

## COB-2019-1998

# UNCERTAINTY QUANTIFICATION ON THE ADHESIVE LAYER FOR PASSIVE SHUNT CONTROL: CANTILEVER BEAM

**Luiz Felipe Ribas Motta**  
**Guilherme Silva Prado**  
**Venicio Silva Araujo**  
**Heinsten Frederich Leal dos Santos**

Universidade Federal de Mato Grosso.

luizf.r.motta@gmail.com

guilhermep Prado@ufmt.com

venicios\_silva@hotmail.com

heinsten.leal@gmail.com

**Abstract.** *This work presents a quantification on performance under uncertainties on the adhesive layer, for passive vibration control of a cantilever beam using piezoelectric materials, bonded with epoxy adhesive. The piezoelectric patch is connected to independent resonant shunt circuit to provide a tuned optimal shunt control. To optimize the configuration of the electronic parameters were used a neural network trained with back-propagation, tuned to define an optimal value for Resistance and Inductance to produce damping for the first natural mode of the beam. In the present work, special attention is given to the thickness and Young's Modulus for an adhesive layer with a parametric analysis is performed to evaluate the influence in shunt control. For uncertainties quantification for the adhesive layer, was used a distribution Gaussian p.d.f with 95% of the interval of confidence.*

**Keywords:** *Piezoelectric, Smart Structures, Uncertainties, Shunt Control, Neural Network...*

## 1. INTRODUCTION

In several fields of engineering, analyzes are carried out in search of a project that presents a high performance and, at the same time, reducing mass. Thus, the use of piezoelectric for the energy harvesting (through the direct effect), and vibration attenuation has a wide application as sensors and actuators, still highlighting the high energy efficient conversion between displacement and electric charge. That is why surface-mounted piezoelectric sensors and actuators are also known as extension or extension-bending sensors and actuators and were widely used an active (Sunar and Rao, 1999), passive (Moheimani, 2003) and hybrid active-passive (Tang *et al.*, 2000; Trindade and Benjeddou, 2002; Santos and Trindade, 2011) control applications.

The performance of piezoelectric patches for these types of applications is very much dependent on the adequate tuning between resonant, circuit and operation frequencies and on the effective electromechanical coupling between patches and host structure. Therefore, variability and/or uncertainties on material properties, boundary conditions, and bonding effectiveness may have a major effect on reducing the expected or predicted performance of such devices (Santos and Trindade, 2012; Godoy and Trindade, 2012)

In this work was present an analysis of the effect of uncertainties in stiffness and thickness properties for a configuration of bonding layer with a focus on the performance of piezoelectric sensors and actuators. This patch piezoelectric was used in application to passive shunted control damping.

## 2. FINITE ELEMENT MODEL OF PIEZOELECTRIC BEAMS

The structure is a fixed beam of the aluminum of dimension 220 mm in length, width of 25mm and thickness of 3 mm, the piezoelectric has a variable length, width of 25mm and thickness of 0.5 mm, as we can see in the Figure 1. The extension piezoceramics are made of PZT-5H material whose properties are:  $\bar{C}_{11}^D = 97.767$  GPa,  $\rho = 7500$  Kg.m<sup>3</sup>, piezoelectric coupling constants  $\bar{h}_{31} = 1.3520 \times 10^9$  N.C<sup>-1</sup>, and dielectric constants  $\bar{\beta}_{33}^E = 57.830 \times 10^6$  m.F<sup>-1</sup>. For the beam has:  $\rho = 2700$  Kg.m<sup>-3</sup> and  $E = 70 \times 10^9$  MPa. (Santos, 2008).

The optimization had the focus in resistance and inductance values of the circuit, wherever the resistance (R) is responsible in damping by means of Joule effect and the inductance (L) is responsible to control resonant frequency of

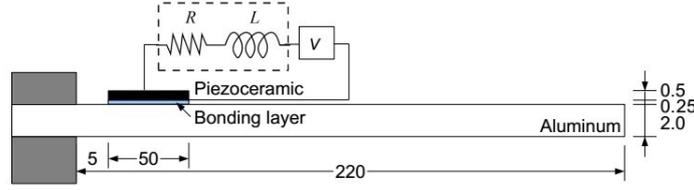


Figure 1: Representation of cantilever beam with bonded extension piezoceramic patch.

the structure, this form had use a shunt circuit, with can see in Figure 1.

## 2.1 Equation of motion for beam structure

For the study of the stiffness of the layer was adopted the classic sandwich model (piezoceramic - bonding layer - host). This allows the shear effect that will be responsible for one of the displacement losses of the layers. Thus, Bernoulli-Euler theory is retained for the outer layers, while the bonding layer (resin) is assumed to behave as a Timoshenko beam, as shown in figure 2.

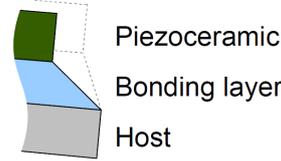


Figure 2: Configuration of the classic sandwich structure

With the applied theory of the Bernoulli and Timoshenko can be developed the equation of movement for the structure, this form the structure-patches-circuits coupled equations of motion can be written as

$$\begin{bmatrix} M & 0 \\ 0 & M_q \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{D}_p \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & C_q \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{D}_p \end{Bmatrix} + \begin{bmatrix} K_m & -\bar{K}_{me} \\ -\bar{K}_{me}^t & \bar{K}_e \end{bmatrix} \begin{Bmatrix} u \\ D_p \end{Bmatrix} = \begin{Bmatrix} F \\ F_q \end{Bmatrix}, \quad (1)$$

where  $\underline{M}_q$  is the inertial vector due to the presence of resistance and inductance,  $\underline{u}$  and  $\underline{D}_p$  are the vectors global mechanical displacement and electric displacement dofs.  $\underline{M}$ ,  $\underline{K}_m$ ,  $\underline{K}_{me}$ ,  $\underline{K}$  are the mass and mechanical, piezoelectric and dielectric stiffness matrices and  $\underline{F}$  is the mechanical excitation force vector.  $\underline{C}_q$  and  $\underline{F}_q$  are the matrix of the damping and the vector of force dues the presence of resistance and inductance, but how in this work is studied only the output mechanical the value of the vector of electric voltage is equal zero.

## 3. PASSIVE SHUNT CONTROL

Once even a minimal vibration is capable cause large destruction in mechanical systems, techniques to suppress or establish control over the systems vibration is widely researched.(Araujo *et al.*, 2019).

Inside the concept of a additional system designed for damping, stands out the usage of piezoelectrics as secondary system for vibration absorption as were made in experiments on optimal vibration control of a flexible beam containing piezoelectric sensors and actuators (Abreu *et al.*, 2003), where instead of transform the mechanical absorbed energy into the range of motion, the structure transform mechanical energy into electrical and disperses it.

For the analysis of harmonic vibration of this work, the proposed model (Santos, 2008) is used to evaluate the mobility (velocity/force) frequency response function of the base structure. The resistive (R) or resonant (RL) shunt circuit affects both the passive control performance. In this way, it became necessary to use the circuit that will dissipate the energy or to storage for later use. How this work analyze a purely mechanical excitation, such as  $\underline{F}_q = 0$  and  $\underline{F} = bf e^{j\omega t}$ , the amplitude of a displacement output  $y = c_p u$  can be written as  $y = G(\omega) f$ , where the FRF  $G(\omega)$  is

$$G(\omega) = c_p \{ (-\omega^2 M + K_m - \bar{K}_{me} (\omega^2 M_q + i\omega C_q + \bar{K}_e)^{-1} \bar{K}_{me}^t) \}^{-1} b \quad (2)$$

Analyzing the equation 2 it can be noted that the resistance and the inductance have the capacity to change the rigidity properties of the piezoelectric material, in this way it will be applied to the case types i) open-circuit when  $R_c$  tending to infinity and ii) short-circuit when  $Lc = Rc = 0$ . For the open circuit it has

$$G^{oc}(\omega) = c_p \{ -\omega^2 M + K_m \}^{-1} b \quad (3)$$

To the closed circuit

$$G^{sc}(\omega) = c_p \{ -\omega^2 M + [K_m - \bar{K}_{me} \bar{K}_e^{-1} \bar{K}_{me}^t] \}^{-1} b \quad (4)$$

You may note that no structural modification is observed in the open circuit box, whereas in the case of a short circuit, the rigidity of the piezoelectric patches is reduced.

### Vibration Control using piezoelectric actuators and state feedback

This way is necessary to rewrite the motion equations in the form of state space, containing the displacements and modal velocities of the piezoelectric patches and their derivatives of time.

$$\dot{z} = \hat{A}z + \hat{B}V_c + \hat{B}_f f, \quad y = \hat{C}_y z, \quad (5)$$

where

$$z = \begin{bmatrix} \alpha \\ q_p \\ \dot{\alpha} \\ \dot{q}_p \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -\Omega^2 & K_p & -\Lambda & 0 \\ L_c^{-1} K_p^t & -\Omega_e^2 & 0 & -\Lambda_e \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ L_c^{-1} \end{bmatrix}, \quad \hat{B}_f = \begin{bmatrix} 0 \\ 0 \\ b_\phi \\ 0 \end{bmatrix}, \quad \hat{C}_y = [c_\phi \quad 0 \quad 0 \quad 0]. \quad (6)$$

The modal displacements are such that  $u = \phi \alpha$  and, for mass normalized vibration modes,  $\Omega^2 = \phi^t K_m \phi$  and  $\Lambda = \phi^t C \phi$ .  $\Omega$  is a diagonal matrix which elements are the undamped natural frequencies of the structure with piezoelectric patches in open-circuit.  $\Omega_e^2 = L_c^{-1} \bar{K}_e$  and  $\Lambda_e = L_c^{-1} R_c$  are both diagonal matrices which elements stand, respectively, for the squared natural frequencies of the electric circuits and the ratio between the resistances and inductances. The electromechanical coupling stiffness matrix projected in the undamped modal basis is defined as  $K_p = \phi^t \bar{K}_{me}$ . Input  $b$  and output  $c_y$  distribution vectors are also defined, with modal projections  $b_\phi = \phi^t b$  and  $c_\phi = c_y \phi$ , and  $f$  is a vector of the amplitudes of each mechanical force applied to the structure (Santos and Trindade, 2011).

A linear state feedback for the applied voltages  $V_c$  is assumed such that  $V_c = -gz = -g_{dm}\alpha - g_{de}q_p - g_{vm}\dot{\alpha} - g_{ve}\dot{q}_p$ , where  $g$  is a matrix of control gains for each state variable. Therefore, the state space equation (5) becomes

$$\dot{z} = (\hat{A} - \hat{B}g)z + \hat{B}_f f, \quad y = \hat{C}_y z. \quad (7)$$

For a single-input mechanical excitation  $f$ , the closed-loop or controlled amplitude of a single displacement output  $y$  can be written such that  $\tilde{y} = G_h(\omega)\tilde{f}$ , where the FRF  $G_h(\omega)$  is

$$G_h(\omega) = \hat{C}_y (j\omega I - \hat{A} + \hat{B}g)^{-1} \hat{B}_f, \quad (8)$$

which can also be derived from the second order equations of motion projected into the undamped modal basis leading to

$$G_h(\omega) = c_\phi \left\{ -\omega^2 I + j\omega(\Lambda + K_p \underline{D}_{cc}^{-1} g_{vm}) + [\Omega^2 + K_p \underline{D}_{cc}^{-1} (g_{dm} - K_p^t)] \right\}^{-1} b_\phi, \quad (9)$$

where the closed-loop dynamic stiffness of the electric circuit  $\underline{D}_{cc}$  is

$$\underline{D}_{cc} = -\omega^2 L_c + j\omega(R_c + g_{ve}) + (\bar{K}_e + g_{de}). \quad (10)$$

In this work, the control gain  $g$  is calculated using the standard optimal LQR control theory applied to a single-input/single-output case, that is with only one active-passive patch-circuit pair for the control to minimize the vibration amplitude at one specific location of the structure, such that the following objective function is minimized

$$J = \frac{1}{2} \int_0^\infty (\dot{y}^2 + rV_c^2) \, t, \quad (11)$$

where  $\dot{y}$  is the velocity at one location of interest and  $V_c$  is the control voltage applied to the active-passive shunt circuit in all cases following an iterative routine proposed in (Trindade *et al.*, 1999).

## 4. ARTIFICIAL NEURAL NETWORK

To be possible to establish this kind of control, the parameters of the resonant circuit associated with the piezo layer has to be set up to convert the external energy properly. For that were used a neural network method.

Neural network consist in models of parallel algorithms that try to reproduce the principle of animal thinking, learning by experience, being very flexible for adaptive problems, once it is able to re-adapt any of your parameters to meet the needs of the systems.

Changing parameters of Resistance and Inductance for the resonant circuit we have different frequency responses for the beam. To this work, were give these different configurations, and their respective frequency responses, for the algorithms of a neural network. It try to seek connections between the received information and, once it get that, were simply asked for the configuration that provides the minor amplitude answer for the first natural mode of the beam.

This model was adopted for get the configuration of Resistance and Inductance for the system. The flowchart of neural network used follows in the Fig. 3

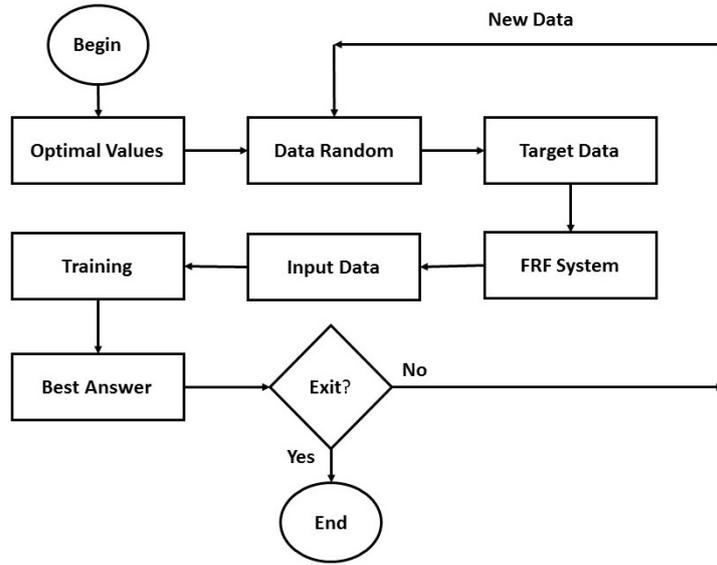


Figure 3: Flowchart for Neural Network.

## 5. UNCERTAINTY QUANTIFICATION

The Monte Carlo method was used to analyze the uncertainties performing 5000 simulations. The frequency response functions were used to evaluate the mean responses for 95% interval of confidence and 100% standard deviation.

The random variables chosen for the adhesive layer are its thickness and Young's modulus. In this sense the study was based on a Probability Density Function *p.d.f* followed by restrictions given by the physical nature of the chosen variables. Those variables are physically restricted to a positive interval. There is not a *p.d.f* known for such random variables, but the mean of the random variables are known and, based on the most expected value for a optimal scenario it is possible to choose non-arbitrarily a existing distribution. With interval for the variables in between  $]0, +\infty[$  and an ideal mean can be constructed a *p.d.f* through the Gaussian function located in the Eq. (12). The Gaussian *p.d.f* exists in the interval of  $]-\infty, +\infty[$  therefore to respect the limits of the random variables the distribution should be truncated in 5%.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]}, \& \quad x \in (-\infty, +\infty) \quad (12)$$

Similarly, based on the mean (or nominal) value estimated, that should be positive, another reasonable stochastic model can be constructed from a Gamma probability density function (*p.d.f*), present in Eq. (13).

$$p_E(E) = \mathbb{I}_{]0, +\infty[} \left( \frac{1}{\delta_E^2 \bar{E}} \right)^{\delta_E^{-2}} \frac{E^{\delta_E^{-2}-1}}{\Gamma(\delta_E^{-2})} \exp\left(-\frac{E}{\delta_E^2 \bar{E}}\right), \quad (13)$$

in which  $\delta_E = \sigma_E/\bar{E}$  is the relative dispersion of stochastic bonding layer Young's modulus  $E_b$  and  $\sigma_E$  the standard deviation. The Gamma function is defined as  $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ .

## 6. RESULTS

### 6.1 Gaussian Distribution

In Figure 5, can be observed the first mode from configuration presented in Figure 4 using a *p.d.f* Gaussian developed in Eq. 12, where (a) is made for thickness and (b) for Young's Modulus. For this configuration, Figure 5(a) shows the mean and 95% confidence interval for the frequency response of the shunted cantilever beam subjected to uncertainties of the thickness of the bonding layer. The expected value of damping for case Figure 5(a) gives 28 *Db* damping with the interval of confidence between 3.5 *Db* for minimal and 8.6 *Db* for maximum value to zero. In the Figure 5(b) shows the mean and 95% confidence interval for the frequency response of the shunted cantilever beam subjected to uncertainties of Young's Modulus of bonding layer revealing 27 *Db* damping with the interval of confidence between 2.8 *Db* for minimal and 19.5 *Db* for maximum value to the zero.

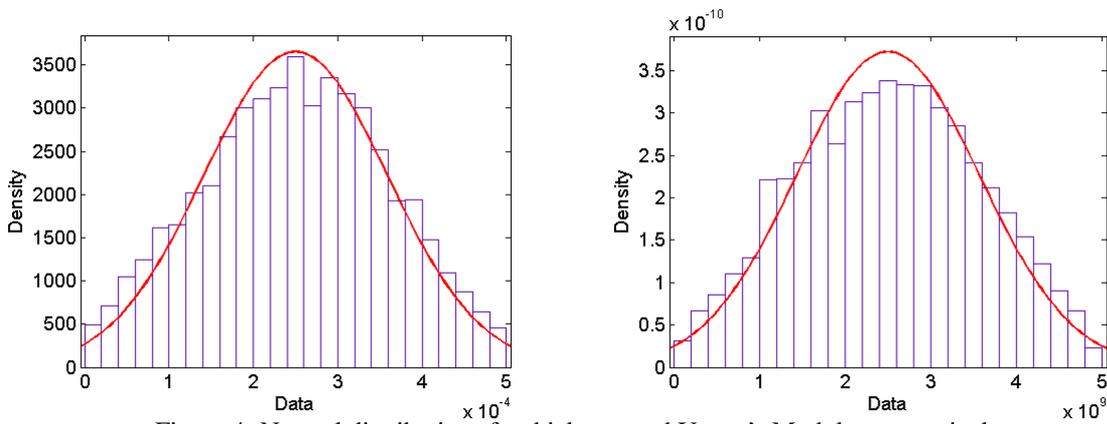


Figure 4: Normal distributions for thickness and Young's Modulus respectively.

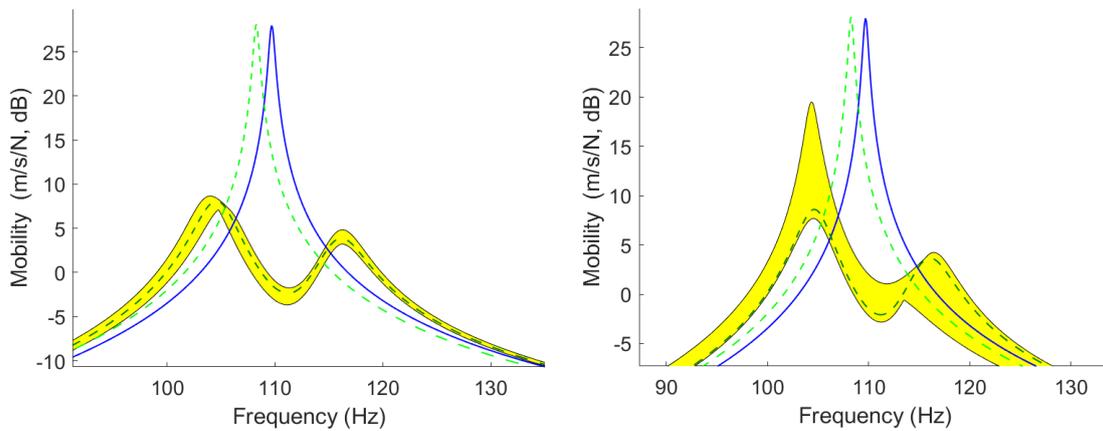


Figure 5: Range of confidence for the frequency response for thickness and Young's Modulus respectively.

## 6.2 Gamma Distribution

For comparing purpose the results were also generated respecting a *p.d.f* Gamma which can be viewed in Figure 6(a) for the bonding layer's thickness and Figure 6(b) for the bonding layer's Young's Modulus. The Gamma distribution is based on parameters of shape  $\alpha$  and scale  $\beta$ . For such configuration the results shown are in Figure 7(a) with the beam subjected to uncertainties of thickness of the bonding layer with 100% confidence interval for the frequency response of the shunted cantilever beam giving the damping of 27.5 *Db* with the interval of confidence between 3.5 *Db* for minimal and 8.6 *Db* for maximum value to zero. The Figure 7(b) shows 100% confidence interval for the frequency response of the shunted cantilever beam. The damping was 27.2 *Db* with the interval of confidence between 3.1 *Db* for minimal and 12.9 *Db* for maximum value to zero. Figures 7(a) and (b) also show the frequency response for the mean values of thickness and Young's Modulus of the cantilever beam.

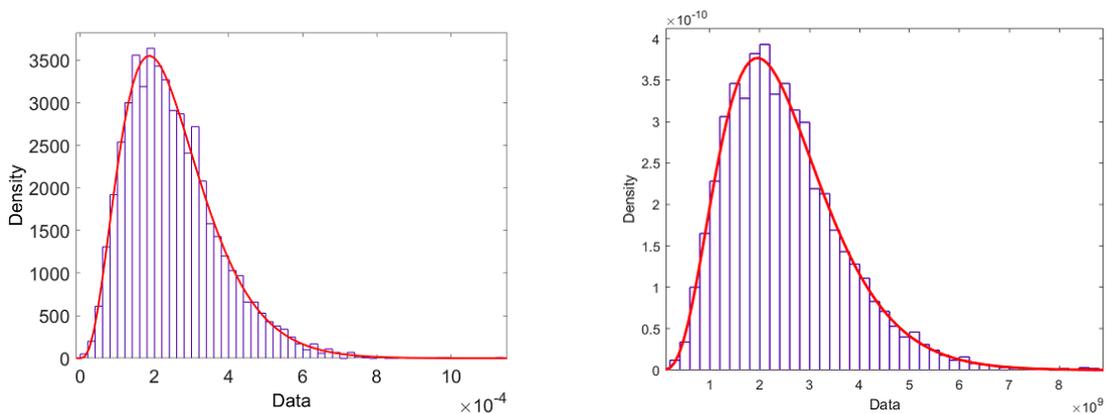


Figure 6: Gamma distributions for thickness and Young's Modulus respectively.

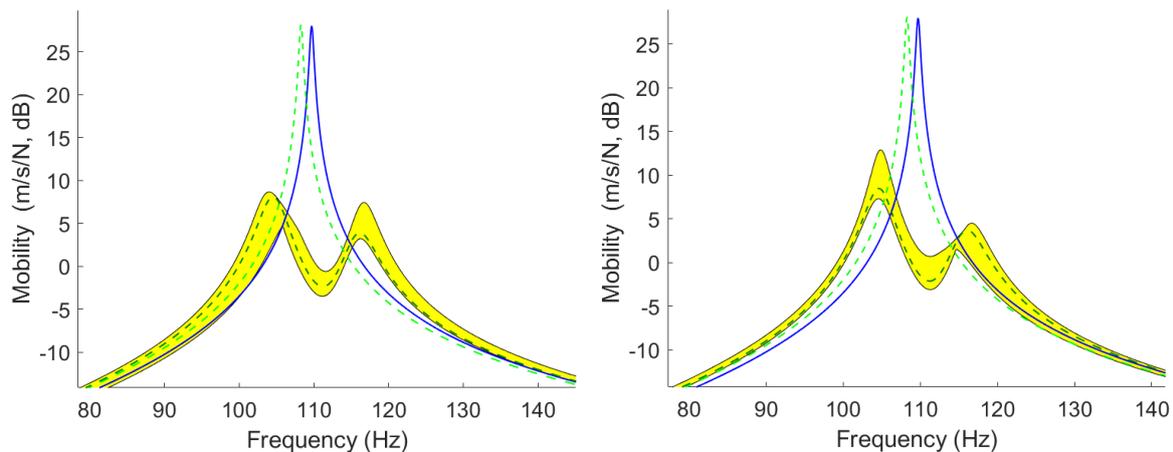


Figure 7: Range of confidence for the frequency response for thickness and Young's Modulus respectively.

## 7. CONCLUSION

Through the methodology used to quantify the uncertainties associated with the adhesive layer and its effects on the frequency response of a shunt circuit was possible to visualize graphically the interval of confidence. The intervals of confidence for the Young's Modulus of both distributions shown that the larger density for a smaller number of data found in a Gamma distribution provides a smaller interval of confidence. By contrast, the same influence does not appear significantly for the adhesive layer thickness.

It is of interest for future works to compare the results of a Neural Network trained for each case of a distribution that alters the same properties of the adhesive layer creating responses with individual values of resistance (R) and Inductance (L) for the circuit for every single structure.

## 8. REFERENCES

- Abreu, G.L.C.M., Ribeiro, J.F. and Steffen Jr, V., 2003. "Experiments on optimal vibration control of a flexible beam containing piezoelectric sensors and actuators". *Shock and Vibration*, Vol. 10, No. 5-6, pp. 283–300.
- Araujo, V.S., Prado, G.S. and Santos, H.F.L., 2019. "Shunt control on smart structures using genetic algorithm and neural network method". In *XVIII International Symposium on Dynamic Problems of Mechanics*. Búzios, Brazil.
- Godoy, T.C. and Trindade, M.A., 2012. "Effect of parametric uncertainties on the performance of a piezoelectric energy harvesting device". *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. XXXIV, No. Special Issue 2, pp. 552–560.
- Moheimani, R.S.O., 2003. "A survey of recent innovations in vibration damping and control using shunted piezoelectric-transducers". *IEEE Transactions on Control Systems Technology*, Vol. 11, No. 4, pp. 482–494.
- Santos, H.F.L., 2008. *Controle de vibrações estruturais usando cerâmicas piezoelétricas em extensão e cisalhamento conectadas a circuitos híbridos ativo-passivos*. Master's thesis, Escola de Engenharia de São Carlos da Universidade de São Paulo, São Carlos, Brasil.
- Santos, H.F.L. and Trindade, M.A., 2011. "Structural vibration control using extension and shear active-passive piezoelectric networks including sensitivity to electrical uncertainties". *Journal of the Brazilian Society of Mechanical Sciences*, Vol. 33, No. 3, pp. 287–301.
- Santos, H.F.L. and Trindade, M.A., 2012. "Performance of active and passive vibration control using piezoelectric materials subjected to uncertainties on electrical and material properties". In *Proceedings of the 1st International Symposium on Uncertainty Quantification and Stochastic Modeling*. Maresias, Brazil.
- Sunar, M. and Rao, S.S., 1999. "Recent advances in sensing and control of flexible structures via piezoelectric materials technology". *Applied Mechanics Review*, Vol. 52, No. 1, pp. 1–16.
- Tang, J., Liu, Y. and Wang, K.W., 2000. "Semiactive and active-passive hybrid structural damping treatments via piezoelectric materials". *The Shock and Vibration Digest*, Vol. 32, No. 3, pp. 189–200.
- Trindade, M.A. and Benjeddou, A., 2002. "Hybrid active-passive damping treatments using viscoelastic and piezoelectric materials: Review and assessment". *Journal of Vibration and Control*, Vol. 8, No. 6, pp. 699–745.
- Trindade, M.A., Benjeddou, A. and Ohayon, R., 1999. "Parametric analysis of the vibration control of sandwich beams through shear-based piezoelectric actuation". *Journal of Intelligent Materials Systems and Structures*, Vol. 10, No. 5, pp. 377–385.