be seen from the results presented in table 5, corroborating with the findings made by Cruchaga (1998) and Santos *et al.* (2010).

Table 5. Validation of results for $Re = 800$ with previous investigations.

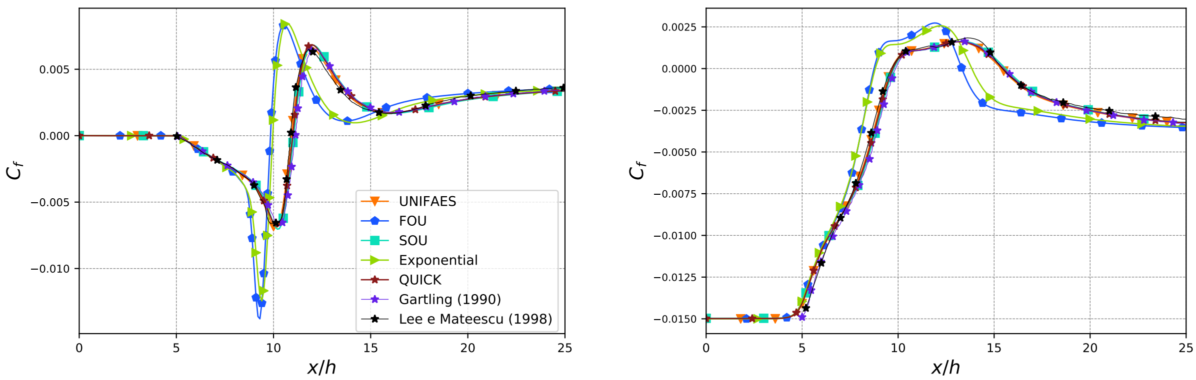
Author	**Scheme**	**Mesh**	**Refinement**	**Input channel**	X_1/h	X_2/h	X_3/h
Lee and Mateescu (1998)	Experimental	-	-	-	12.97	10.58	20.54
Barton (1995)	SOU	staggered	200x100	no	12.09	9.54	22.21
	SOUD			no	12.17	9.61	22.07
	QUICKER			no	12.20	9.64	20.01
Barton (1997)	QUICK	staggered	240x100	no	12.03	9.64	20.96
				yes	11.51	9.14	20.66
Chiang and Sheu (1999)	QUICK	staggered	100x85	no	12.49	9.78	20.68
Abu-Nada *et al.* (2007)	SOU	staggered	250x125	no	12.00	9.62	20.30
Santos *et al.* (2010)	UNIFAES	semi-staggered	240x120	no	12.18	9.81	20.92
				yes	11.72	9.25	20.58
Present results	UNIFAES	semi-staggered	250x150	yes	11.83	9.33	20.75
	SOU			yes	12.08	9.55	21.19
	QUICK			yes	11.97	9.44	20.96

A possible cause for this underestimation of lengths, by all authors, is due to the consideration that the flow is two-dimensional in $Re = 800$, which Armaly *et al.* (1983) makes it very clear that it is not, thus neglecting the three-dimensional effects induced by the sidewalls. Thus, it is impossible to use the experimental results as a reference for the estimation of the numerical error in the two-dimensional case, because the experimental data correspond to an experiment with three-dimensional effects. A more appropriate error analysis for this problem can be made from the use of Richardson extrapolation in estimating the exact solutions. The use of this technique in the estimation of the numerical error generated by the schemes in this problem will be approached in the second part of this investigation in the companion paper of Rodrigues *et al.* (2019).

Finally, a final analysis in the table 5, it is possible to observe the success of the use of the semi-staggered mesh in predicting the recirculation lengths, which were close to when not identical to those obtained in studies in the literature with them schemes that were used in the present investigation with the staggered mesh.

Another validation parameter used in the literature is the coefficient of friction along the channel walls. The coefficient of friction is defined by $C_f = \tau_w / \frac{1}{2}\rho U_{med}^2$ where τ_w is the shear stress on the wall. Thus, from the fields of converged velocities it was possible to calculate the coefficients of friction along the channel walls for the discretizing schemes used, allowing the comparison with the results obtained previously in the literature.

Figure 5 shows the coefficient of friction along the channel walls in $Re = 800$, for the six discretizing schemes used, as well as the results obtained by Gartling (1990) and Lee and Mateescu (1998).



(a) Lower wall. (b) Upper wall.

Figure 5. Coefficient of friction on the walls of the BFS for $Re = 800$.

The results obtained by the SOU, QUICK and UNIFAES schemes show good agreement with the numerical results obtained by Gartling (1990) and Lee and Mateescu (1998). FOU and EXP show the most discrepant results due to the characteristic numerical excess of the same. It should be noted that Gartling (1990) used the finite element method and Lee and Mateescu (1998) used the artificial compressibility method.

4. CONCLUSIONS

For low Reynolds numbers all the discretization schemes applied in medium or refined meshes produce very similar results between themselves. In the case of flows with high Reynolds numbers, the EXP scheme and FOU fail to provide physically realistic results. The CDS scheme converges only for refined meshes up to $Re = 600$, after which the scheme does not converge for any other case. QUICK also fails in almost all intermediate meshes, obtaining results only in the most refined mesh up to $Re = 1000$.

The performance of the SOU and UNIFAES schemes is very similar, these two converge to any number of Reynolds and present better results than the other schemes. The closest results of the experimental data were obtained in intermediate meshes and not in refined meshes, justifying the optimum performance of the SOU in predicting the recirculation lengths due to its slower convergence rate. The diagonal dominance of the system of equations is impaired due to the input length of the channel geometry, which is a possible cause for instability presented by the FOU, CDS and QUICK schemes.

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6. REFERENCES

Abu-Nada, E., Al-Sarkhi, A., Akash, B. and Al-Hinti, I., 2007. “Heat transfer and fluid flow characteristics of separated flows encountered in a backward-facing step under the effect of suction and blowing”. *Journal of Heat Transfer*, Vol. 129, No. 11, p. 1517.

Armaly, B.F., Durst, F., Pereira, J.C.F. and Schonung, B., 1983. “Experimental and theoretical investigation of backward-facing step flow”. *Journal of Fluid Mechanics*, Vol. 127, No. 1, p. 473.

Barton, I.E., 1995. “A numerical study of flow over a confined backward-facing step”. *International Journal for Numerical Methods in Fluids*, Vol. 21, No. 8, pp. 653–665.

Barton, I.E., 1997. “The entrance effect of laminar flow over a backward-facing step geometry”. *International Journal for Numerical Methos in Fluids*, Vol. 25, No. 25, pp. 633–644.

Chiang, T. and Sheu, T.W., 1999. “Time evolution of laminar flow over a three-dimensional backward-facing step”. *International Journal for Numerical Methods in Fluids*, , No. 31, pp. 721–745.

Courant, R., Isaacson, E. and Rees, M., 1952. "On the solution of nonlinear hyperbolic differential equations by finite differences". *Communications on Pure and Applied Mathematics*, Vol. 5, No. 3, pp. 243–255.

Cruchaga, M.A., 1998. "A study of the backward-facing step problem using a generalized streamline formulation". *International Journal for Numerical Methods in Biomedical Engineering*.

Erturk, E., 2008. "Numerical solutions of 2-d steady incompressible flow over a backward-facing step, part i: High reynolds number solutions". *Computers & Fluids*, Vol. 37, No. 6, pp. 633–655.

Ferziger, J.H. and Peric, M., 2002. *Computational Methods for Fluid Dynamics*. Springer-Verlag Berlin Heidelberg, 3rd edition.

Figueiredo, J.R., 1997. "A unified finite-volume finite-differencing exponential-type scheme for convective-diffusive fluid transport equations". *Journal of the Brazilian Society of Mechanical Sciences*, Vol. V 19, pp. pp. 371–391.

Gartling, D.K., 1990. "A test problem for outflow boundary conditions—flow over a backward-facing step". *International Journal for Numerical Methods in Fluids*, Vol. 11, No. 7, pp. 953–967.

Harlow, F.H. and Welch, J.E., 1965. "Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface". *Physics of Fluids*, Vol. 8, No. 12, p. 2182.

Kuznetsov, B., 1968. "Numerical methods for solving some problems of fluid flow". *Fluid Dynamics Transactions*, , No. 4, pp. 85–89.

Lee, T. and Mateescu, D., 1998. "Experimental and numerical investigation of 2-d backward-facing step flow". *Journal of Fluids and Structures*, Vol. 12, No. 6, pp. 703–716.

Leonard, B., 1979. "A stable and accurate convective modelling procedure based on quadratic upstream interpolation". *Computer Methods in Applied Mechanics and Engineering*, Vol. 19, No. 1, pp. 59–98.

Nascimento, A.H.G., Rodrigues, G.S. and Figueiredo, J.R., 2018. "Performance of finite volume discretization schemes for the convective-diffusive linear transport equation. part i: Low eigenvalue-peclet ratios". *17th Brazilian Congress of Thermal Sciences and Engineering*.

Nascimento, A.H.G., Rodrigues, G.S. and Figueiredo, J.R., 2019. "Incompressible flows bifurcation phenomena in symmetric channels". *25th International Congress of Mechanical Engineering*.

Patel, M.K. and Markatos, N.C., 1986. "An evaluation of eight discretization schemes for two-dimensional convection-diffusion equations". *International Journal for Numerical Methods in Fluids*, Vol. 6, No. 3, pp. 129–153.

Peyret, R. and Taylor, T.D., 1983. *Computational Methods for Fluid Flow*. Springer series in computational physics. Springer-Verlag, 2nd edition.

Pollard, A. and Siu, A.L., 1982. "The calculation of some laminar flows using various discretisation schemes". *Computer Methods in Applied Mechanics and Engineering*, Vol. 35, No. 3, pp. 293–313.

Raithby, G. and Torrance, K., 1974. "Upstream-weighted differencing schemes and their application to elliptic problems involving fluid flow". *Computers & Fluids*, Vol. 2, No. 2, pp. 191–206.

Rodrigues, G.S., Nascimento, A.H.G. and Figueiredo, J.R., 2018. "Performance of finite volume discretization schemes for the convective-difusive linear transport equation. part ii: High eigenvalue-peclet ratios". *17th Brazilian Congress of Thermal Sciences and Engineering*.

Rodrigues, G.S., Nascimento, A.H.G. and Figueiredo, J.R., 2019. "Performance of finite volume discretization schemes in the backward-facing step problem - part ii: Numerical error of schemes". *25th International Congress of Mechanical Engineering*.

Santos, R. and Figueiredo, J., 2011. "Numerical simulation study in a three-dimensional backward-facing step flow". *21st International Congress of Mechanical Engineering*.

Santos, R.G., Oliveira, K.P.M. and Figueiredo, J.R., 2010. "Influence study of the entrance channel in a two-dimensional backward-facing step flow". *Mecánica Computacional*, Vol. XXIX, No. 33.

Sharif, M. and Busnaina, A.A., 1988. "Assessment of finite difference approximations for the advection terms in the simulation of practical flow problems". *Journal of Computational Physics*, Vol. 74, No. 1, pp. 143–176.

Shyy, W., Tong, S.S. and Correa, S.M., 1985. "Numerical recirculating flow calculation using a body-fitted coordinate system". *Numerical Heat Transfer*, Vol. 8, No. 1, pp. 99–113.

Shyy, W., 1985. "A study of finite difference approximations to steady-state, convection-dominated flow problems". *Journal of Computational Physics*, Vol. 57, No. 3, pp. 415–438.

Spalding, D.B., 1972. "A novel finite difference formulation for differential expressions involving both first and second derivatives". *International Journal for Numerical Methods in Engineering*, Vol. 4, No. 4, pp. 551–559.

Warming, R.F. and Beam, R.M., 1976. "Upwind second-order difference schemes and applications in aerodynamic flows". *AIAA Journal*, Vol. 14, No. 9, pp. 1241–1249.

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