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A Gap Metric Approach for Controller Certification of a Hydraulically-Actuated Legged Robot

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Abstract.*In the last decade robotics became extremely popular among students, researchers as well as in industry due to the advances in technology, and it seems to have promising expectations for the future. The hydraulic quadruped robot, or HyQ, is a class of legged robot with a high power output, responsible for interactions with humans and tools in a dynamic environment where it systems control has to handle different boundary conditions (seeing as uncertainties) at every step of the way. The existence of these uncertainties can cause instability and loss of performance in the system. Therefore, a modern control paradigm is required in order to find controllers that can guarantee stability and performance even in the face of uncertainties. This paper presents a procedure to achieve robust proportional–integral–derivative (PID) tuning based on the gap metric approach combined with a branch and bound algorithm for the controller certification of a HyQ. The case study in this letter is the abrupt mass variation on the legs of the robot during the walk type movement. It is obtained a robust PID controller that can be an alternative to robust controllers like the* H_∞ *and some PID tuning.*

Keywords: *Hydraulic quadruped robot, controller certification, gap metric theory, robust control*

1. INTRODUCTION

Robotics has an important role in engineering and it has been experiencing large increase in technology over the last decades with the improvement of hardware, software as well as mathematical tools in the form of modern systems control (Bajcsy, 2019). Physical interactions like manipulation and legged locomotion are a requirement for robotics systems which means being able to deal with different and difficult environments. Specifically, the hydraulic quadruped robot HyQ deals with substantial variations in the mass of each leg during the walk and these uncertainties inherent of the movement can complicate the controller design in the process of mathematical modeling generating a system control with poor performance and unstable for some operating points. Robust control tries to overcome these problems because deals directly with the existence of uncertainties in complex systems (Vinnicombe, 2001). However there are some drawbacks in the robust control theory such as the mathematical complexity required to design the controller and conservativeness. The gap metric approach alongside the generalized stability margin proves to be an excellent and practical tool that can create strategies to overcome some of the robust control problems, although it is applications in the control literature are relatively new.

The gap metric measures the distance between two systems (possibly unstable) in terms of dynamic response and exhibits robustness properties. One of this properties being the theorem stating that if two systems are close in the gap metric sense there is at least one controller that stabilizes both systems (El-Sakkary, 1981, 1985), therefore, the gap metric can sustain an open loop analysis like the Nyquist stability criterion which is a great advantage and can be viewed as a generalization of the operator norm. The controller certification problem revolves around the assurance of stability and performance by a controller or a set of controllers when implemented in a system subject to different types of uncertainty (Park, 2007). The controller certification is a complex problem to solve due to it is intrinsic robust control nature which implies in a trade off between stability, performance and feasibility. (Zhou and Doyle, 1998).

This letter intends to attack some of the robust control issues by developing a practical and relatively simple procedure to obtain a robust PID controller using the gap metric approach which can be an alternative to high order and complex controllers like the H_∞ and simple PID tunings which often generate poor results. The initial problem treated here is a HyQ system subject to an intense parametric variation characterized by the oscillation of the mass in one of the legs of the robot due to the walking type movement and a multiplicative unstructured uncertainty applied directly into the system. Each leg corresponded by an actuator experiences a maximum mass condition when the leg is pressing against the ground and a minimum mass condition when the leg is in the air. In fact the mass on the legs have their own dynamic represented by the multiplicative unstructured uncertainty which can cause a rather dramatic instability in the system depending on the operating point. A range of admissible uncertainty is created for the linearized HyQ plant from a first gap metric analysis. Then based on the range of admissible uncertainties several candidate controllers are generated by the PID tuning process and a combinatorial search for the best controller that satisfies the stability and performance requirements in full range of uncertainties is done by the combination of a second gap metric analysis with a branch and bound algorithm. This methodology enabled the achievement of a final robust PID controller.

2. BACKGROUND OF THE GAP METRIC

There is a considerable quantity of gap metric extensions in the control literature, each one displaying advantages and disadvantages which depends on the context where it is used. In this paper a gap metric based on (Georgiou, 1988) was chosen due to it is easy implementation in the MATLAB environment and for being more conservative concerning the robustness analysis.

Let P be a rational transfer function having a normalized right coprime factorization:

$$P(s) = NM^{-1} \; with \; M^*M + N^*N = I \tag{1}$$

Where N and M are transfer functions that belongs to H_2 (standard Hardy space) and (*) denotes the complex conjugate operator. The gap between two finite-dimensional linear system P_1 and P_2 with the same dimensions is defined by:

$$\delta(P_1, P_2) = max\left\{ inf_{Q\in\, H_\infty} \left\| \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} - \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} Q \right\|_\infty , \; inf_{Q\in\, H_\infty} \left\| \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} - \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} Q \right\|_\infty \right\} \tag{2}$$

One solution for Eq. 2, which is a calculus of variations problem, is presented in (Georgiou and Smith, 1990).

Theorem 1 *There is a robust stabilizing controller C for both P_1 and P_2, if and only if:*

$$\delta(P_1, P_2) < \gamma(P_1, C) \tag{3}$$

With γ as a measure of stability margin

In the control literature there are a few stability factors that satisfies 1, however the most used is the generalized stability margin $\gamma := b$.

Given the sensitivity function:

$$T(P, C) = \begin{pmatrix} P\,(I + CP)^{-1}\,C & P\,(I + CP)^{-1} \\ (I + CP)^{-1}\,C & (I + CP)^{-1} \end{pmatrix} \tag{4}$$

Then the generalized stability margin b(P,C) is:

$$\begin{cases} \|T(P, C))\|_\infty^{-1} , \; if \; (P, C) \; is \; stable, \\ \qquad\qquad 0, \; else. \end{cases} \tag{5}$$

An easy way to calculate Eq. 5 is presented in (Cantoni and Vinnicombe, 1999). Both the gap metric and the generalized stability margin have values between [0,1].

3. CONTROLLER CERTIFICATION PROCEDURE

Given an uncertain plant described by the set P_Δ, a nominal plant $P_{nom} \in P_\Delta$, indexes $i, j \in \mathbb{N}$ and a PID controller C of the type:

$$C = \frac{K_d s^2 + K_p s + K_i}{s} \tag{6}$$

The closed loop configuration (P_i,C) is stable for every $P_i \in P_\Delta$ if

$$\delta(P_{nom}, P_i) < b(P_{nom}, C) \tag{7}$$

Now consider only a nominal plant P_{nom} and two PID controllers, C_p and C_q ($p, q \in \mathbb{N}$), with C_i stabilizing P_{nom}. The control loop (P_{nom}, C_q) is stable if

$$\delta(C_p, C_q) < b(P_{nom}, C_i) \tag{8}$$

The performance specifications can be established from the calculation of the generalized stability margin with the following inequalities:

$$GM(P_i, C) \geq \frac{1 + b(P_i, C)}{1 - b(P_i, C)} \tag{9}$$

and

$$PM(P_i, C) \geq 2arcsin\, b(P_i, C) \tag{10}$$

Where GM and PM are the gain and phase margin respectively. There is a margin guarantee for P_i defined by the next equation:

$$b(P_{nom}, C) - \delta(P_{nom}, P_i) \leq b(P_i, C) \tag{11}$$

Meaning that a controller C that stabilizes P_{nom} and P_i also produces a minimum of stability margin in P_i.
Now it is presented the steps for the certification procedure:

- **Step 1**: Create a finite set of plants around the initial nominal plant in the range of operating points $P_{\triangle} = \{P_{nom}, P_{i-1}\}$. $i = nom \rightarrow i = 0$

- **Step 2**: The gap metric is calculated for the set $P_{\triangle}$ using Eq. 2

- **Step 3**: Evaluate the robustness of the initial controller C with the set of plants using theorem 1. If the gap metric conditions are satisfied, this initial controller is certified, otherwise, go to the next step

- **Step 4**: A new nominal plant is chosen based on the biggest value assumed by the gap metric calculated in step 2

$$\begin{cases} sup_{i \neq j} \delta(P_i, P_j) \\ P_{nom} \rightarrow P_i \end{cases} \tag{12}$$

- **Step 5**: Based on Eq. 9 and Eq. 10 a desired generalized stability margin is chosen to produce a specific phase margin (or gain margin) in the closed loop configuration of the controlled uncertain system

- **Step 6**: The desired generalized stability margin is used in Eq. 11 along with the biggest value of the gap metric obtained in step 2 to obtain a conservative generalized stability margin $b_{conservative}(P_{nom}, C)$. The objective is to maximize the conservative generalized stability margin and the admissible uncertainty, represented by the gap metric, to guarantee performance and safety in the system operation range. The maximization of the admissible uncertainty is done through the minimization of the gap metric values. The optimization problem is stated below:

$$\begin{cases} max\; b_{conservative}(P_{nom}, C) - min\; \delta(P_{nom}, P_{i-1}) < b_{desired}(P_{i-1}, C) \\ \delta(P_{nom}, P_{i-1}) < b_{conservative}(P_{nom}, C) \end{cases} \tag{13}$$

  As it is seen in the equation above, there is a trade-off between $b_{conservative}(.)$ and $\delta(.)$. This trade-off can be improved by loop-shaping and multi- objective optimization techniques (Okle, 2016).

- **Step 7**: A initial candidate PID controller C_p is designed to stabilize P_{nom}. The controller C from step 3 can be used as a initial candidate controller in this step

- **Step 8**: A PID tuning is done for a candidate controller C_p in order to achieve the desired generalized stability margin. This part occurs from the combination of Eq. 8 with a branch and bound algorithm represented by the following flow chart:

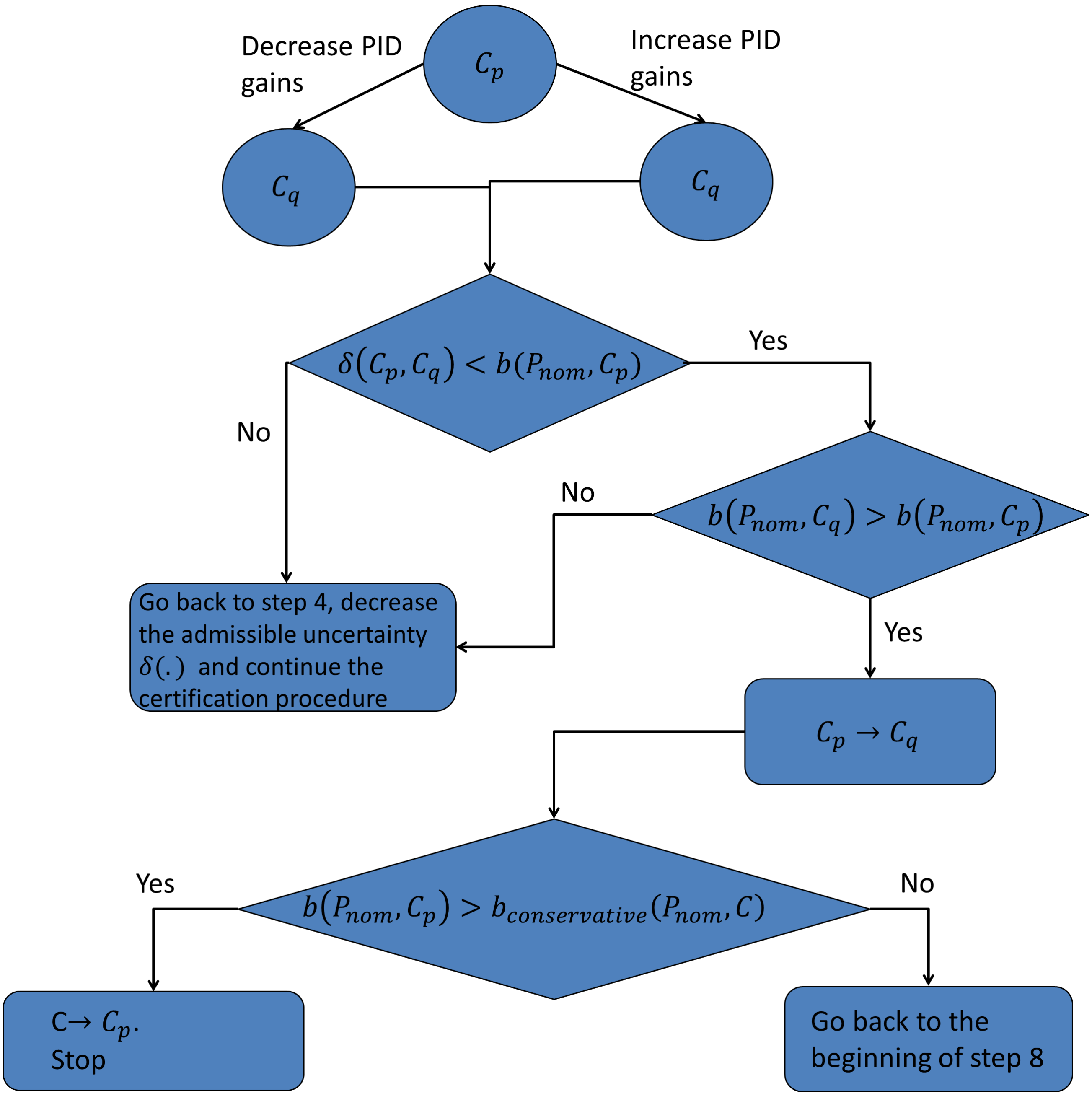

Figure 1: Flow chart of the controller tuning process

4. CONTROLLER CERTIFICATION OF THE HyQ SYSTEM

Each actuator of the robot leg is coupled with a sensor and a load mass M_l. The load mass M_l is measured when the four legs are standing still on the ground. An illustrative model of the robot leg is shown in Fig. 2.

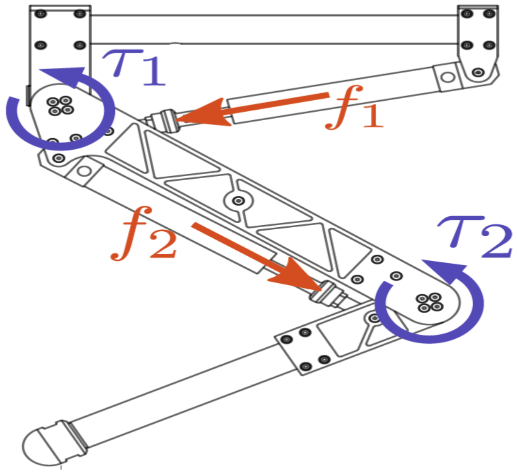

Figure 2: Schematic of the robot leg taken from (Boaventura, 2013)

The transfer function of the system to be controlled is:

$$\frac{\Delta f}{\Delta u} = (\frac{1}{\frac{s^2}{w_v^2} + \frac{2D_v}{w_v} + 1})(\frac{K_{u_v}(M_l s + B_l)}{(s - K_{f_h})(M_l s + B_l + B) - K_{\dot{x}_p}}) = P_{nom} \tag{14}$$

Where Δu is the control law and Δf is the linear load force to be controlled. The block diagram of Eq. 14 can be seen in Fig. 3.

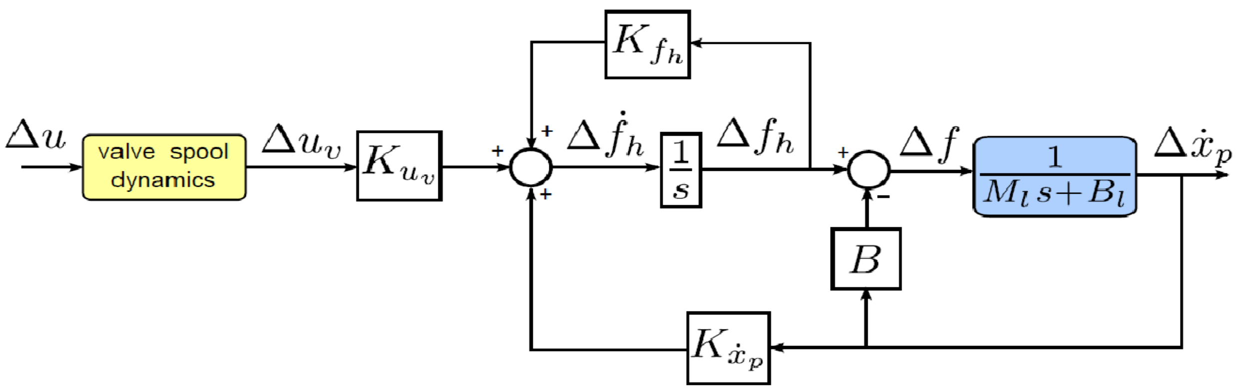

Figure 3: Block diagram of the transfer function

The parameters of the transfer function described by Eq. 14 are:

K_{u_v} (hydraulic force constant)	$7,56 \times 10^6$ [m^2/s]
M_l [load mass]	30 [kg]
B_l [load damping]	20 [Ns/m]
w_v [valve spool angular frequency]	1570 [rad/s]
D_v [valve spool damping]	0,5 [Ns/m]
K_{f_h} [hydraulic force constant]	$-2,88 \times 10^{-2}$ [1/s]
B [cylinder damping]	1000 [Ns/m]
$K_{\dot{x}_p}$ [hydraulic force constant]	$5,86 \times 10^4$ [N/m]

Table 1: Transfer function parameters [SI units]

The uncertainty analysis is done throughout the following arrangement:

$$\begin{cases} M_{l,\Delta i} = M_l + M_{\Delta_i} \\ M_{l,\Delta i} \Rightarrow \{P_i\} \end{cases} \tag{15}$$

Where M_l is some nominal value for the load mass and M_{Δ_i} is the parametric uncertainty symbolizing the variation of the mass during the walk type movement of the robot. A decrease in mass felt by the sensor implies in the leg moving towards the air and an increase in mass implies in the leg pressing the ground. Also a multiplicative uncertainty that creates an unstable pole is considered in the model for two reasons:

1. To emulate an exogenous dynamic variable capable of destabilize the system
2. To show that the gap metric approach can be applied to unstable uncertainties

The multiplicative uncertainty chosen is presented below:

$$P_\Delta \Delta(s) \Rightarrow \Delta(s) = \frac{1}{s-1}. \tag{16}$$

The objective is to certify a controller that can guarantee stability for the maximum unstructured uncertainty$P_{\Delta_{max}}$, during the walk type movement, while producing a phase margin of at least 60° in the closed loop configuration. The 60° phase margin was chosen purposely to attenuate the oscillations of the system.

Based on the procedures presented in the third topic of this letter the following results are obtained:

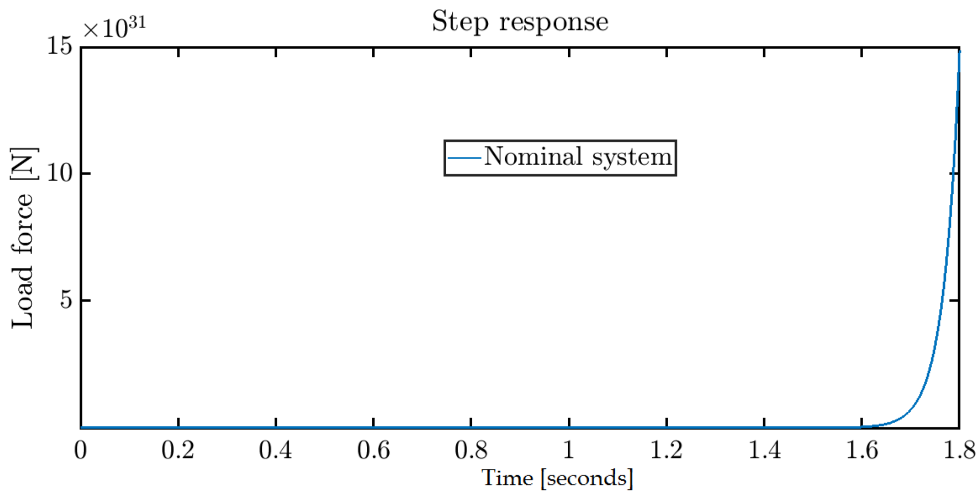

Figure 4: Open loop response of the nominal system

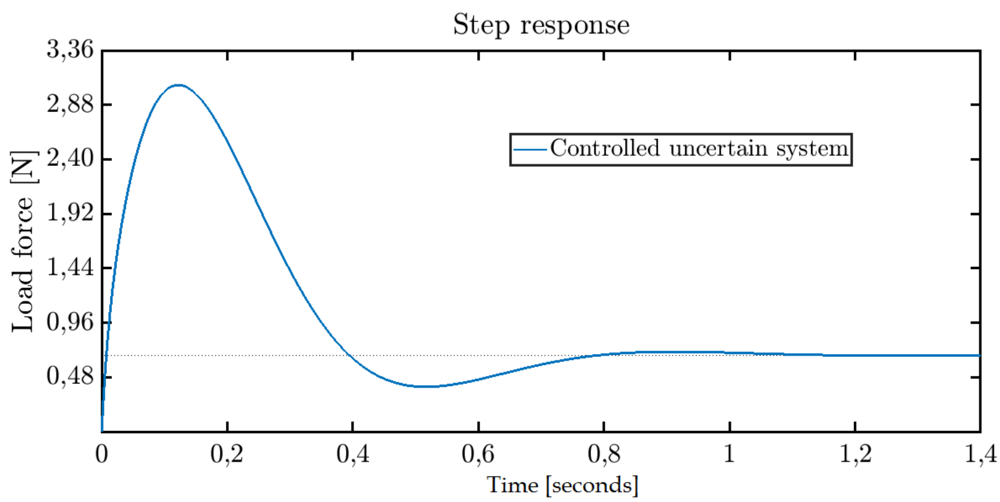

Figure 5: Closed loop response of the controlled system subject to the multiplicative uncertainty

Figure 5 shows the controlled uncertain system without the mass variation. Initially a simple PID was designed to stabilize the nominal system using the Ziegler-Nichols method. Then the certification procedure took place in order to ensure stability for the feedback system after the insertion of the multiplicative uncertainty. The initial PID controller was able to handle the first situation.

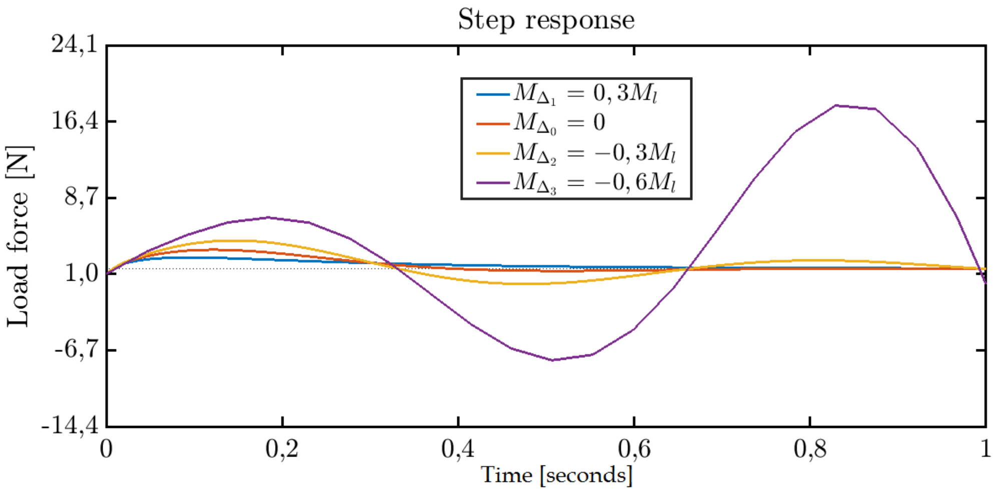

Figure 6: Closed loop response for the controlled uncertain system subject to parametric uncertainty

Now considering the variation in the mass during the robot movement it is observed that the controlled uncertain system goes unstable for the $-0,66M_l$ operation point. Therefore, the initial controller is not appropriate for this situation. As it will be shown in the next tables, the controller certification procedure can predict the failure of the controller, without having to simulate the closed loop response done in Fig. 6, just by knowing the types and range of the uncertainties of the open loop system and the structure of the controller. One of the great features of controller certification is the capacity to evaluate the quality of a given controller.

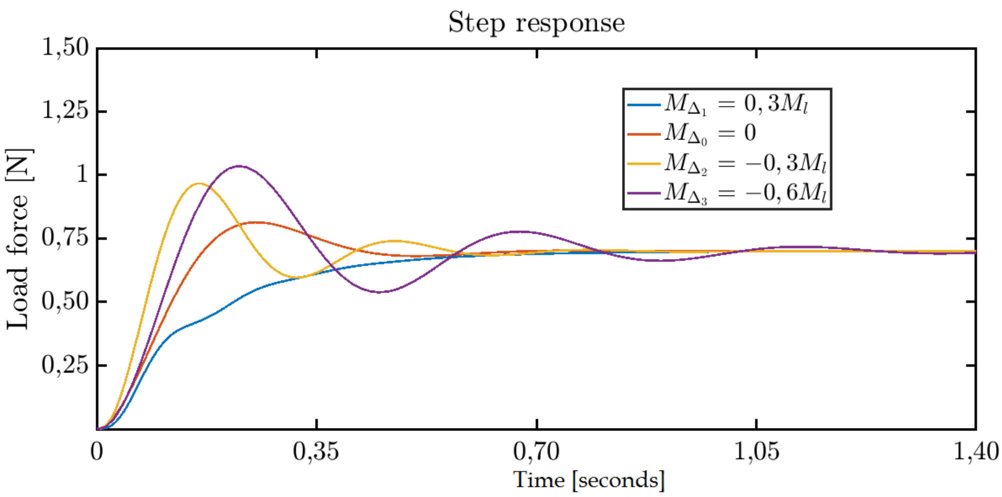

Figure 7: Closed loop step response for the system with the final robust PID controller

After the controller certification which considers the multiplicative uncertainty and the parametric uncertainty, a final PID controller, capable of stabilizing the plant for the whole range of uncertainties, is obtained. The gains of the robust PID controller are: Kp = $3,5 \times 10^{-3}$; Kd = 5×10^{-6}; Ki = 10^{-4}. The phase margin for the worst case of load uncertainty (-0,66M_l) is 80°.

Now it is presented the values of the gap metric and generalized stability margins before and after the certification procedure:

$\delta(P_{nom}, P_i)$	i
0,0079	1
0,0099	2
0,0123	3

Table 2: Values for the gap metric, for a small set of plants around a nominal plant which produces a maximum gap metric

-	Before certification	After certification
$b(P_{nom}, C)$	0,0117	0,43

Table 3: Generalized stability margin values

Before the certification procedure the generalized stability margin had a smaller value than the gap metric calculated for the plant P_3 which is equivalent to the -0,66M_l operation point. Therefore, the instability condition is already expected and was confirmed by Fig. 6. After the certification procedure, a generalized stability margin much higher was obtained which could guarantee the fulfillment of Eq. 7 for the whole range of uncertainties, also producing the desired phase margin.

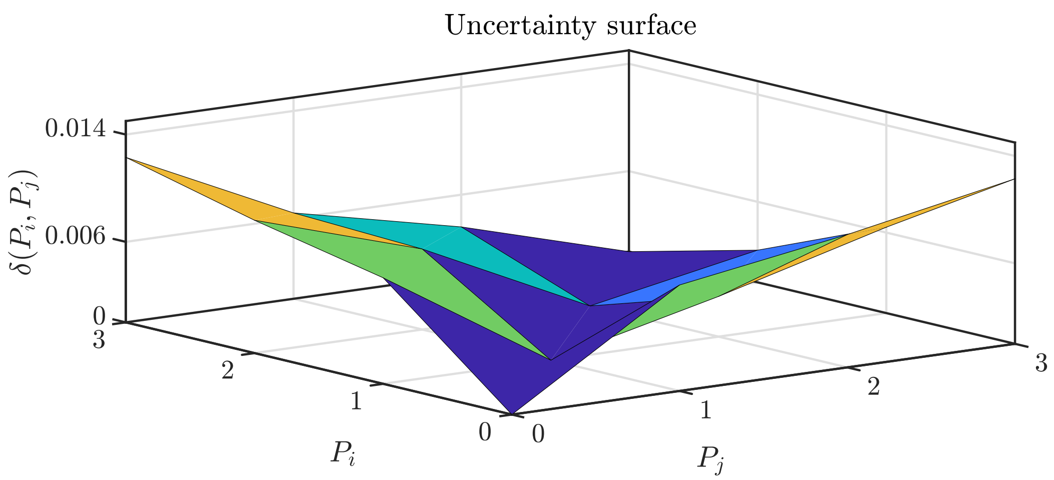

Figure 8: Distribution of gap metric values

Figure 8 shows that there are an infinite quantity of plants that were not considered in the initial set P_i. However, because of the symmetry of the gap for this particular system, the set of plants created around a nominal plant could be smaller which implied in less computational effort, therefore the right choice of operating points and the knowledge of the uncertainty structure have a tremendous importance for the success of the gap metric analysis and controller certification.

5. CONCLUSION

The controller certification procedure using the gap metric approach led to the synthesis of a robust PID controller capable of dealing with a certain range of uncertainty admissible in the gap metric sense and also preserving a performance requirement. Some simplifications were made such as the linearization of the HyQ, the type of modeling uncertainty and the heuristic optimization method chosen (Branch and Bound). However, the controller certification procedure showed potential and was able to illustrate some of the applications of the gap metric and the generalized stability margin in a hydraulically-actuated legged robot. The procedure can be viewed as an alternative to robust or classical controller analysis and synthesis. For future works a broader modeling of the HyQ system considering nonlinear dynamics and more realistic types of uncertainties is necessary. The nonlinear gap metric may be more adequate to this situation. And finally an overall improved certification procedure capable of producing better controllers that can ensure stability and performance when implemented on hardware.

6. ACKNOWLEDGEMENTS

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