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INFLUENCE OF RESIDUAL STRESSES IN FATIGUE LIFE OF PARTIALLY YIELDED MECHANICAL PARTS

Danielle Lopes Zaidan
Paulo Pedro Kenedi
Departamento de Engenharia Mecânica – CEFET/RJ - Av. Maracanã, 229 – RJ – Brazil.
Programa de Pós-graduação em Engenharia Mecânica e Tecnologia de Materiais - PPEMM - CEFET/RJ; Av. Maracanã, 229 – RJ – Brazil.
danielle.zaidan@gmail.com, paulo.kenedi@cefet-rj.br.

Ana Beatriz Werneck Gonçalves Mano
Departamento de Engenharia Mecânica – CEFET/RJ - Av. Maracanã, 229 – RJ – Brazil.
abwgmano@gmail.com.

Abstract. *Residual stresses usually play an important role in partially yielded mechanical parts. In fact, its deleterious effects need to be considered, in machine design, to prevent unexpected mechanical failures. In this paper, it is analyzed the effect of residual stresses in partially yielded beams, with rectangular cross section, submitted to fatigue loading. To accomplish this objective an analytical model is proposed, for estimating both residual stress distribution in partially yielded beams and its respective fatigue life.*

Keywords: *residual stresses, fatigue, partially yielded parts*

1. INTRODUCTION

Residual stress is a wide matter, which can be accessed by different approaches. For instance, Li *et al*. (2018) studied the effect of compressive residual stresses in crack growth performance, through the utilization of Mechanical Fracture concepts, as crack closure. Lopez-Jauregi *et al*. (2015) analyzed the effect of residual stresses in fatigue response of multipass welded joints, estimating the residual stress distribution through the utilization of numerical methods.

Although analytical approaches can be used to address residual stresses, they usually have some limitations as: geometry (only simple geometries have been used) and type of loading (it is mostly used for a single type of loading as, for instance, bending moments). Nevertheless, the analytical approach has an important advantage of estimating the residual stress cross section distribution, not only superficial stress values, as it is the case for well-established experimental techniques, as can be seen, for instance, in Schajer (2013). Castro and Meggiolaro (2009) and Jirásek (2002) are good references for analytical models of residual stress; as well an interesting article of Stok (2008). Also, Castro *et al*. (2019), Castro (2018), Vargas (2014) and Lopes (2013) have examples of application of the analytical approach model to estimate the residual stress distribution of partially yielded static structures of rectangular cross sections. Yet, Riagusoff *et al*. (2010) used a hollow circular cross section to apply their analytical formulation.

In this work, it is proposed an analytical model to describe the effect of residual stresses in fatigue life of partially yielded structures. To achieve this objective it is proposed submit two specimens to a two sequential loading phases: Phase 1 it is characterized by the application of monotonic pure bending moment through the utilization of a four-point bending apparatus, sufficient to partially yield its middle region cross sections (between internal rollers); and Phase 2 it is characterized by the application of constant amplitude, time variable, bending moment through the utilization of a three-point bending apparatus. The phase 1, produces the same residual stress distribution for both specimens at a middle cross section (between internal rollers), with tensile residual stress at intrados and compressive residual stress at extrados. At phase 2 one specimen is positioned with negative curvature and the other specimen is positioned with positive curvature, both on a three-point bending apparatus.

The analytical model shows that the specimen stress with negative curvature, will be submitted to a more severe loading, at the middle section, at a point of bottom surface, because the additive effect between the tensile residual stress (of phase 1) and the tensile variable loading (of phase 2). Consistently, the specimen with positive curvature, will be submitted to a less severe loading, at the middle section, at a point of bottom surface, because the subtractive effect between compressive residual stress (of phase 1) and the tensile variable loading (of phase 2). Resulting in a longer fatigue life of specimen 1, with positive curvature, in comparison of specimen 2, with negative curvature, leading to the conclusion that the cross section residual stress distribution is the main responsible for this behaviour.

2. ANALYTICAL MODEL

The analytical model, based in mechanics of solids, deals with two phases. Phase 1 estimates the cross section residual stress distribution and phase 2 estimates the effect of the residual stress distribution in specimens fatigue life. The mechanics of solids theory, as in Crandall *et al*. (1988), was used in the proposed analytical model. The material was supposed to behave as elastic perfectly plastic:

$$\sigma_x = \begin{cases} -S_y & -\varepsilon_R \le \varepsilon_x < -\varepsilon_y \\ E\varepsilon_x & -\varepsilon_y \le \varepsilon_x \le \varepsilon_y \\ +S_y & \varepsilon_R \ge \varepsilon_x > \varepsilon_y \end{cases} \tag{1}$$

Where σ_x is the normal stress in x direction (shown in Fig.1.b), E is the modulus of elasticity, S_y is the yield strength, ε_x is the strain in x direction, ε_y is the yield strain and ε_R is the rupture strain. The beam has a rectangular cross section, as shown in Fig.1.a (partially yielded). It was used two specimens through phases 1 and 2.

Fig.1.b shows, schematically, a specimen positioned on a four-point bending fixture to be loaded with P_1 monotonic load, to induce residual stress distributions at its cross sections, during phase 1. Fig.1.c and Fig.1.d shows, schematically, the specimens with, respectively, positive and negative curvatures being positioned on a three-point apparatus, during phase 2. To generate P_2 of phase 2 it is necessary the utilization of a servo-mechanical testing machine, with constant amplitude, time variable loading application, as shown in Fig.5.b.

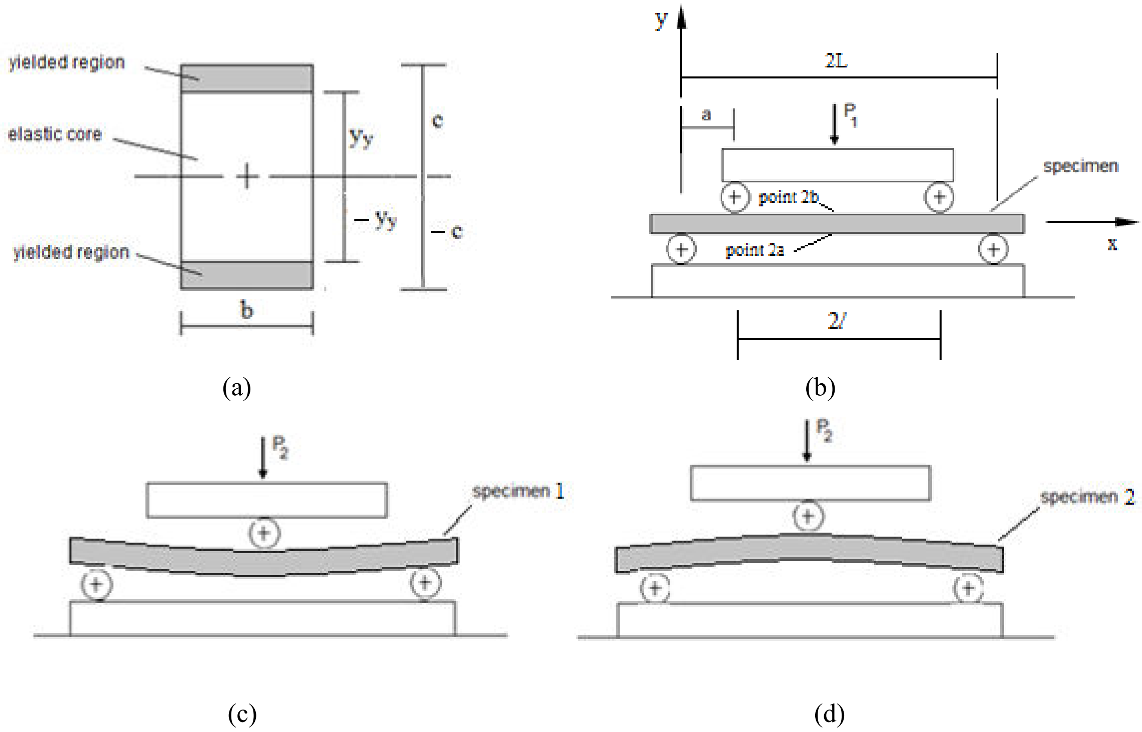


Figure 1. (a) Specimen partially yielded rectangular cross section. Schematic drawing of two testing apparatus (lateral view): (b) four-point bending fixture with a specimen, three-point bending (c) for specimen 1 and (d) for specimen 2.

Where y_y is the vertical distance between the elastoplastic border and the neutral line; b and c are, respectively, the width and the half height of the specimen. $2L$ is the length of specimens between external rollers and $2l$ is the length of specimens between internal rollers.

2.1 Residual stress model – phase 1

To a specimen, of rectangular cross section, loaded in pure bending moment (M), the beginning of yield moment M_y and the fully plastic moment M_p can be estimated, as in Crandall *et al*. (1988):

$$M_y = \frac{2}{3}bc^2S_y \qquad M_p = bc^2S_y \qquad \alpha_{max} = \frac{M_p}{M_y} \qquad \text{for} \quad M_y \le M \le M_P \tag{2}$$

Where α_{max} is a ratio between equation 2.b and 2.a.

The bending moment at the medial section can be accessed by equation 3.a, for four-point fixture and by equation 4.a, for three-point fixture:

$$M_{four} = a\frac{P}{2} \qquad P_{y\_four} = \left(\frac{2}{a}\right)M_y \qquad P_{p\_four} = \left(\frac{2}{a}\right)M_p \qquad \text{for} \qquad P_{y\_four} \le P \le P_{P\_four} \tag{3}$$

$$M_{three} = L\frac{P}{2} \qquad P_{y\_three} = \left(\frac{2}{L}\right)M_y \qquad P_{p\_three} = \left(\frac{2}{L}\right)M_p \qquad \text{for} \qquad P_{y\_three} \le P \le P_{P\_three} \tag{4}$$

Where, *a* is the distance between inner and outer rollers, *P* is the applied load (P_1 for phase 1 and P_2 for phase 2). P_y and P_P are, respectively, the beginning of cross section yielding load and the fully cross section yielding loading load, with additional index *three* for three-point fixture and additional index *four* for four-point fixture. The P_1 value is calculated as:

$$P_1 = \alpha P_{y\_four} \qquad \text{for} \qquad 1 \le \alpha \le \alpha_{max} \tag{5}$$

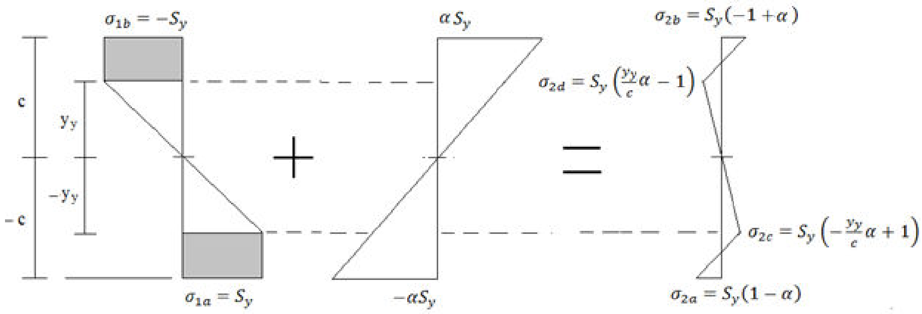
For rectangular cross the maximum applied load in phase 1 is $P_{1\_max} = \alpha_{max}P_y$, where $\alpha_{max} = 1.5$. The elasto-plastic border y_y can be obtained, as in Crandall *et al*. (1988):

$$\frac{y_y}{c} = \sqrt{3\left(1 - \frac{M}{M_p}\right)} \tag{6}$$

As well as, the specimen initial curvature radius during loading ρ_o and the final curvature radius after load removal ρ_f at the end of phase 1:

$$\rho_o = \frac{y_y}{\varepsilon_y} \qquad \rho_f = \frac{1}{\left(\frac{c}{y_y} - \alpha\right)}\frac{c}{\varepsilon_y} \qquad \text{for} \qquad 1 \le \alpha \le \alpha_{max} \tag{7}$$

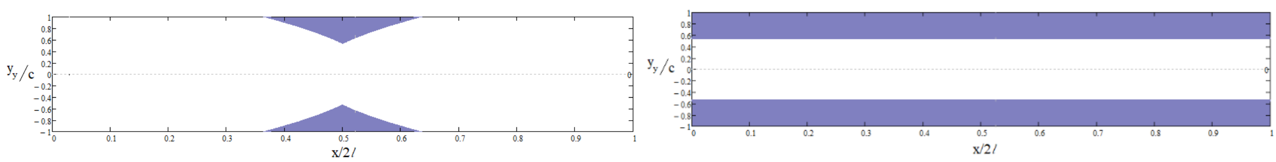
Fig.2 shows the lateral view of the residual stress generation sequence of phase 1. Fig.2.a show the cross-section stress distribution at P_1 applied load (P_{1_ap}); Fig. 2.b shows the spring back effect of the removal the P_{1_ap} and Fig. 2.c is the cross section residual stress distribution resultant.



(a) (b) (c)

Figure 2. Residual stresses generation sequence (lateral view): (b) loading, (c) unloading (spring-back) and (d) resultant residual stress distribution.

The selection of a four-point bending fixture to impose the residual stress distribution in phase 1 rather than using the same three-point bending fixture for phases 1 and 2, is justified in Fig. 3, where the lateral view of the yielded region (marked in blue) at 2*l* length of a specimen, generated with the utilization of equation (6). It was used $P_{1\_ap} = 1.35P_y$ for both fixtures (three-point and four-point). Note that the yielded region resultant of application of P_{1_ap} with a three-point bending fixture generates a concentrated pattern in the middle of the specimen, making this choice force positioning dependent, whereas the application of the P_{1_ap} on a four-point bending fixture produces a continuous pattern, that is, therefore, not force positioning dependent.



(a) (b)

Figure 3. Lateral view of a specimen, of yielded regions (marked in blue), at 2*l* length. P_{1_max} load, with $y_y/c = 0.548$ (see equation 6) for: (a) a three-point apparatus and (b) a four-point apparatus.

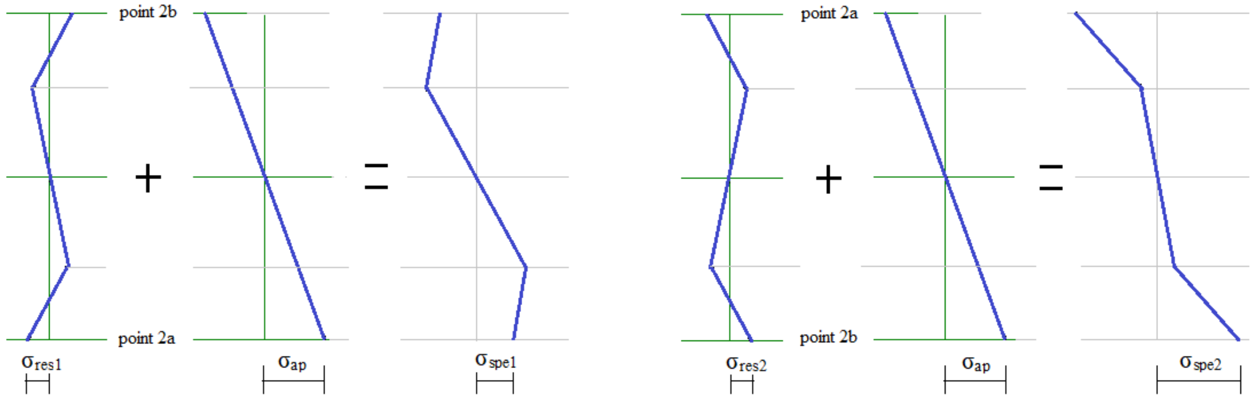
2.3 Fatigue life model – phase 2

After the two specimens have passed through phase 1, and they have been partially yielded, generating residual stress distributions (after spring back), generates specimens with a curved format with same residual stress distributions. For the phase 2, the specimens are used in distinct ways:

Specimen 1, of Fig.1.c, is positioned on the three-point bending apparatus has a positive curvature. So, the point 2a at the central bottom point of the specimen, the compressive residual stress generated in phase 1 will be algebraically added to the tensile stresses of phase 2 (σ_{ap}), diminishing the specimen bottom resultant stress.

Specimen 2, of Fig.1.d, is positioned on the three-point bending apparatus has a negative curvature. In this case the point 2b at the central bottom point of the specimen, the tensile residual stress generated in phase 1 will be algebraically added to the tensile stresses of phase 2 (σ_{ap}), augmenting the specimen bottom resultant stress.

Figure 4 show, schematically, for both specimens, the lateral view of cross section stress distribution for phases 1 and 2. Note the differences, for instance, at up and bottom surfaces, of the resultant stresses (Fig. 4.c and 4.f)



(a) (b) (c) (d) (e) (f)

Figure 4. Cross section stress distribution (lateral view). Specimen 1: (a) phase 1 residual stresses, (b) phase 2 stresses (c) phase 2 resultant stresses. Specimen 2: (d) phase 1 residual stresses, (e) phase 2 stresses (f) phase 2 resultant stresses.

So, as shown in Fig. 2.c, the residual stress at a point of the specimen bottom (point 2a for specimen 1 and point 2b for specimen 2), of a cross section inside the inner rollers, can be stated as:

$$\sigma_{res1} = \sigma_{2a} = S_y(1-\alpha) \text{ for specimen 1} \qquad \sigma_{res2} = \sigma_{2b} = -S_y(1-\alpha) \text{ for specimen 2} \tag{8}$$

The phase 2 stresses relative to applied load P_2 at a point of the specimen bottom can be estimated as:

$$\sigma_{ap} = -\frac{M_{three}(-c)}{I} \tag{9}$$

Where I is the moment of inertia of specimen cross section. The resultant stress at a bottom point of the specimen can be estimated as:

$$\sigma_{spe1} = \sigma_{res1} + \sigma_{ap} \quad \text{for specimen 1} \qquad \sigma_{spe2} = \sigma_{res2} + \sigma_{ap} \quad \text{for specimen 2} \tag{10}$$

Note that although σ_{ap} has the same value for both specimens, $\sigma_{spe2} > \sigma_{spe1}$ because σ_{res2} has the same signal as σ_{ap} resulting in a higher value of stress if compared with σ_{res1} that has the opposite signals (see Figs. 4.c and 4.f).

The fatigue part of the model uses SN diagram, as shown in Fig. 5.a. The sinusoidal load, schematically shown in Fig. 5.b, oscillates between $P_2/P_{max} = 0.2$ and $P_2/P_{max} = 1$. The resultant stresses at a bottom point of the specimen 1 can be estimated as:

$$\sigma_{min1} = 0.2\sigma_{spe1} \qquad \sigma_{max1} = \sigma_{spe1} \qquad \sigma_{m1} = 0.6\sigma_{spe1} \qquad \sigma_{a1} = 0.4\sigma_{spe1} \tag{11}$$

The stresses at a bottom point of the specimen 2 can be estimated as:

$$\sigma_{min2} = 0.2\sigma_{spe2} \qquad \sigma_{max2} = \sigma_{spe2} \qquad \sigma_{m2} = 0.6\sigma_{spe2} \qquad \sigma_{a2} = 0.4\sigma_{spe2} \tag{12}$$

Where the indexes of stresses *min*, *max*, *m* and *a* signifies, respectively, minimum, maximum, mean and alternate.

To use SN diagram of Fig. 5.a, it is necessary to obtain an alternate equivalent stress (σ_{a_eq}):

$$\sigma_{a\_eq1} = \left(\frac{1}{1-(\sigma_{m1}/S_{ut})}\right)\sigma_{a1} \quad \text{for specimen 1} \qquad \sigma_{a\_eq2} = \left(\frac{1}{1-(\sigma_{m2}/S_{ut})}\right)\sigma_{a2} \quad \text{for specimen 2} \tag{13}$$

Note that as comment previously $\sigma_{spe2} > \sigma_{spe1}$, which induces $\sigma_{a\_eq2} > \sigma_{a\_eq1}$. In other words, the residual stress value, and in this case if it is in tension or in compression at the critical point, defines the fatigue performance of partially yielded mechanical parts.

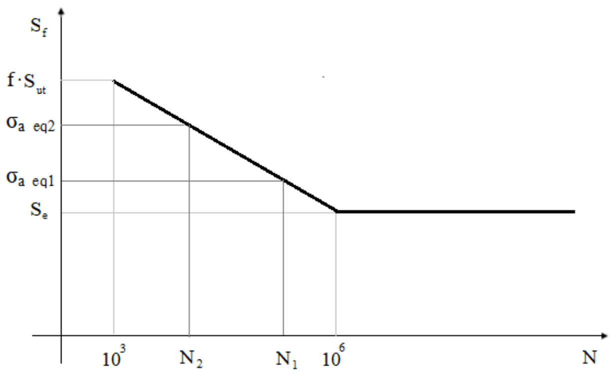
To estimate the specimen finite life *N*, Castro and Meggiolaro (2009):

$$S_f = aN^b \qquad a = \frac{(fS_{ut})^2}{S_e} \qquad b = -\frac{1}{3}log\frac{(fS_{ut})}{S_e} \tag{14}$$

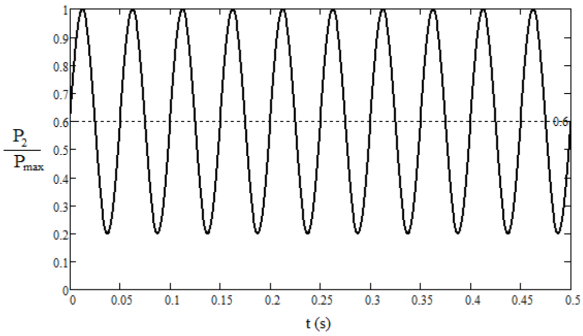
Where S_f is the fatigue strength and S_e is the fatigue limit, which is estimated for a cold rolled material, a non-rotative specimen with rectangular cross- section, submitted to bending load:

$$S_e = 4.51(S_{ut})^{-0.265} \cdot \left(\frac{0.808\sqrt{bh}}{7.62}\right)^{-0.107} \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 0.5 \cdot S_{ut} \tag{15}$$

The SN diagram, show in Fig. 5.a, presents a graphical representation of equations (13), (14) and (15):



(a)



(b)

Figure 5. (a) SN Diagram and (b) phase 2 loading.

Fig.5.a show, schematically, the effect of residual stress in fatigue life of partially yielded specimens. In this case both specimens have finite life. Fig. 5.b shows a sinusoidal loading, with a 20 Hz frequency, of phase 2 loading. Note that in limit cases, the residual stress distribution can make such a great difference in the fatigue results that for partially yielded mechanical parts, submitted to the same alternate loading, some parts could have finite life while others infinite life.

3. CONCLUSIONS

The analytical model, based in mechanic of solids, was presented and divided in two phases. The phase 1 deals with quantification of residual stress distribution of specimens, with rectangular cross section, submitted to monotonic pure bending loading that exceed the yield resistance. The phase 2 imposes a variable bending load for two specimens of the same material and dimensions but different stress residual distributions. It was show that although both specimens were submitted to the same fatigue loading, they could have different fatigue life performance, in function of the critical point the residual stress (for instance, tension or compression), respectively augmenting or diminishing the resultant stress. The overall conclusion is that residual stresses do influence the fatigue life of partially mechanical parts. The experimental part of this research is, also, under implementation.

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