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PREDICTING LANDING GEAR LIMIT CYCLE OSCILLATIONS BY DESCRIBING FUNCTIONS

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Abstract. *The vibration of landing gear due to the interaction between the tire and road called shimmy is an important topic in engineering mainly because it can cause catastrophic conditions during the aircraft ground operation. The presence of nonlinear forces can lead to marginally stable oscillations, with deterministic behavior in some cases, also defining amplitudes and frequency of motion. To evaluate these characteristics an eigenvalue-based method applied to the quasi-linear system, combined to describing functions quasi-linearization approaches are employed herein to calculate the amplitude and frequency of limit cycle oscillations as function of a parameter of designing. Time domain simulations are carried out to demonstrate how a describing function is an efficient alternative to predict the dynamic of this kind of motion.*

Keywords: *shimmy in landing gears, eigenvalues analysis, describing functions, nonlinear characteristics*

1. INTRODUCTION

Shimmy is a combination of lateral and torsional vibration which occurs in motorcycles, automobiles and landing gear of aircraft. This phenomenon is potentially dangerous during operation because the pilot can loss the vehicle directional control and affect the performance and life span of the mechanical components. The rolling motion of the wheels distorts the tire contact patch, due to the rubber elasticity, a lateral force and a moment appears that under certain conditions leads to undesirable self-excited vibrations. This problem is often discovered during the later stages of development of an aircraft when is difficult to change drastically the designed parameters. The study of this phenomenon during the early stages can help the designer reduce the cost of changes by predicting beforehand the dynamical behavior of the system.

The shimmy phenomenon is treated using both linear and nonlinear techniques. The linear methods like the linearization presented in the introdutory content in Besselink (2000) is used to characterize the overall stability of the system, where one is not concerned about the magnitude of the vibration and only whether the system is stable or unstable. Using different nonlinear techniques like bifurcation theory in Terkovics *et al.* (2014) and Ran *et al.* (2014) or the numerical integration of the differential equations like Takács and Stépán (2009) use, one can analyze the influence of different design parameters in the magnitude of vibration and establish tolerable levels of vibration before maintenance takes place.

The interest of this work is to predict the shimmy amplitude and frequency associated to different design parameters of a nonlinear single degree of freedom landing gear shimmy model. The tire forces in the contact patch are assumed as nonlinear functions of the slip angle and the sinusoidal input describing function (SIDF) method is used to give a *quasi*-linear description of the initially nonlinear system. Using the eigenvalue method proposed by Somieski (2001). It is possible to characterize the amplitude and frequency of limit cycle oscillations (LCO) that occur during the shimmy event. Although time domain simulations can be applied to verify the LCO phenomenon, an alternative strategy involving describing function based-approach is employed and results demonstrate that it is a suitable choice to aid the design of these systems, mainly in early stages of designing.

2. LANDING GEAR MODEL

Figure 1 shows a single degree of freedom shimmy model. The model consists of a king-pin moving with velocity V. An arm of length e rotates around the king pin vertical axis with linear stiffness k_ψ and viscous damping c_ψ. At the end of the arm, a wheel of mass m and inertia I_z is attached. Thus, the rotational inertia at the king-pin is $I_t = I_z + me^2$. The degree of freedom ψ is the angle between the arm and the velocity. The cornering forces, $F_y(\alpha)$ and $M_z(\alpha)$, originated in the contact patch are shown. The brackets express the function relation between the forces and the tire variable α, which is called "slip angle".

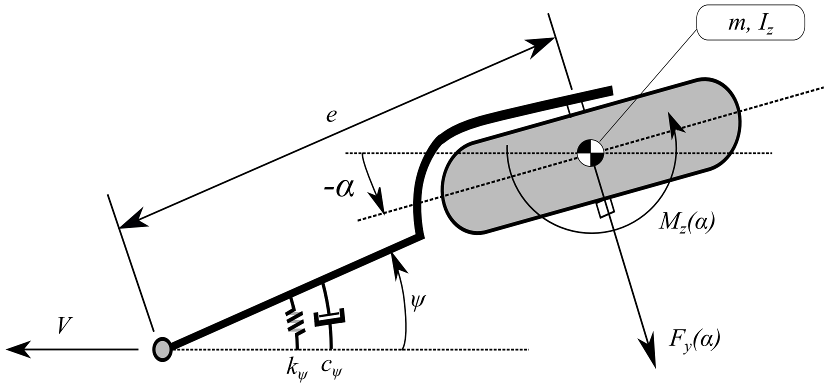


Figure 1: One degree of freedom landing gear model.

2.1 Equations

Equation (1) is the differential equation of motion from the model in Fig. (1). Somieski (2001) uses Eq. (2) and Eq. (3) to model the tire cornering forces. These are nonlinear functions that models the resultant of the distributed forces in the tire contact patch.

$$I_t\ddot{\psi}(t) + c_\psi\dot{\psi}(t) + k_\psi\psi(t) = M_z(\alpha) - eF_y(\alpha) \tag{1}$$

$$F_y(\alpha) = \left\{ \begin{array}{cc} F_zC_{f\alpha}\alpha & |\alpha| < \alpha_c \\ F_zC_{f\alpha}\alpha_c sign(\alpha) & |\alpha| \geq \alpha_c \end{array} \right\} \tag{2}$$

where F_z is the vertical force in the landing gear, $C_{f\alpha}$ is the lateral force stiffness coefficient, $sign(x)$ is the signal function.

$$M_z(\alpha) = \left\{ \begin{array}{cc} F_zC_{m\alpha}\dfrac{\alpha_g}{\pi}sen\left(\dfrac{\pi\alpha}{\alpha_g}\right) & |\alpha| < \alpha_a \\ 0 & |\alpha| \geq \alpha_a \end{array} \right\} \tag{3}$$

with $C_{m\alpha}$ the aligning moment stiffness coefficient and α_g a saturation parameter. These can be estimated from the tire experimental characteristics. To represent the dynamic of the contact patch, the straight-tangent tire model equation is used as shown in Eq. (4).

$$\sigma\dot{\alpha}(t) + V\alpha(t) = (e-a)\,\dot{\psi}(t) + V\psi(t) \tag{4}$$

3. DESCRIBING FUNCTIONS

The sinusoidal input describing function (SIDF) is a *quasi*-linear representation of a nonlinear element with sinusoidal input, used for the determination of the amplitude and frequency of the so called LCO (Limit Cycle Oscillations). It is a form of transfer function with amplitude dependence. This information is only obtained using nonlinear techniques and are helpful to determine quantitative values of shimmy characteristics. For example, one can use to establish tolerable levels of shimmy before maintenance of components.

The SIDF from a nonlinear function y of the variable x can be determined solving the following equations:

$$N(A,\omega) = n_p + jn_q \tag{5}$$

where $j = \sqrt{-1}$ and

$$n_p = \frac{1}{\pi A}\int_0^{2\pi} y(Asin\theta, A\omega cos\theta)sin\theta d\theta \tag{6}$$

$$n_q = \frac{1}{\pi A} \int_0^{2\pi} y(Asin\theta, A\omega cos\theta) cos\theta d\theta \tag{7}$$

where θ is the phase angle which is taken as a random variable with uniform distribution over 2π radians according to Gelb and Vander Velde (1968).

Simple functions like the saturation function, used to describe the tire lateral force, can be solved analytically. As the functions involve more terms or complicated relations of variables, the integral becomes difficult to be evaluated. This is the case of the tire aligning moment approximation, which is a complicated function to evaluate analytically and does not worth the effort, since it is a simplification of the tire cornering data characteristics. Thus, from this considerations, the numerical approach is required to obtain the SIDF of the complex nonlinear functions.

3.1 Numerical quadrature

To obtain the numerical value of the SIDF, the method of Newton-Coates using the Simpson rule is used. The interval $[a, b]$ of integration is divided into N evenly spaced intervals of length $h = (b - a)/N$. The integral I is the sum of the area $I_{j \to j+2h}$ of each interval $[j, j + 2h]$ calculated using the Simpson rule as show in Eq. (8). The sum of all intervals can also be expressed as Eq. (9).

$$I_{j \to j+2h} = \frac{h}{3} [f(j) + 4f(j + h) = f(j + 2h)] \tag{8}$$

$$I = \int_a^b f(x)dx \approx \frac{h}{3} \left[f(a) + 4 \sum_{j=1}^{N} f(a + (2j - 1)h) + 2 \sum_{j=1}^{N-1} f(a + (2j)h) + f(b) \right] \tag{9}$$

Once the value of I is numerically calculated, the number of intervals N is multiplied by 2 and a new value of I_{new} is computed. This procedure is repeated until the convergence criteria to be satisfied. For this work, the criteria is $|I_{new} - I|$ between the consecutive iterations lower than 10^{-1}. Using this procedure, it is possible to obtain the numerical value of the SIDF from the complicated nonlinear functions which are present in the shimmy model of landing gears.

3.2 Numerical calculation of the SIDF of the nonlinear elements

Applying the numerical quadrature to obtain the describing function from the nonlinear functions in Eq. (2) and Eq. (3), it is possible to obtain the values for the describing functions for multiple values of amplitude. Figure 2 shows the real part of the lateral force SIDF as function of the amplitude of vibration. The blue line plots the exact value of the describing function, since it has analytical solution as shown in Gelb and Vander Velde (1968). It is possible to see that both methods have good agreement. The aligning moment SIDF is shown in Fig. 3. Since the exact value of the real part is not know, only the numerical value is plotted in Fig. 3. The SIDF imaginary part for both nonlinear functions is zero since the function is odd.

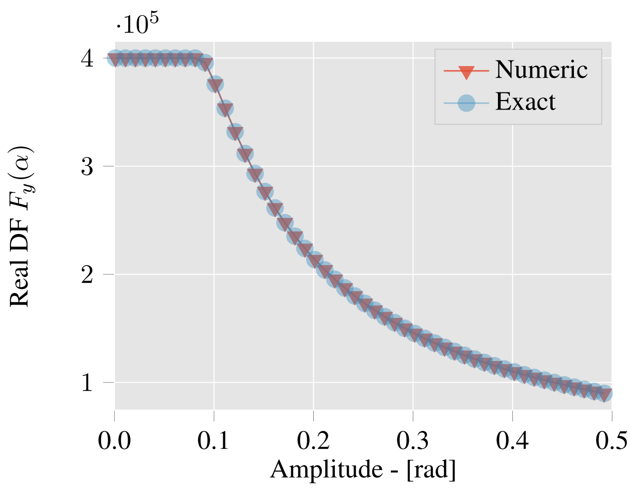


Figure 2: Real part of the lateral force SIDF.

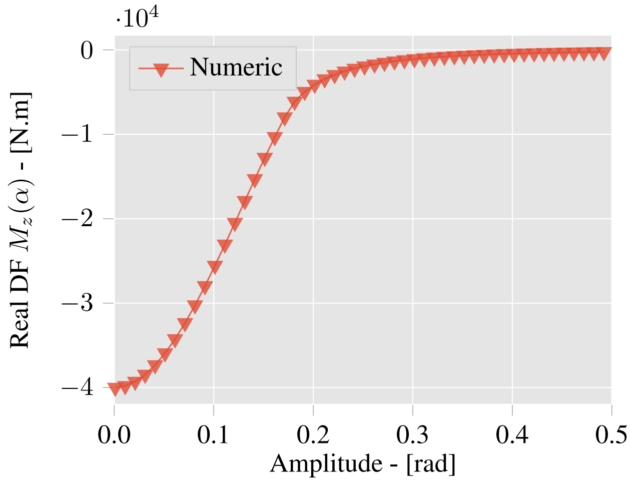


Figure 3: Real part of the aligning moment SIDF.

4. EIGENVALUE METHOD

According to Somieski (2001), a nonlinear system can be separated into a linear and nonlinear part according to Eq. (10).

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) \tag{10}$$

where $\mathbf{x}$ is an n-state vector, $\dot{\mathbf{x}}$ is the time derivative vector, $\mathbf{F}$ is the $n \times n$ system matrix, $\mathbf{f}$ is an $n \times n$ matrix with the nonlinearities. Using the SIDF, the nonlinearities vector can be replaced according to Eq. (11).

$$\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{N}(\mathbf{A}, \omega)\mathbf{x} \tag{11}$$

$\mathbf{N}(\mathbf{A}, \omega)$ is a matrix that contains the sinusoidal input describing function of the system nonlinearities. For the tyre nonlinearities, this matrice is real valued and depends only on amplitude vector $\mathbf{A}$. The stability of the *quasi*-linear system can be determined by solving Eq. (12) for the eigenvalues. The states variable are stable if all the eigenvalues are positive and unstable in the case of at least one eigenvalue negative.

$$(s\mathbf{I} - (\mathbf{F} + \mathbf{N}(\mathbf{A})))\mathbf{x} = \mathbf{0} \tag{12}$$

In the case of the instability, the vibration increases and the *quasi*-linear system dynamics change and, consequently, the eigenvalues too, since the nonlinearities SIDF are function of the amplitude of vibration. In the case of decreasing eigenvalue with amplitude, the system dynamic characteristics may change from unstable to marginally stable with amplitude and frequency of vibration correspondent to the system with null real part eigenvalue.

4.1 Time response and spectral analysis

To compare the result given from the describing function analysis, the parameters e and V were arbitrary chosen in a region that exhibits limit cycle oscillations. To evaluate the time response, the Runge-Kutta Fehlberg (RKF) method was implemented. One advantage of this method is the variable time step that is changed depending on the error estimation and the accuracy, this considerations leads to a reduced number of computations to obtain the desired time response. This advantage is a disadvantage for the signal FFT algorithms, that requires an fixed time step to calculate the frequency contents of the temporal signal.

4.1.1 Numerical integration of a system of differential equations

To solve the system of differential equations, a numerical method was implemented to solve for the state variables as function of the time. The method was a time adaptive Runge-Kutta. This methods control the time step used for the numerical computations from the error estimation compared with a given tolerance. This error is calculated from the difference between the states values given from the 4th and 5th order Runge-Kutta.

4.1.2 Spectral analysis

To compare the quantitative aspects of the methods used in the work, the steady state information of amplitude and frequency is desired. A time interval where the amplitude response is constant is select with their amplitude information.

Due to the variable step method used for the numerical integration, a new data set is created with a constant time step and the amplitude vector is linearly interpolated to send the data to the FFT specialized algorithms.

5. RESULTS AND DISCUSSIONS

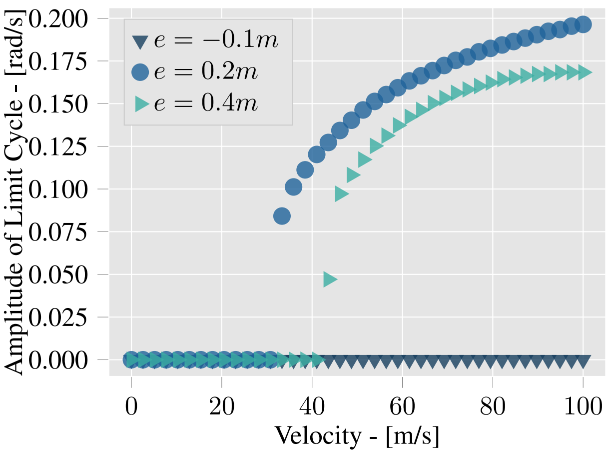
From the nature of the aircraft mission, the velocity varies during the ground operations while others parameters in the model are in a reasonably manner fixed. Given a set of design parameters over a velocity range, the linearized system is stable or unstable. In case of an unstable system, with what amplitude and frequency it becomes marginally stable, *i.e.*, the ocurrence of shimmy. With the real part value from the eigenvalues of the linearized system, is possible to answer the first question. If the value is negative, then the solution is stable and the state variables of the system goes to the equilibrium state. For the positive value, the system is unstable and the amplitude of vibration grows until the eigenvalues of the *quasi*-linear system are marginally stable. At this condition, the amplitude of the marginally stable system is the shimmy vibration amplitude.

The numerical simulation of the system requires the determination of the model parameters. Those are arbitrary defined in the Table 1 to occur shimmy in a specific region for analysis.

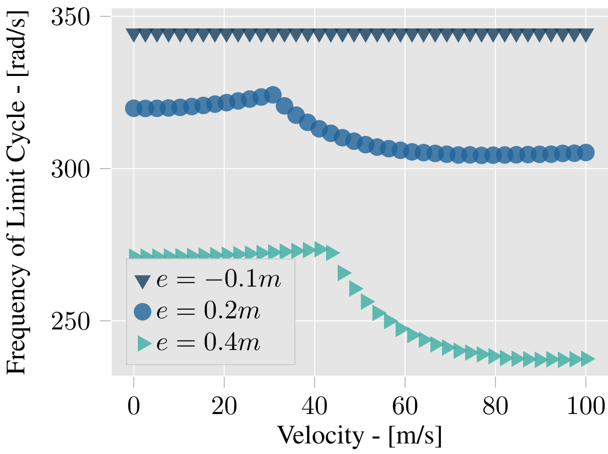
Table 1: Geometrial and physical parameters used for the calculations.

Parameter	Value	Dimension
V	$[0-100]$	$m.s^{-1}$
e	$[0; 0.4]$	m
F_z	20000	N
k_ψ	100000	$N.rad^{-1}$
c_ψ	50	$N.s.rad^{-1}$
$C_{f\alpha}$	20	$-$
$C_{m\alpha}$	-2	$-$
m	17.65	kg
I_z	0.66	$kg.m^2$
a	0.05	m
σ	0.448	m
α_g	0.174	rad
α_c	0.087	rad

Applying the discussed eigenvalue method for the shimmy dynamical model with the nonlinear tyre forces, the amplitudes and frequencies of oscillations is shown in Figs. (4a) and (4b) respectively for different values of arm length and variable velocity, which represent different landing gear configuration operating under the ground envelope of the aircraft. For the arm length value of $e = -0.1m$, the system is stable over the selected velocity range, while for values of $e = 0.2$ and $e = 0.4$ the system is marginally stable with vibration amplitudes varying with velocity. Frequency of oscillation decays with augmenting amplitudes.



(a)



(b)

Figure 4: Limit cycle oscillations characteristics as function of the velocity for three different values of arm length e. (a) indicates the amplitude and (b) the frequency of oscillation.

The numerical integration is realized with fixed landing gear configuration for $e = 0.2m$, $V = 100m/s$ and the parameters presentedv in Table (1). Figure (5) shows the variable state $\alpha(t)$ from the system of Eq.(1-4). The state is initially unstable and grows until it becomes marginally stable with bounded amplitude. A time window where the response is steady was selected and is represented with the dashed vertical lines, Fig. (6a) shows the data within the time window. After the integration, the state vector were interpolated to obtain a new state vector with constant time interval between points. This is necessary due to the standard FFT algorithms libraries.

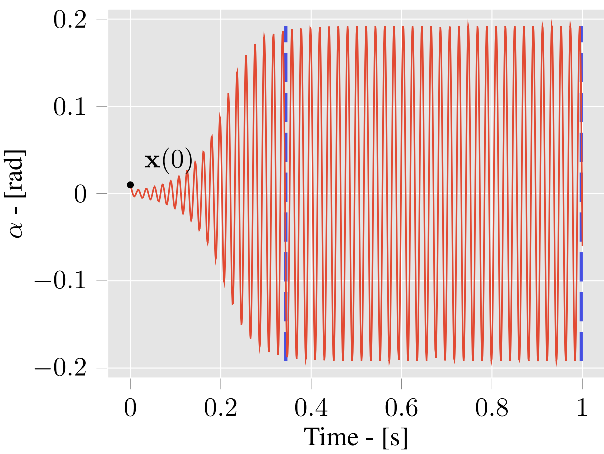


Figure 5: Numerical integration using RKF method for $e = 0.2m$ and $V = 100m/s$ with initial conditions: $\mathbf{x}(0) = [0; 0; 0.01]^T$.

Figure (6a) shows the state variable within the time window and Fig. (6b) the Fourier transform of this state to obtain the frequency content. The amplitude of vibration is determined from the maximum value of the state within the time frame and the frequency from the spike marked with a circle in Fig. (6b), which is the principal frequency of the signal.

The obtained values between the numerical integration method and using the eigenvalue method with *quasi*-linearization are presented in Table (2). The numerical integration of system equations does not take into account the describing function of the nonlinearities, since the values are similar, both methods are useful to obtain information of the steady shimmy characteristics of each landing gear configuration.

Table 2: Amplitude and frequency of the system obtained from different methods

Method	Amplitude $[rad]$	Frequency $[rad/s]$
Numerical Integration	0.192	308.0
Eigenvalue Method	0.196	305.3

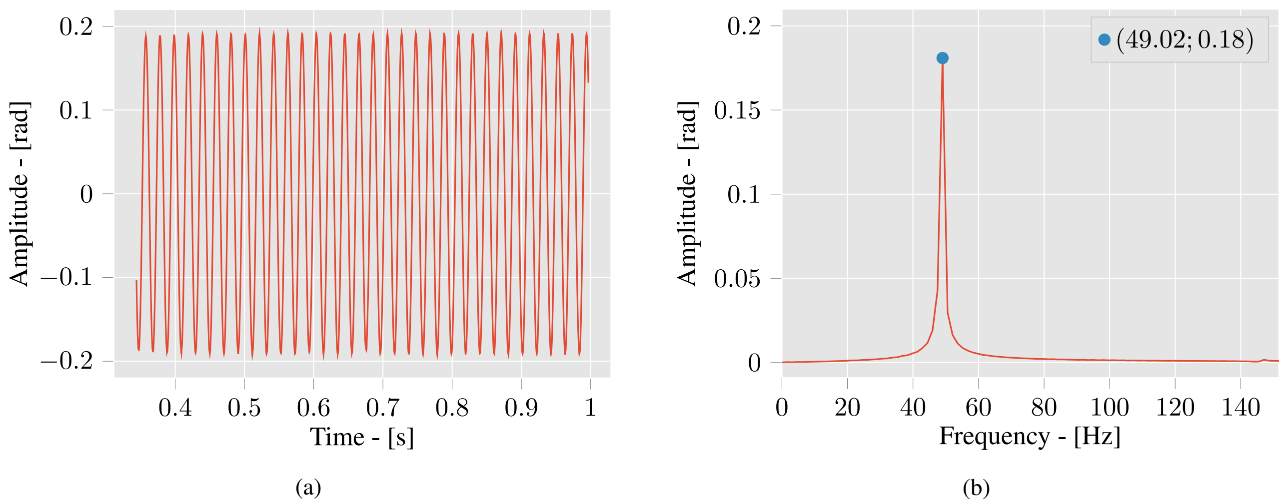


Figure 6: Time response and the frequency content of the variable state $\alpha(t)$.

The hypothesis of filtering of the system as mentioned by Gelb and Vander Velde (1968) in the describing function calculation does not create a relevant difference in the results show by the numerical integration. While the numerical integration analysis need to be realized for each landing gear configuration, the eigenvalue method provide way to broadly visualize the dynamic of the system as function of a design parameter, permitting the analysis of different landing gear configurations.

6. FINAL REMARKS

Using the describing function and the eigenvalue method, it is possible to know the characteristics of motion as amplitude and frequency of a landing gear shimmy vibration. The numerical simulations of the landing gear in a wide variety of physical and geometrical properties allow to carefully specify physical and geometrical parameters during the design to avoid dangerous shimmy oscillations. The studied method can also be applied for multi degrees of freedom system, and because of this it can easily be employed to evaluate more complex mechanical landing gears, also including additional nonlinear elements or a nonlinear tire model.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

Besselink, I., 2000. *Shimmy of Aircraft Main Landing Gears*. Ph.D. thesis, Delft University of Technology.

Gelb, A. and Vander Velde, W.E., 1968. *Multiple-input describing functions and nonlinear system design*. McGraw-Hill.

Ran, S., Besselink, I. and Nijmeijer, H., 2014. "Application of nonlinear tyre models to analyse shimmy". *Vehicle System Dynamics*, Vol. 52, No. sup1, pp. 387–404. doi:10.1080/00423114.2014.901542. URL `https://doi.org/10.1080/00423114.2014.901542`.

Somieski, G., 2001. "An eigenvalue method for calculation of stability and limit cycles in nonlinear systems". *Nonlinear Dynamics*, Vol. 26, No. 1, pp. 3–22. ISSN 1573-269X. doi:10.1023/A:1017384211491.

Takács, D. and Stépán, G., 2009. "Experiments on quasiperiodic wheel shimmy". *Journal of Computational and Nonlinear Dynamics*, Vol. 4, No. 3, pp. 031007–031007–7. ISSN 1555-1415. doi:10.1115/1.3124786. URL `http://dx.doi.org/10.1115/1.3124786`.

Terkovics, N., Neild, S., Lowenberg, M. and Krauskopf, B., 2014. "Bifurcation analysis of a coupled nose-landing-gear-fuselage system". *Journal of Aircraft*, Vol. 51, No. 1, pp. 259–272. ISSN 0021-8669. doi:10.2514/1.c032324. URL `https://doi.org/10.2514/1.C032324`.