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TORSIONAL STIFFNESS ANALYSIS OF A FORMULA SAE CHASSIS BASED ON VEHICLE DYNAMICS

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Abstract. *The main objective of this work is analyzing a Formula SAE chassis based on vehicle dynamics and its performance. This analysis was made using some suspension and chassis parameters and the main parameter of the chassis stiffness, the torsional stiffness. This work determined the chassis torsional stiffness for two chassis. It was also analyzed two methods for determining of the torsional stiffness, where the first was the model developed by Deakin et al. and the second the model of Sampò. Both methods use the lateral load transfer for determining the torsional stiffness, but the model of Deakin et al. uses two masses, the front and rear mass in the analysis. Differently, the Sampò one separates in sprung and unsprung masses, then the analysis has four masses, and the lateral load transfer is obtained using the mass height center of these four masses. This work intends to obtain the chassis torsional stiffness of the Fênix Racing Formula SAE Team vehicle used in 2018 and 2019 years and also analyzing which the two models behave better and differences between them. In addition to finding out until when it could be adopted the Deakin et al. simplified considerations for the chassis design and from when the Sampò model necessarily should be.*

Keywords: *Torsional stiffness; Chassis; Vehicle dynamics; Formula SAE*

1. INTRODUCTION

The study of vehicle dynamics was developed principally because of the car races. Understand the car, the movements and the answers to commands are fundamental for have more velocity, control and stability, and these are very important for win races. Therefore, with the advance of great competitions, the vehicle dynamics became more important; and so, the developers obtained by competitions to be used in the automotive industry for design car more comfortable and safer. In this context the chassis is one of the most important parts of a vehicle design, because the chassis is responsible for integration and union of the other parts of vehicle besides housing the pilot, protecting him from accidents. Therefore, the chassis have an important structural function, because it supports all components and loads externals and internals (Belote and Menezes, 2019).

1.1 Vehicle Dynamics

Vehicle dynamics in its broadest sense encompasses all forms of conveyance-ships, airplanes, railroad trains, track-laying vehicles, as well as rubber-tired vehicles (Gillespie, 1992). This work study a little more about this theme in a Formula SAE car, specifically the comportment of chassis with suspension and its relation with vehicle dynamics. The relation between these two areas will be studied in a Formula SAE car, using the chassis designed by Fênix Racing Formula SAE Team in 2018 and 2019 years.

Vehicle dynamics can be described as the science that studies two behavioral aspects of a machine, the first would be the isolation which separated occupants from external disturbances, and the second is the control, which is the answer of vehicle the actions of the driver (Blundell and Harty, 2004). The knowledge about the dynamics of car, is important for the design of car because is possible predict the behavior of car and the forces acting in suspension and transfer for chassis, and thereby find a geometry of suspension able of obtain the maximum contact with asphalt. For a good operation of the suspension is necessary a good chassis because the operation of the suspension depends on the interaction with chassis. The main parameter in this interaction is the torsional stiffness that is responsible for the correct lateral load transfers, and thus allowing the control of car by the driver during turns and change directions.

1.2 Torsional Stiffness

It is generally thought that if torsional and vertical bending stiffness are satisfactory then the structure will generally be satisfactory. Torsional stiffness is generally the most important as the total cornering traction is a function of the lateral weight transfer (Riley & George, 2002). Owing this importance of the torsional stiffness uses an approach involving the vehicle dynamics for determine the value of torsional stiffness for the chassis. Knowing this value is important because, besides be a parameter for the correct operating of the suspension, a chassis more rigid than is necessary, is a heavy chassis, because you need more locking for obtain the stiffness and this cause a gain of mass. However, if the torsional stiffness is not sufficient, it is not possible control the lateral load transfer in the car and it becomes difficult the adjustments and gain of the performance making an unpredictable car.

The determination of torsional stiffness is very important becouse is the key for being able to obtain a good handling balance. The lateral load transfer distribution can only be controlled however, if the chassis is stiff enough to transmit the torques (Deakin et al., 2000). Therefore, in this work, this parameter, torsional stiffness was analyzed following two different models for the two chassis. These two methods were selected because are more commonly in Formula SAE chassis design, that is the object of study in this present work.

1.2.1 Model of lateral load transfer by Deakin et al. (Deakin et al., 2000)

In their work, the authors (Deakin et al., 2000) studied the torsional stiffness of chassis in function of the lateral load transfer using for determined the value of torsional stiffness parameters of the suspension like the front and rear mass, the front and rear roll stiffness, the mass height center. Following the equations and considerations of Milliken (Milliken and Milliken, 1994) and its variations (Botosso, 2015) for calculus of moments acting in chassis and with these, was established a relation involving these parameters with the torsional stiffness. The scheme used is shown below, where the front and rear roll stiffness are connected by the stiffness of chassis.

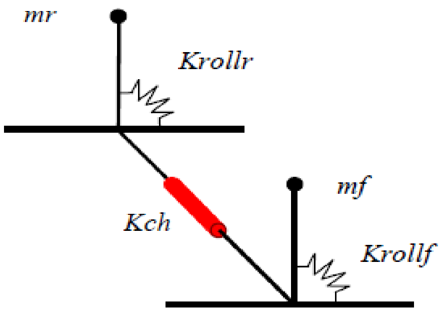

Figure 1. Scheme of chassis-suspension. (Deakin et al., 2000)

This model was analyzed follow the equations bellow:

$$M_f = K_{rollf}\Phi_f - K_{ch}\Phi_c \quad (1)$$

$$M_r = K_{rollr}\Phi_r + K_{ch}\Phi_c \quad (2)$$

$$\Phi_f + \Phi_c = \Phi_r \quad (3)$$

In these equations, Φ_f is the front roll angle suspension, Φ_r is the rear roll angle suspension; Φ_c is the chassis roll angle. K_{rollf} and K_{rollr} are the front and rear roll stiffness respectable, the K_{ch} is the torsional stiffness of chassis and the M_f and M_r are front and rear moments due the lateral acceleration. The lateral load transfer is obtained using this:

$$lt = \frac{m a h_{cg}}{t} \quad (4)$$

or

$$lt = \frac{\Phi K_{roll}}{t} \quad (5)$$

Where the lateral load transfer is lt, a is the lateral acceleration, m is the axis mass, h_{cg} is the mass height center and the track width is t. The roll angles are calculated using the follow expressions.

$$\Phi_c = \frac{m_r a h_{cg} - K_{rollr}\Phi_f}{K_{rollr} + K_{ch}} \tag{6}$$

$$\Phi_f = \frac{(K_{rollr} m_f) + K_{ch} m_t a h_{cg}}{K_{rollr}K_{rollf} + K_{ch}K_{rollf} + K_{ch}K_{rollr}} \tag{7}$$

m_f, m_r and m_t are respectable front mass, rear mass and the total mass of car. Using the Eq. (6), (7) and (3), obtains the roll angles and with this the lateral load transfer using the Eq. (5). Then the suspension roll stiffness and chassis torsional stiffness are variated and found a graph of percentage of front roll stiffness and the percentage of front lateral load transfer and the value of torsional stiffness for chassis like in Figure 2.

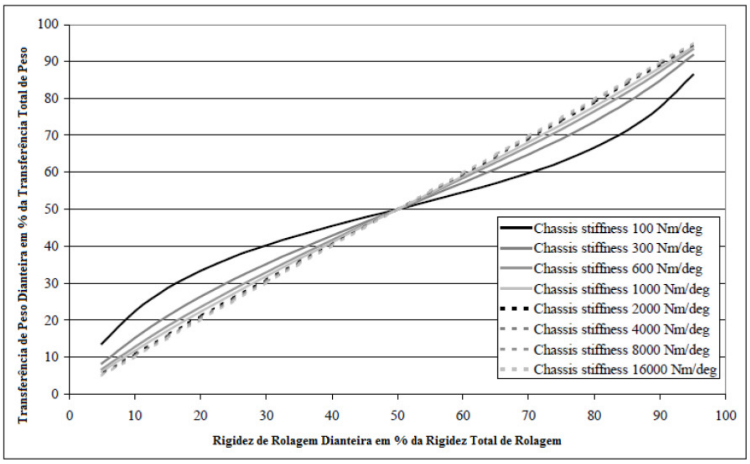

Figure 2. Graph of lateral load transfer, roll stiffness and torsional stiffness. (Deakin et al., 2000)

Analyzing the graph, is possible find the torsional stiffness the chassis needs to occurring the lateral load transfer generated in turns and directions change. According (Deakin et al., 2002) the goal is to determine a chassis stiffness that ensures the vehicle's handling is sufficiently sensitive to changes in the roll stiffness distribution. A large percentage of the difference in front to rear roll stiffness must therefore result in a difference in front to rear lateral load transfer, for example 80%. Looking at Figure 2, for example the point where the roll stiffness distribution is 30:70, the lateral load transfer distribution can be anything from 30:70 to 40:60. If the difference between front and rear lateral load transfer is to be 80% of the difference between front and rear roll stiffness, then the lateral load transfer distribution must be at least 34:66.

1.2.2 Model of lateral load transfer by Sampò (Sampò, 2011)

In this case, the author used a model very similar to that of Deakin et al., but its analysis are based in four masses, where have a front and rear sprung mass, and a front and rear unsprung mass, thereby, it has a mass height center for the sprung masses and the mass height center for the unsprung masses and the analysis of the lateral load transfer allows see the influence of theses masses separately.

According Sampò (Sampò, 2011), Deakin et al. (Deakin et al., 2000) presented an analytical model in order to evaluate the chassis torsional stiffness that ensures a good handling sensitivity when the roll stiffness distribution is changed. Some figures were given, inconsistencies however were found in the equations and in their comments. Furthermore, the model described in the paper does not consider the fact that load transfer distribution is partially controlled by roll axis position and by unsprung masses.

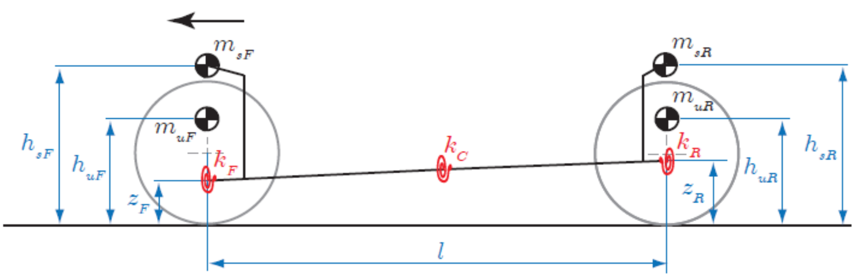

Figure 3. Scheme of chassis-suspension. (Sampò, 2011)

Besides of division of masses, Sampò used the height of roll center in its analysis, like is showed in Figure 3. The graph for analysis is similar with the Figure 2, but the analysis is more real and precise than first analysis proposed previously, and is possible verify the influence of unsprung mass in the lateral load transfer.

For obtain the graph was used the follow equations, obtained the front and rear lateral load transfer and roll angles.

$$llt_f = \frac{a}{t_f}\left(\frac{K_{rollf} d_f m_{sf}}{K_{rollf} + \frac{K_{rollr} K_{ch}}{K_{rollr} + K_{ch}}} + \frac{\frac{K_{rollf} K_{ch}}{K_{rollf} + K_{ch}} d_r m_{sr}}{\frac{K_{rollf} K_{ch}}{K_{rollf} + K_{ch}} + K_{rollr}} + z_f m_{sf} + h_{uf} m_{us}\right) \tag{8}$$

$$llt_r = \frac{a}{t_r}\left(\frac{K_{rollr} d_r m_{sr}}{K_{rollr} + \frac{K_{rollf} K_{ch}}{K_{rollf} + K_{ch}}} + \frac{\frac{K_{rollr} K_{ch}}{K_{rollr} + K_{ch}} d_f m_{sf}}{\frac{K_{rollr} K_{ch}}{K_{rollr} + K_{ch}} + K_{rollf}} + z_r m_{sr} + h_{ur} m_{ur}\right) \tag{9}$$

The roll angles can be calculated with the equations below.

$$\Phi_f = \frac{K_{ch}(m_{sf} d_f + m_{sr} d_r) + K_{rollr} m_{sf} d_f a}{K_{rollf} K_{rollr} + K_{ch} K_{rollf} + K_{ch} K_{rollr}} \tag{10}$$

$$\Phi_r = \frac{K_{ch}(m_{sf} d_f + m_{sr} d_r) + K_{rollf} m_{sr} d_r a}{K_{rollf} K_{rollr} + K_{ch} K_{rollf} + K_{ch} K_{rollr}} \tag{11}$$

In these equations, the m_{sf} and m_{sr} are the front and rear sprung masses, m_{uf} and m_{ur} are front and rear unsprung masses, z_f and z_r are these height roll centers in front and rear, h_{uf} and h_{ur} are the mass height center of front and rear unsprung masses and d_f and d_r are difference between the mass height center of the sprung mass and the height of roll center.

$$d_f = h_{sf} - z_f \tag{12}$$

$$d_r = h_{sr} - z_r \tag{13}$$

Then, was obtained a graph similar that is shown in the Figure 4, where the loads transfer, roll stiffness and chassis torsional stiffness are related. Sampò (Sampò, 2011) use a normalization in the axes for show your graph, but the axes are the same present in previous model.

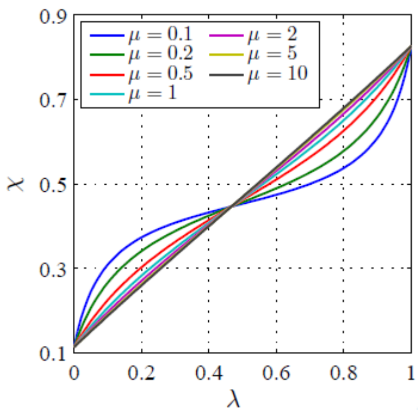

Figure 4. Graph of lateral load transfer, roll stiffness and torsional stiffness normalized. (Sampò, 2011)

The difference between the graphs of Deakin et al. and Sampò will be the inclination of curves, expected that in Sampò model, the curves are more inclined than in Deakin et al. model because the influence of the unsprung mass and its mass height center. The behavior of curves is the same of the first model, where if, it has an increase of the torsional stiffness, the distribution of the lateral load transfer approaches of a linear behavior.

Finally, the last motivation for this work is the fact that have few works in literature about this theme making this comparation using real data and making a numerical analysis, then this work is important for aggregate more information in this area and so, it can be specified information about a vehicle type. It is important know that model differences depending on conditions acting on each vehicle and application because each vehicle have many specify characteristics of performance and track conditions.

2. METHODOLOGY

This work has the main objective of finding values of torsional stiffness for chassis of 2018 and 2019 years of the Fênix Racing Formula SAE Team vehicle following the two methods shown in the introduction, and then compare the results; besides, find in which condition is necessary use the methodology of Sampò (Sampò, 2011) or the model of Deakin et al. (Deakin et al., 2000).

For this, it will be obtained the data necessaries for the model of Deakin et al. (Deakin et al., 2000) where it is needing the roll stiffness of front and rear suspension, the front and rear mass (mass distribution), the mass height center and the track width. With this data, is possible find the roll angles of suspension and chassis, and with the roll angles, is found the moments acting in lateral load transfer and the lateral load transfer. The calculus is released with assistance of MATLAB®, and the graph shows the variation of roll stiffness and lateral load transfer according the torsional stiffness of chassis like in Figure 2.

Following this, the data necessaries for model of Sampò (Sampò, 2011) are the same data, but the masses are separated in sprung and unsprung masses, and are necessary the mass height center of each mass, four mass height centers. Besides, it needs of height of front and rear roll centers. The all data are showing in Table 1.

Table 1. Data of 2018 and 2019 Fênix Racing car

Parameters	Data of 2018	Data of 2019
Front track	1.223 m	1.21 m
Rear track	1.18 m	1.19 m
Total mass	349 kg	311 kg
Front mass	177.99 kg	152.5 kg
Front sprung mass	141.27 kg	122.5 kg
Front unsprung mass	36 kg	30 kg
Rear mass	171.01 kg	158.48 kg
Rear sprung mass	135.73 kg	126.48 kg
Rear unsprung mass	36 kg	62 kg
Height center of mass	0.29 m	0.275 m
Center of front sprung mass	0.543 m	0.555 m
Center of front unsprung	0.1951 m	0.1951 m
Center of rear sprung mass	0.543 m	0.555 m
Center of rear unsprung	0.1951 m	0.1951 m
Front roll center	3.45 mm	5.32 mm
Rear roll center	33.01 mm	13.48 mm
Suspension roll stiffness	956.36 Nm/º	609.06 Nm/º

Data provided by Fênix Racing Formula SAE Team

The torsional stiffness can be considerate ideal for the design, when the percentage of the lateral load transfer in graph is the same percentage of the roll stiffness, that is, when the torsional stiffness have a linear comportment, but this is impossible. Then, the analysis is made following variations in lateral load transfer for each value of torsional stiffness, when changings of torsional stiffness cause a little increase of lateral load transfer for a big change in torsional stiffness is not recommended increase the stiffness owing the gain of mass and the insignificant gain of load transfer. This relation between increasing of torsional stiffness and gain of lateral load transfer will be used as a reference to determine the torsional stiffness in the two analyzed cases.

For choose the value of chassis torsional stiffness, it should be considered another factor beside the described in (Deakin et al., 2002). If you increase the torsional stiffness is necessary attention with the weight of structure because a chassis heavier is harmful for general performance of the vehicle, mainly because the chassis is one of the heavier components in a car, being important consider its mass during design. Therefore, for determining the value of the chassis torsional stiffness, it should be also considered the increases of the lateral load transfer according the increases in torsional stiffness and the probably addition in chassis mass that this increase in stiffness could cause, and then verify if was really beneficial increase the stiffness depending on these all factors.

Besides that, after these comparations and choose the chassis torcional stiffness, the chassis stiff was fixed and the unsprung mass was increased of 10% in each step, and with this was plot a graph showing the curves variation due this increase in unsprung mass, being possible seen the influence of this mass portion in lateral load transfer distribution. The intension of make this is verify if the increase of unsprung mass in the total mass of car have a significant influence in distribution of lateral load transfer and consequently in the chassis torsional stiffness in Formula SAE cases.

3. RESULTS AND DISCUSSIONS

Following the explanations, were constructed the graphs. Figures 5 and 6 show the relation between front suspension roll stiffness, chassis torsional stiffness and front lateral load transfer for 2018 car according (Deakin et al., 2002) model. Following the descript in paper of (Deakin et al., 2002), the chassis torsional stiffness must have been 3000 Nm/º of stiffness. Considering the analysis involved the mass of chassis, determined that 1500 Nm/º is a good value for chassis torsional stiffness in this case, because with this value, the lateral load transfer is 53,77%, and with stiffness of 3000 Nm/º is 54%, that is a little difference, principally if you consider the big increase in torsional stiffness, and according simulations, the chassis with this stiffness would be increased in 5 kg, or 10,5% of chassis mass with 1500 Nm/º. Was considered that the gain in lateral load transfer does not justify the addition in weight and the choice of the torsional stiffness of 1500 Nm/º in chassis design, because have a good relation between torsional stiffness, lateral load transfer and chassis weight.

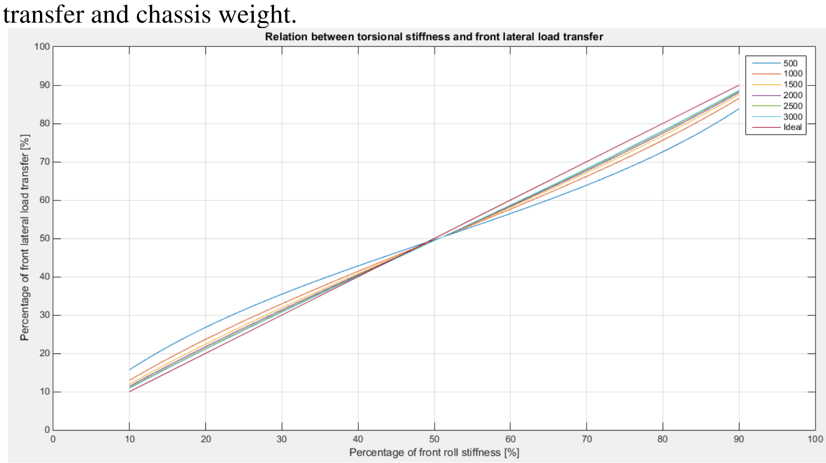

Figure 5. Graph of front lateral load transfer and front roll stiffness of suspension by Deakin model for 2018 car. From authors

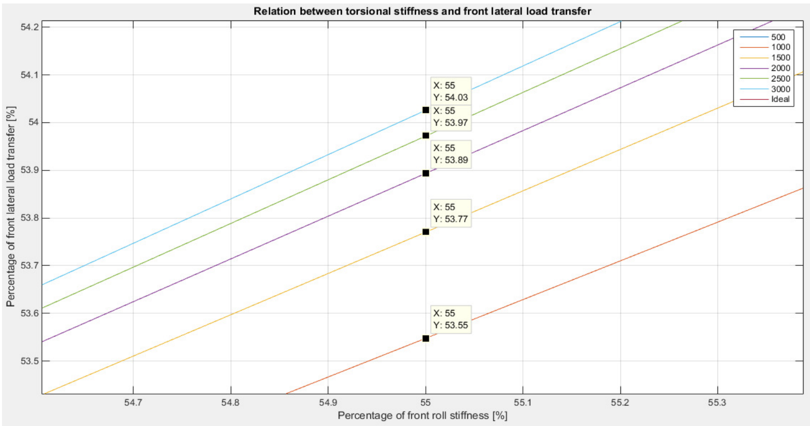

Figure 6. Detail of graph of Figure 5. From authors

After, the analysis was made following the model used by (Sampò, 2011) for 2018 car and the results are showed in Figures 7 and 8. In this case, it is seen that for a chassis torsional stiffness of 1500 Nm/º the frontal lateral load transfer is 52.21%, and for a chassis with 3000 Nm/º of torsional stiffness, the front lateral load transfer is 52.36%, then the difference between these values is even smaller than in (Deakin et al., 2002) model for a same chassis, with same weight. Therefore, the choice for a chassis with 1500 Nm/º of torsional stiffness is maintained.

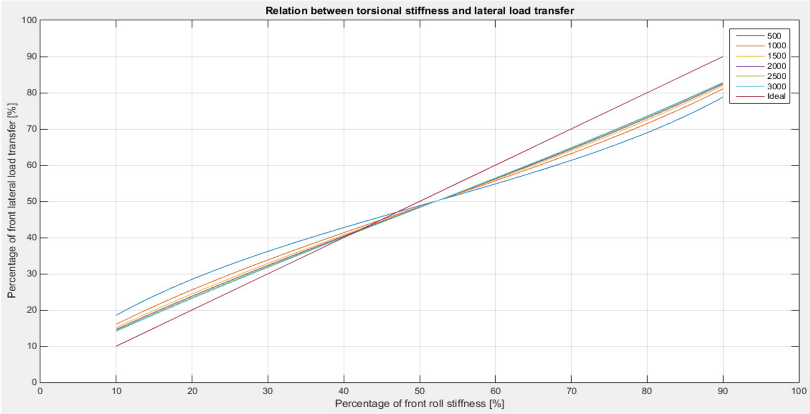

Figure 7. Graph of front lateral load transfer and front roll stiffness of suspension by Sampò model for 2018 car. From authors

Comparing the two models analyzed, was obtained the Figure 9. Comparing these models, it is seen that curves using the model of (Sampò, 2011) are more inclined than the curves obtained using the (Deakin et al., 2002) model like was supposed and the difference between them was of only 1.56% in the front lateral load transfer. Then, in the case it is possible using any of the models without losing any design information, because in this case, the influence of unsprung mass on the lateral load transfer is small.

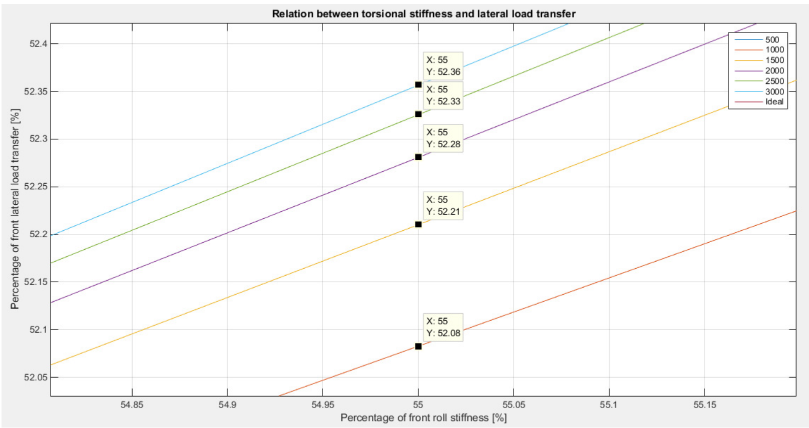

Figure 8. Detail of graph of Figure 7. From authors

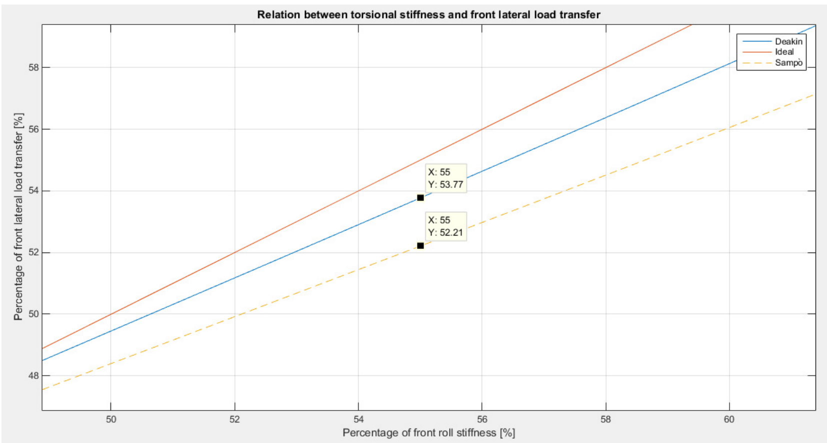

Figure 9. Comparation of two models for 2018 car. From authors

Furthermore, it was made a similar analysis using the data of 2019 car, and the results are shown in figures 10 and 11 for (Deakin et al., 2002) model. Then analyzing the results, in the first case using a simpler model, the front lateral load transfer suffers a very little increase according the chassis torsional stiffness is increased. The difference between lateral load transfer in chassis with 1000 Nm/º and 3000 Nm/º is only 0.17%; therefore, the choice was made in question involved the mass addition in chassis according the increase in torsional stiffness depending on chassis design and model.

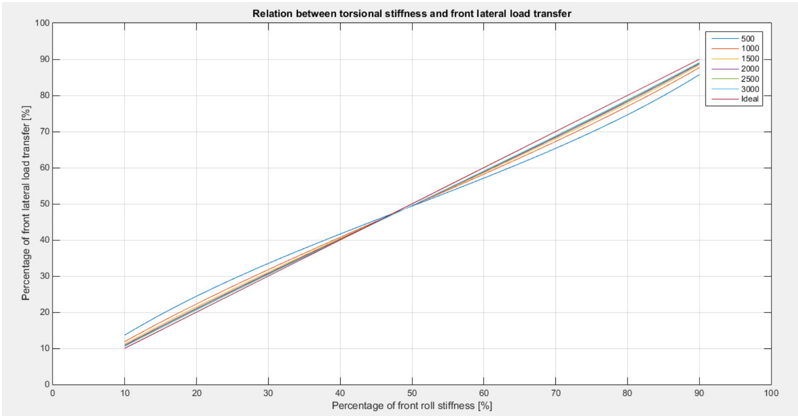

Figure 10. Graph of front lateral load transfer and front roll stiffness of suspension by Deakin model for 2019 car. From authors

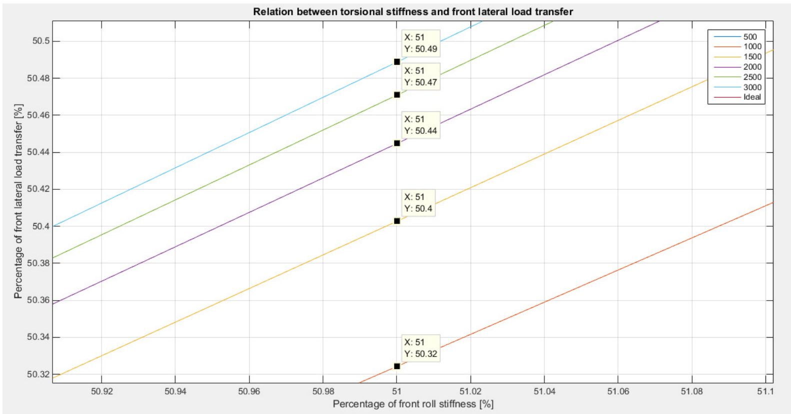

Figure 11. Detail of graph of Figure 10. From authors

The results are also shown in figures 12 and 13 for (Sampò, 2011) model. In this case, the difference between lateral loads transfer for the same values of chassis torsional stiffness is smaller than in the first model; for 3000 Nm/º of chassis torsional stiffness, the lateral load transfer is 49.94% and for 1500 Nm/º of torsional stiffness, the lateral load transfer is 49.84%, or 0.1% less; then, the choice of the chassis one more time was based on the chassis model and its weight. One more time was seen that the curves of torsional stiffness in the second model was more inclined than the curves of Deakin et al. (Deakin et al., 2002) model, because the consideration of unsprung masses and the roll centers.

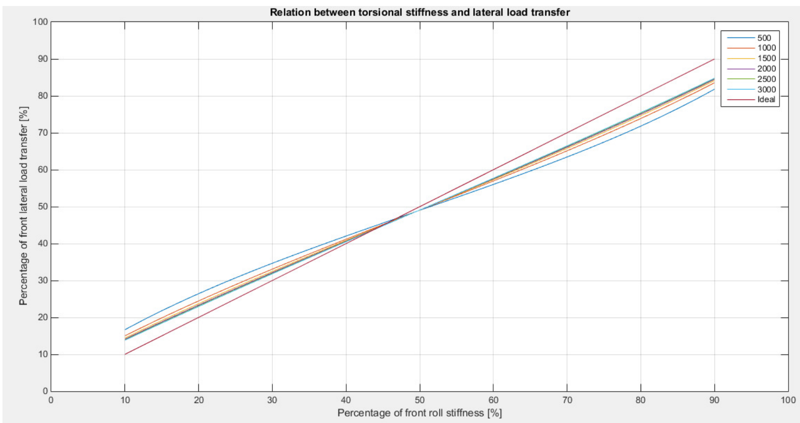

Figure 12. Graph of front lateral load transfer and front roll stiffness of suspension by Sampò model for 2019 car. From authors

Therefore, because the chassis design and restraint of the car, it was opted for a torsional stiffness around 1400 Nm/º, having a good value of stiffness and having a good balance of mass for the chassis and car in general.

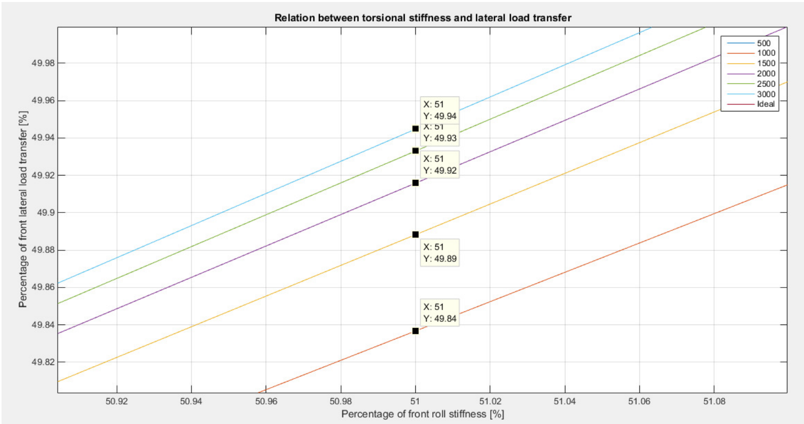

Figure 13. Detail of graph of Figure 12. From authors

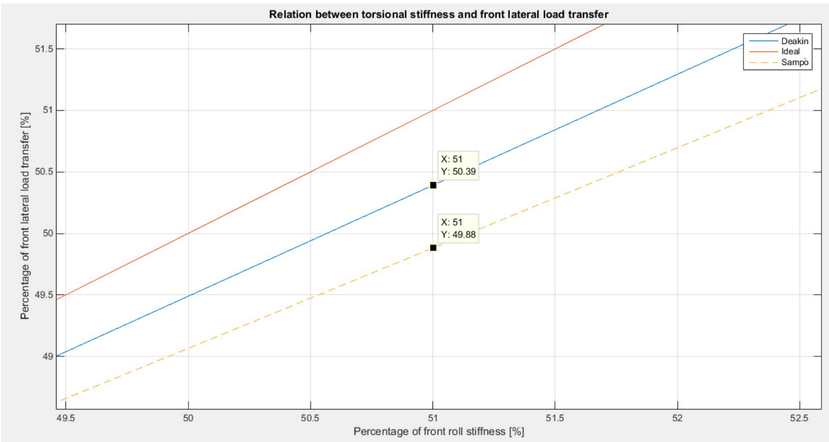

Figure 14. Comparation of two models for 2019 car. From authors

Then, the values of front lateral load transfer using the two methods were showed in Figure 14, obtained 50.39% of lateral load transfer with Deakin et al. model and 49.88% of lateral load transfer with Sampò model, that is, the difference between the models was even lower than the same situation using the data of 2018 chassis; for 2019 chassis the difference was 0.5% and 2018 1.57%. Therefore, the two cases analyzed showed that for similar conditions of this work, either one or other model can be used without loose information in chassis design and car.

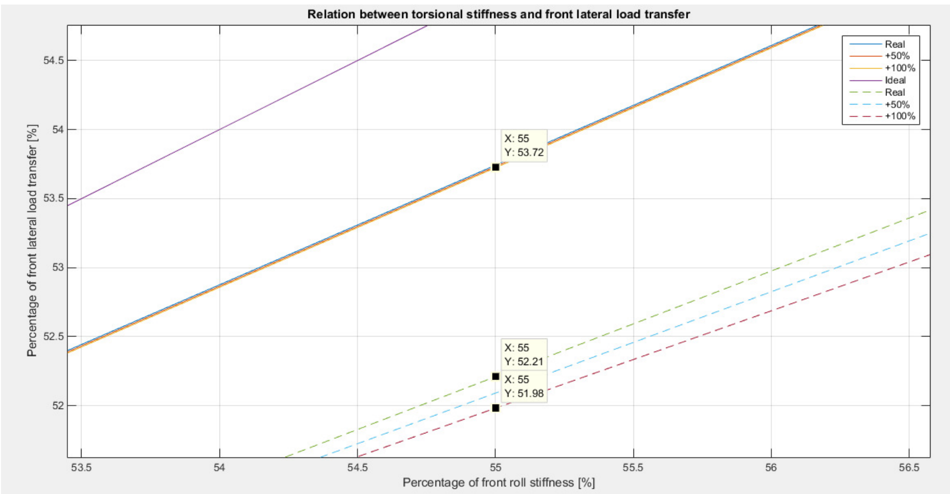

Figure 15. Influence of increase unsprung mass in 2018 chassis. From authors

Finally, in Figure 15, was made a comparation between the two models using the data of 2018 car and increasing the unsprung mass. The result using Deakin et al. (Deakin et al., 2002) model do not suffer influence, the continues lines remained in same place, overlapping, but in case of Sampò (Sampò, 2011) model, is possible to see in Figure 15, the dashed lines, that according the unsprung mass increases, for a same value of torsional stiffness and roll stiffness, the lateral load transfer decreases as expected.

4. CONCLUSIONS

Analyzing the results, was concluded that the case in question, the use Deakin et al. model (Deakin et al., 2002) or model proposed by Sampò (Sampò, 2011) for determine the chassis torsional stiffness, do not have many differences, in two cases analyzed, 2018 and 2019 chassis, the final values of torsional stiffness were very similar, with the model of Sampò tending to have a lower lateral load transfer, but in these case this lower value was not substantial, therefore, it can use the two models without loose significant information for chassis design and consequently for car, in cases of a Formula SAE, as was analyzed here.

The use of two models do not have many differences in results because the unsprung mass is not a large percentage of the car total mass, but the difference between the models tends to increasing according the percentage of unsprung mass is increased in the car total mass and in this situation, the curves descript by Sampò model (Sampò, 2011) tends to be more inclined according the percentage of growing up, increasing the difference of lateral load transfer for a same torsional stiffness and roll stiffness. Similarly, if the unsprung mass decreases, the difference between the two methods tends to decreasing too. Then, for vehicles different of a Formula SAE, where the unsprung mass represents a big percentage of the car total mass, is advisable using the Sampò method (Sampò, 2011).

Besides, it was clear the importance of reduction of mass, improving the car performance and the possibility of simplify the design with models and considerations simpler.

5. ACKNOWLEDGEMENTS

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