25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COBEM2019-0684
INTERVAL UNCERTAINTY AND SENSITIVITY ANALYSES DEDICATED TO THE NEAR-FIELD ACOUSTIC LEVITATION FORCE

Geisa Arruda Zuffi[1]
geisazuffi@ufu.br

Fabian Andres Lara Molina
Department of Mechanical Engineering, Federal University technology - Paraná, Av. Alberto Carazzai, 1640, Cornélio Procópio, PR, 86300-000, Brazil.
lara.f8@gmail.com

Fran Sérgio Lobato[1]
Aldemir Ap. Cavalini Jr.[1]
Valder Steffen Jr.[1]
[1]Laboratory of Mechanics and Structures, School of Mechanical Engineering, Federal University of Uberlândia, Av. João Naves de Ávila, 2121, Uberlândia, MG, 38408-196, Brazil.
aacjunior@ufu.br, vsteffen@ufu.br.

Abstract. *This work aims to analyze the effect of uncertain parameters on the levitation force obtained using a mechanical system subjected to the phenomenon of near-field acoustic levitation. The system consists of two discs, namely the driving surface and the object to be levitated. In this case, the object to be levitated is fixed, that is, the gap between the discs varies only according to the vibration amplitude of the driving surface. The pressure distribution generated between the discs is described by the Reynolds equation. The uncertainties are introduced as variations in the environmental conditions and in the actuation system operational parameters. Thus, the atmospheric pressure, fluid viscosity, initial height of the object to be levitated, amplitude of vibration of the driving surface, operational frequency, and the size of both discs are considered as uncertain information. In the present contribution, the levitation force was obtained from the solution of the Reynolds equation associated with the so-called interval uncertainty method. A sensitivity analysis was also performed to determine the uncertain parameters which most affects the levitation capability of the levitation system.*

Keywords: *near-field acoustic levitation, Reynolds equation, interval uncertainties, force capacity.*

1. INTRODUCTION

The near-field acoustic levitation, also known as ultrasonic levitation, occurs when an object is positioned next to a vibrating surface (driving surface), excited by an ultrasonic frequency. This way, the air between the object and the driving surface is pressurized, creating a pressure field in the gap whose resultant force balance the weight of the object enabling its levitation (Ilssar and Bucher, 2015).

This technique does not offer any restrictions to the size of the object or its composition and can guarantees the non-contact between the object to be levitated and the driving surface during all the process. This characteristic is an interesting alternative to rotating machines that operate in high speeds, as well as in handling, storage, and transportation of components and substances that cannot be contaminated by handle contact, such as silicon wafers (Ilssar *et al.*, 2017). However, the near-field acoustic levitation approach has restricted applicability when high load capacity is required.

Several applications have successfully used the near-field acoustic levitation approach. A journal bearing was constructed with three piezoelectric transducers mounted in a circle with 120 degree among each other, promoting the near-field acoustic supporting force of 5.6 N (Stolarski *et al.*, 2011). Axial and radial loads of 6 and 15 N, respectively, were obtained by using the same system by Li *et al.* (2016). Meanwhile, a radial maximum force of 51 N was generated by Zhao (2010). As an alternative device, a bearing system based on the deformation of the journal operating with six piezoelectric transducers was proposed by Wang and Au (2012). The piezoelectric transducers were coupled in pairs, acting longitudinally. In this case, a maximum levitation force of 3.5 N was obtained approximately.

Large-amplitude and high frequency oscillations are required to promote the ultrasonic levitation. It is usually achieved by using ultrasonic actuators operating at resonance condition. Nevertheless, these ultrasonic actuators have low damping ratios meaning that slight deviations from resonance, for which they were projected, will reduce their efficiency substan-

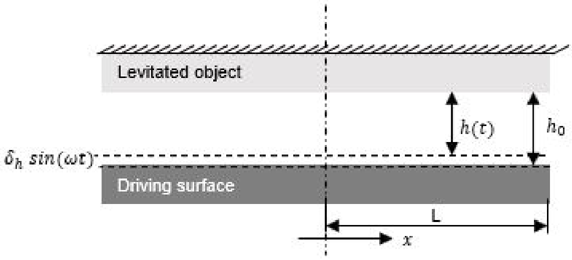

Figure 1. Near-field acoustic levitation system.

tially (Ilssar and Bucher, 2017). Consequently, operational parameters of the ultrasonic actuator, as resonance frequency (ω) and amplitude vibration (δ_h), can suffer small variations around their nominal values.

Moreover, variations in fluid proprieties (μ) have a direct effect on the load capacity of the technique. Environmental conditions, as the initial pressure in the gap (P_0), can also modify the performance of the system. Likewise, the initial distance between the driven surface and the object to be levitated (h_0) and due to manufacturing, such as the discs sizes (L), may also provoke variations in the levitation force. Consequently, it is necessary to quantify the influence of variations in the geometric, environmental, and actuation parameters on the force generated by the near-field acoustic levitation. In this context, interval uncertainty and sensitivity analyses will be applied to the mentioned parameters, regarding force capacity.

In this context, the application of uncertainty and sensitivity analyses concerning the mentioned parameters is interesting. The uncertain parameters analyzed in this paper are the amplitude of vibration of the driving surface δ_h, the frequency of operation ω, related to the ultrasonic actuator, the initial distance between the object to be levitated and the driving surface h_0 (initial gap), the parameters related to the environmental conditions, such as the initial pressure p_0 and viscosity μ of the air trapped in the gap, and the size of the discs L. It is worth mentioning that the interval uncertainty and sensibility analyses were used in the present work, as proposed by Moore, Kearfott, and Cloud Moore *et al.* (2009) and Moens and Vandepitte Moens and Vandepitte (2006b), respectively. These techniques allow to compute numerically the uncertain envelopes and the sensitivity of the interval uncertainty parameters on the behavior of the system.

2. MATHEMATICAL MODEL

The mathematical model that represents the phenomenon of near-field acoustic levitation is described assuming a mechanical system composed of two discs with size L. Both discs are immersed in air at atmospheric pressure with an initial distance between them of h_0. In the present contribution, the upper disc (object to be levitated) is considered fixed. Additionally, a sinusoidal vibration at ω frequency and δ_h amplitude is applied in the lower disc (driving surface). The considered near-field acoustic levitation system is presented in Fig. 1.

The time variation distance $h(t)$ between the discs is described by Eq. (1).

$$h(t) = h_0 + \delta_h\ \sin(\omega t) \tag{1}$$

The pressure distribution in the air film present between the discs is obtained from the solution of the Reynolds equation in Cartesian coordinates, as given by Eq. 2. In this case, the distance between the surfaces is considered small when compared to other dimensions of the near-field acoustic levitation system and the temperature of the air film is considered as constant (Zhao, 2010).

$$\frac{\partial}{\partial x}\left(ph^3\frac{\partial p}{\partial x}\right) = 12\mu\frac{\partial(ph)}{\partial t} \tag{2}$$

where p represents the pressure distribution along the air film, t is the time, x is a given position along the discs ($0 \leq x \leq L$), h is given by Eq. (1), and μ is the dynamic viscosity of the air.

Equation (2) is solved in its dimensionless form, which was obtained using the relations as presented by Eq. (3).

$$P = \frac{p}{p_0},\ H = \frac{h}{h_0},\ R = \frac{r}{r_0},\ T = \omega t,\ \epsilon = \frac{\delta_h}{h_0},\ \sigma = \frac{12\omega\mu L^2}{p_0 h_0^2} \tag{3}$$

Table 1. Initial and boundary conditions.

Conditions	Values
Initial	$P(X,T=0)=1$
Dirichlet	$P(X=1,T)=1$
Neumann	$\frac{\partial P(X=0,T)}{\partial X}=1$

where σ is called squeeze number and ϵ is the dimensionless vibration amplitude of the driving surface. Replacing Eq. (3) in Eq. (2) and Eq. (1), Eq. (4) and Eq. (5) are determined.

$$H = 1 + \epsilon\, sin(T) \tag{4}$$

$$\frac{\partial}{\partial R}\left(PH^3\frac{\partial P}{\partial R}\right) = \sigma\frac{\partial (PH)}{\partial T} \tag{5}$$

Equation (5) is discretized along the x coordinate, as presented in Fig. 1. Then, Eq. (6) is obtained.

$$\sigma\left(\frac{\partial P}{\partial T}\right) = -\frac{P_i}{H_i}\left(\frac{dH}{dT}\right)_i + \frac{P_iH_i^2}{R}\left(\frac{\partial P}{\partial R}\right)_i + H_i^2\left(\frac{\partial P}{\partial R}\right)_i^2 + P_iH_i^2\left(\frac{\partial^2 P}{\partial r^2}\right)_i \tag{6}$$

where the subscript i indicates the grid coordinate.

Equation (4) is solved considering that the system is symmetric along the x coordinate, the pressure at the edges of the discs is equal to the atmospheric pressure (p_0; Dirichlet boundary condition), and null pressure gradient at the center of the discs ($x = 0$) (Neumann boundary condition). The considered initial and boundary conditions used for the solution of Eq. (5) are presented in Tab. (1).

Thus, the levitation force can be obtained by integrating the pressure field over the driven disc surface area, as shown in Eq. (7).

$$F_l = 2\pi\int_L^0 (p - p_0)dx \tag{7}$$

3. INTERVAL UNCERTAINTY AND SENSITIVITY ANALYSES

This section presents the theory associated with the uncertainty and sensitivity analyses used to investigate the influence of variation in some parameters of the near-field acoustic levitation system. These methods are based on the formulations proposed by Moore *et al.* (2009), Moens and Vandepitte (2007, 2006a,b), and Modares and Mullen (2013).

3.1 Interval uncertainty analysis

The relationship between the input parameters u and the system output y is defined by the numerical model f, as given by Eq. (8).

$$y = f(u) \tag{8}$$

Considering that the parameters can vary within a uncertain interval range u_I, with an upper limit u_u and a lower limit u_l. The interval analysis aims to quantify the influence of u_I in y in terms of an interval y_I, as represented by Eq. (9) and Eq. (13) (Modares and Mullen, 2013).

$$y_I = f(u_I) \tag{9}$$

$$u_I = [u_l,\ u_u] \tag{10}$$

$$y_I = [y_l,\ y_u] \tag{11}$$

In this case, f is the interval function that represents the relationship between u_I and y_I. The interval limits of the output can be obtained using a global optimization method that determines its maximum and minimum values considering u_I as the search space of the uncertain parameter u, as described in Eq. (13).

$$y_l = \min_{u \in u_I} f(u) \tag{12}$$

$$y_u = \max_{u \in u_I} f(u) \tag{13}$$

3.2 Interval sensitivity analysis

The objective of sensitivity analysis is to quantify the individual contribution of each uncertain interval parameter on the uncertain interval responses as obtained by using the numerical model of the system. According to Moens and Vandepitte (2006b), the interval sensitivity analysis is performed based on the radius associated with the input and output ranges as given in Eq. (14). The relationship between them is defined by the function f^{Δ}.

$$\Delta y = \frac{y_u - y_I}{2} \tag{14}$$

$$\Delta u = \frac{u_u - u_I}{2} \tag{15}$$

$$\Delta y = f^{\Delta}(\Delta u) \tag{16}$$

Thus, the interval sensitivity of an interval output y_I with respect to an interval input u_I is determined from Eq. (17), when only one uncertain parameter is being considered. For more than one uncertain parameter, Eq. (18) is used.

$$\delta_{u_I}^{y_I} = \frac{\partial(\Delta y)}{\partial(\Delta u)} = \frac{\partial f^{\Delta}(\Delta u)}{\partial(\Delta u)} \tag{17}$$

$$\rho_{u_{I\,i}}^{y_I} = \frac{\partial\left(\frac{\Delta y}{\Delta y^*}\right)}{\partial\left(\frac{\Delta u_i}{\Delta u_i^*}\right)} = \frac{\Delta u_i^*}{\Delta y^*} \times \delta_{u_{I\,i}}^{y_I} \tag{18}$$

where Δy^* and Δu_i^* are interval nominal widths.

The relative normalized interval sensitivity is defined as given by Eq. (19). It represents the relative contribution of different input intervals to the total interval sensitivity of the defined problem. The index $v_{u_{I\,i}}^{y_I}$ permits to compare the contribution of each interval uncertain parameter on the interval output of the system model.

$$v_{u_{I\,i}}^{y_I} = \frac{\rho_{u_{I\,i}}^{y_I}}{\sum_{i=1}^{n} \rho_{u_{I\,i}}^{u_I}} \tag{19}$$

4. Numerical results

In order to obtain maximum and minimum values of the levitation force, the Differential Evolution optimization technique was applied (Storn and Price, 1997). In this case, six uncertain variables were considered, namely: δ_h, h_0, p_0, μ, L, and ω. Equation (7) was adopted as the objective function. Table 2 shows nominal values of the uncertain parameters, as well as the associated lower and upper limits. It is worth mentioning that variations of $\pm 15\%$ were applied to the frequency (ω) and $\pm 30\%$ to the other uncertain parameters.

Table 2. Uncertain parameters.

Parameter	Unit	Nominal value	Lower limit u_l	Upper limit u_u
δ_h	m	5×10^{-6}	3.5×10^{-6}	6.5×10^{-6}
h_0	m	50×10^{-6}	35×10^{-6}	5×10^{-6}
p_0	Pa	101325	70928	131720
μ	Pas	1.8253×10^{-5}	1.2777×10^{-5}	2.3729×10^{-5}
L	m	0.03	0.021	0.039
ω	Hz	20000	10681	14451

Regarding the uncertainties analysis, two different scenarios are being simulated. In the first one, all parameters are considered as uncertain information at once. Each uncertain parameter is considered separately in the second case, enabling to evaluate individually its influence on the levitation force.

Figure 2 shows the behavior of the nominal levitation force along the time (3.2×10^{-4} sec of simulation). The uncertain lower and upper limits as determined using the interval analysis also presented. In this case, variations on δ_h, h_0, p_0, μ, L, and ω are being considered. Note that a maximum levitation force of 33.52 N was achieved at 2×10^{-4} sec. Concerning the nominal values of the uncertain parameters, the maximum levitation force was 6.4 N. Thus, small variations applied in the considered uncertain parameters can increase the levitation force 5 times, approximately.

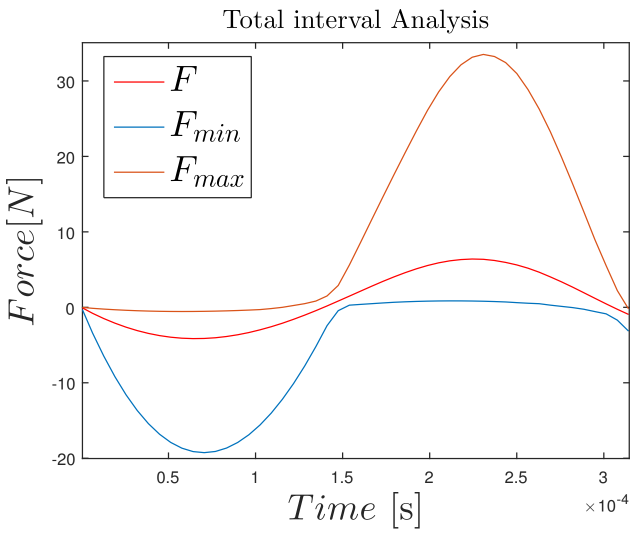

Figure 2. Interval analysis of the system.

Aiming to evaluate the individual effect of each parameter, Figs. 3 to 8 present the levitation force obtained considering δ_h, h_0, p_0, μ, L, and ω as uncertain information, separately. Fig. 3, shows the behavior for the variation of δ_h. The maximum force in this case is 8.73 N, and the minimum is 4.23 N. For h_0 varying, Fig. 4, the maximum obtained is 10.55 N versus a minimum of 4.35 N.

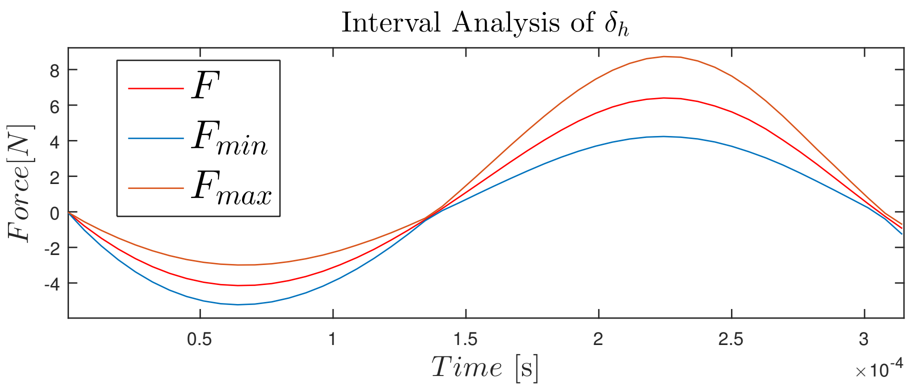

Figure 3. Interval analysis of the system with δ_h varying.

Considering variations in the environmental parameters, Fig. 5 and Fig. 6, there is a maximum of 8.04 N, related to P_0, and 6.6 N to μ, against a minimum of 4.35 N and 6.04 N, respectively.

Inserting uncertainties in the measurement of the size of the discs L, Fig.7, the maximum of 11.48 N can be reached followed by the minimum of 2.75 N. On the other hand, varying ω, increases the force only to $6.51N$, being the minimal 6.27 N, as showed in the Fig. 8.

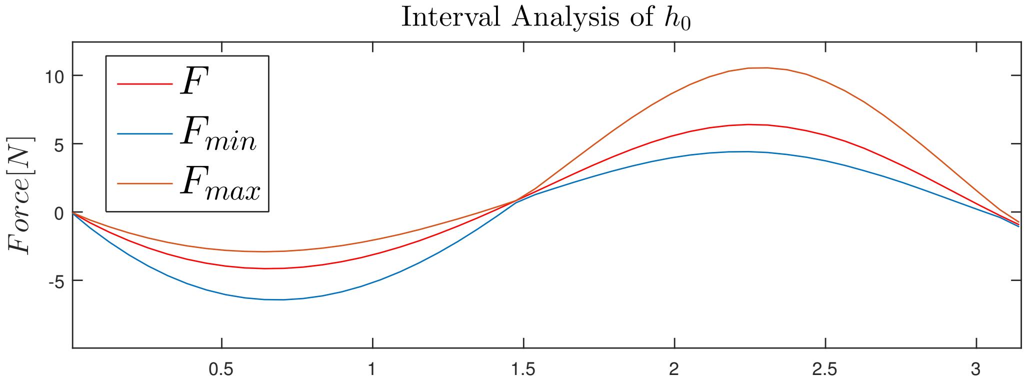

Figure 4. Interval analysis of the system with h_0 varying.

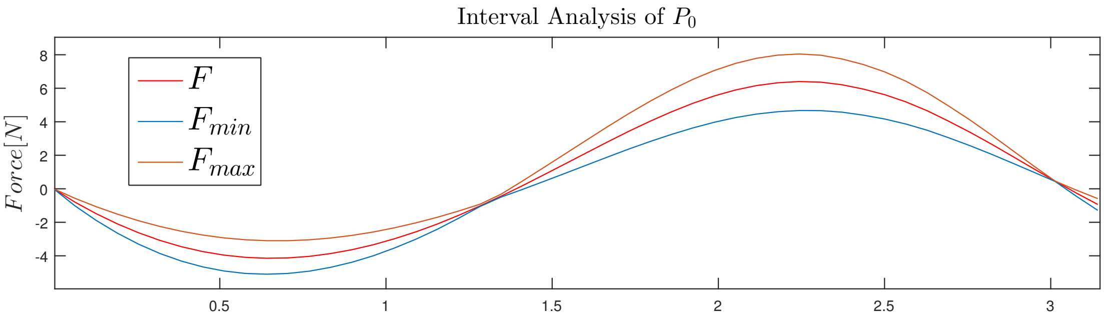

Figure 5. Interval analysis of the system with P_0 varying.

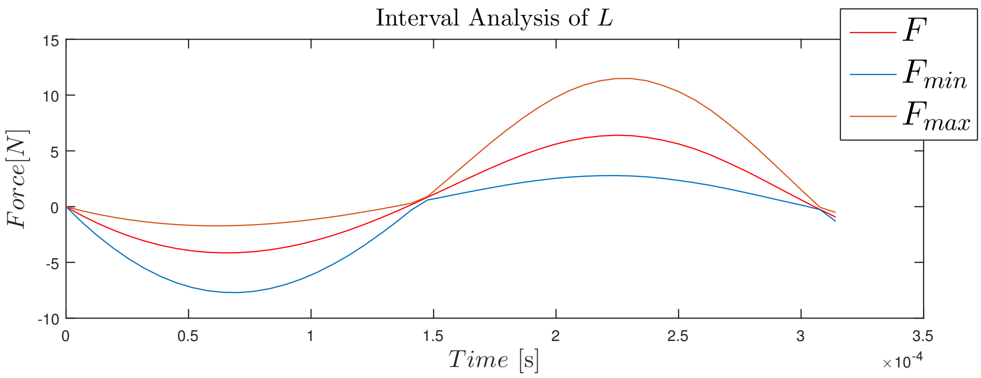

Figure 7. Interval analysis of the system with L varying.

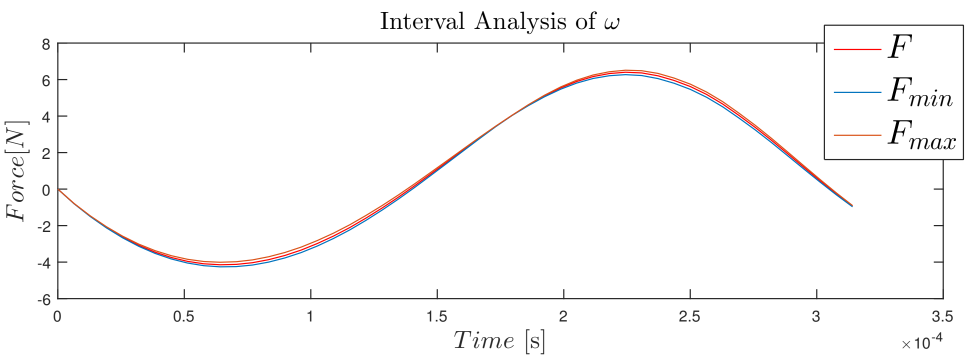

Figure 8. Interval analysis of the system with ω varying.

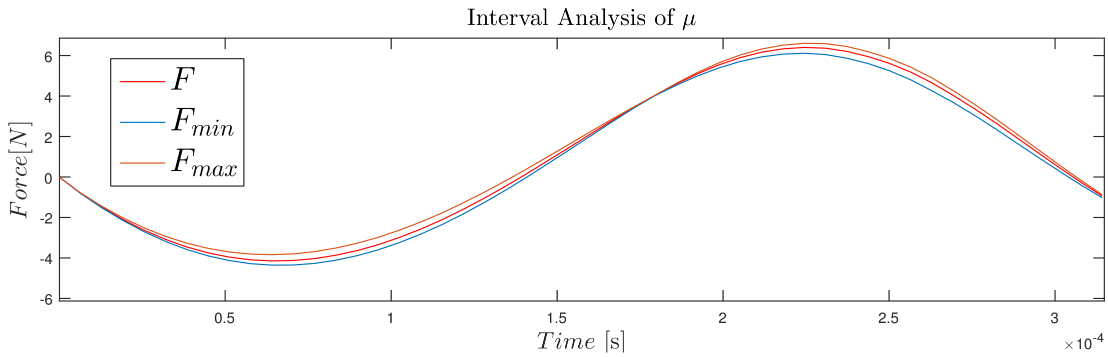

Figure 6. Interval analysis of the system with μ varying.

4.1 Interval Sensitivities Analysis

Using equations 19, 18 and 17, it is possible obtain the Normalized Relative Interval Sensitivity of parameters analyzed along a simulation time of 3.1×10^{-4} s, as shown in Fig. 9. The system is more sensitivity to variations firstly in the environment pressure P_0, reaching a maximum of 0.73, secondly to operational frequency ω, with 0.45, thirdly to variations in the fluid viscosity μ, 0.4, fourthly to the amplitude of vibration δ_h of the driving surface, getting around to 0.35, fifthly to the initial gap h_0, with 0.31 and at last to variations in the size of the discs L, with 0.26. The sensibility analysis , the influence of the variation in the interval range of the parameters in the force are computed for each uncertain parameter acting at a time.

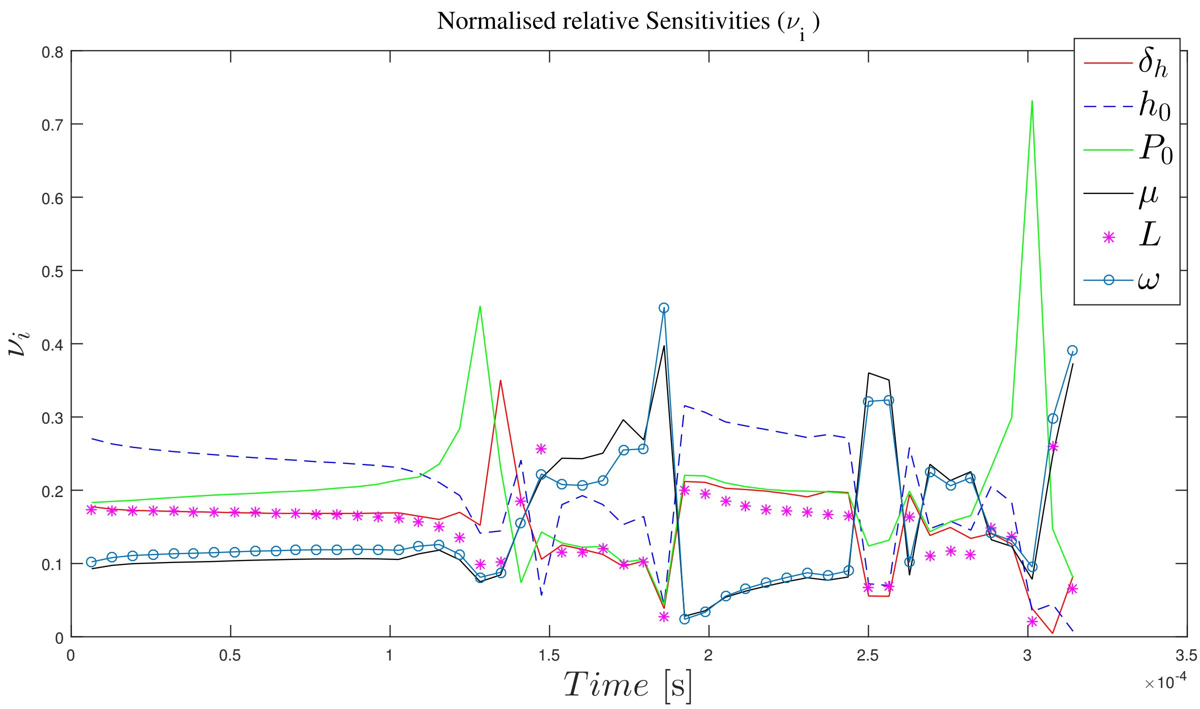

Figure 9. Normalised relative interval sensitivities of the system.

5. CONCLUSION

The interval uncertainty approach presented in this contribution permitted to evaluate the influence of the uncertain parameters of a near-field acoustic levitation system in the generated levitation forces. The uncertainties are produced by environment conditions variations, sensors errors and operational system variations (ultrasonic transducer + driving surface). As evaluated numerically, the variation on these parameters affects the force capacity of the system. It was possible to determine that the force capacity is more sensitive to uncertainties in the environmental conditions (p_0 and μ) and in the operational frequency (ω). Additionally, the interval uncertainty analysis reveled that the maximum levitation force varies significantly with changes on the considered uncertain parameters. The information obtained from the uncertainty analysis and the sensitivity analysis is useful and necessary to investigate which parameters can be altered to improve the force capacity of the near-field levitation. Future work will be dedicated to the construction of experimental tests.

6. ACKNOWLEDGEMENTS

The authors are thankful for the financial support provided to the present research effort by CNPq (574001/2008-5, 304546/2018-8, and 431337/2018-7), FAPEMIG (TEC-APQ-3076-09, TEC-APQ-02284-15, TEC-APQ-00464-16, and PPM-00187-18), and CAPES through the INCT-EIE.

7. REFERENCES

Ilssar, D. and Bucher, I., 2017. “The effect of acoustically levitated objects on the dynamics of ultrasonic actuators”. *Journal of Applied Physics*, Vol. 121, No. 11, p. 114504.

Ilssar, D. and Bucher, I., 2015. “On the slow dynamics of near-field acoustically levitated objects under high excitation frequencies”. *Journal of Sound and Vibration*, Vol. 354, pp. 154–166.

Ilssar, D., Bucher, I. and Flashner, H., 2017. “Modeling and closed loop control of near-field acoustically levitated objects”. *Mechanical Systems and Signal Processing*, Vol. 85, pp. 367–381.

Li, H., Quan, Q., Deng, Z., Hua, Y., Wang, Y. and Bai, D., 2016. “A novel noncontact ultrasonic levitating bearing excited by piezoelectric ceramics”. *Applied Sciences*, Vol. 6, No. 10, p. 280.

Modares, M. and Mullen, R.L., 2013. “Dynamic analysis of structures with interval uncertainty”. *Journal of Engineering Mechanics*, Vol. 140, No. 4, p. 04013011.

Moens, D. and Vandepitte, D., 2006a. “Interval sensitivity analysis of dynamic response envelopes for uncertain mechanical structures”. In *III European Conference on Computational Mechanics*. Springer, pp. 385–385.

Moens, D. and Vandepitte, D., 2006b. “Sensitivity analysis of frequency response function envelopes of mechanical structures with interval uncertainties”. In *Proceedings of ISMA2006: International Conference on Noise and Vibration Engineering, Vols 1-8*. KATHOLIEKE UNIV LEUVEN, DEPT WERKTUIGKUNDE, Vol. 7, pp. 4197–4212.

Moens, D. and Vandepitte, D., 2007. “Interval sensitivity theory and its application to frequency response envelope analysis of uncertain structures”. *Computer methods in applied mechanics and engineering*, Vol. 196, No. 21-24, pp. 2486–2496.

Moore, R.E., Kearfott, R.B. and Cloud, M.J., 2009. *Introduction to interval analysis*, Vol. 110. Siam.

Stolarski, T., Xue, Y. and Yoshimoto, S., 2011. “Air journal bearing utilizing near-field acoustic levitation stationary shaft case”. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, Vol. 225, No. 3, pp. 120–127.

Storn, R. and Price, K., 1997. “Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces”. *Journal of global optimization*, Vol. 11, No. 4, pp. 341–359.

Wang, C. and Au, Y.J., 2012. “Levitation characteristics of a squeeze-film air journal bearing at its normal modes”. *The International Journal of Advanced Manufacturing Technology*, Vol. 60, No. 1-4, pp. 1–10.

Zhao, S., 2010. *Investigation of non-contact bearing systems based on ultrasonic levitation*. PZH, Produktionstechn. Zentrum.