NUMERICAL SIMULATION OF ROLL WAVES IN NEWTONIAN FLUID FLOWS

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Abstract. *This work aims to simulate roll waves in turbulent Newtonian flows with a free surface, using the $k-\varepsilon$ and $k-\omega$ standard turbulence models available in commercial Ansys Fluent software. Brock's classic experiment is used as validation. From these simulations, the kinematic properties of the stabilized roll waves can be determined. These simulations using complete Navier-Stokes equations allowed to visualize and analyze properties necessary for engineering projects, such as the velocity fields and shear stress at the bottom of the channel in the presence of such waves, this being the principal contribution of the present paper. The results show waves with the typical profiles of shock waves, with maximum velocities and shear stress at the bottom of the channel in the shock region. For the studied case of the roll waves profile, the $k-\omega$ turbulence model provided results closer to Brock's experiment.*

Keywords: *roll waves, numerical simulation, turbulence*

1. INTRODUCTION

Roll waves are waves that can appeared in shallow water flow (Balmforth and Mander, 2004; Dressler, 1949; Brock 1969), propagating with well-defined velocity, length and amplitude in the form of a wave train. Roll waves may occur in both Newtonian and non-Newtonian flows (Ng and Mei, 1994; Tamburrino and Ihle, 2013, Maciel et al. 2017). In favorable conditions, these waves appear both in free surface flow and in closed flows, such as pipelines (Aydin et al., 2015; Gaspari, 2013). Therefore, the study of roll waves is of industrial and environmental interest.

Although the phenomenon occurs more frequently in artificial channels, roll waves can also be found in natural environments, such as rivers, debris flows, mudflows and avalanches. During natural disasters the occurrence of this type of wave becomes an aggravating factor in both Newtonian and non-Newtonian flows. For Newtonian fluids, such as water, these waves may have a high propagation velocity and the flow can be visibly turbulent.

The first study of roll waves in free surface flow was begun by Cornish (1910), when turbulent flow in the Merligen channel (Switzerland) was observed. Jefreys (1925) determined the first condition for this phenomenon to occur in turbulent flow, based on the Froude number (Fr), in which roll waves should not appear if Fr < 2.

Following that research, Dressler (1949) developed a study that became a classic of roll wave studies. He described a roll wave as being a series of waves with well-defined lengths interconnected by discontinuities through shock conditions. The theoretical fundamentals of Dressler's work are still used by several researchers.

With regard to experimental turbulent roll waves, the first to measure such waves was Brock (1969), presenting the profile for natural and permanent roll waves occurring in water. The permanent shape is obtained through the imposition of a disturbance at the beginning of the flow. Recently Miao et al. (2017) measured properties such as the propagating velocity, length and amplitude of waves occurring in water.

The disturbance frequency is important for the generation of roll waves. The roll wave period will be identical to the period of the disturbance imposed. When there are multiple disturbances in the same flow, the one with the greater period will prevail on the waves (Kranenberg, 1992; Maciel, 2001).

In the numerical simulations of this phenomenon in turbulent flows, various models have been proposed in trying to reproduce numerically the Brock (1969) experiment. Zanuttigh and Lamberti (2002) used the weighted-average-flux method along with the shallow waters equations (SWE) and tried to reproduce the natural roll waves observed by Brock.

Richard and Gavrilyuk (2012) added entropy terms into the shallow water equations, with the objective of representing the dispersion due to the non-uniform distribution of the vertical velocity profile of the flow and proposed a system of equations (Richard-Gavrilyuk equations) to represent this phenomenon in the flow of Newtonian fluids in turbulent regime. The model fitted very well the experimental results of Brock (1969), both in the shock and in the maximum amplitude.

A modified k-ε turbulence model was proposed by Cao et al. (2014) (a parameter modifying the Reynolds stress and one modifying the mean depth in the standard k-ε model) coupled to the shallow water equations. These authors analyzed the influence of the period and intensity of disturbance imposed on the flow in the generation of roll waves, showing that for the available channel length, the roll waves were already stabilized, that is, in maximum amplitude and length. The results were compared with the experiments of Brock (1969), showing a good agreement on the amplitude of the waves.

More recently, Ivanova et al. (2017) used the RGE turbulence model to reproduce the experiments of Brock (1969). In addition, they proposed a qualitative method to verify the stability of roll waves, called "periodic box." In this method the domain is created with periodic boundary conditions. The results obtained were compared with the experimental results of Brock (1969) and the one-dimensional model of Dressler (1949).

Bazargan and Aghebatie (2015), as well as Aghebatie and Hosseini (2016), also used the Fluent Software to simulate *roll waves,* the Volume of Fluid (VoF) model and the $k-\varepsilon$ turbulence model. The results were in line with the experimental data from an Azad Dam spillway prototype. The authors confirm that the best model to use for such simulations is the standard $k-\varepsilon$, however comparisons with other turbulence models are not shown.

It's worth noting that most mathematical and numerical analysis of roll waves of Newtonian fluids in turbulent flow are based on the shallow water equations (flow depth smaller than the characteristic length) or on permanent roll wave models, based on Dressler's (1949) mathematical model. Furthermore, the properties analyzed are amplitude, length and velocity of propagation of the roll waves. Other properties of the flow, such as velocity fields and stress at the bottom of the channel in the presence of roll waves have not been analyzed, which has been a persistent gap. Toniati et al. (2018) studied the phenomenon in laminar flow, mentioning the importance of determining the shear stress at the bottom of the channel in the presence of roll waves and also discussing possible implications of an erosion process.

In this context, this paper presents a case study of one of Brock's permanent roll waves experiments by means of numerical simulation, using the commercial Ansys Fluent software and the Navier-Stokes equations. The turbulence models used will be the $k-\varepsilon$ and $k-\omega$ models, to verify which one better represents the results obtained by Brock in his experiment. With these simulations, besides the kinematic properties of the waves, the velocity field and the shear stress at the bottom of the channel will be analyzed in the presence of roll waves. These variables may provide better understanding of the phenomenon and possible consequences during an event. The main contribution is the determination of the velocity profile and the shear stress at the channel bottom for flows with roll wave instabilities. It is emphasized that the stress exerted by the flow in the channel bed represents an important design parameter in studies of erosion.

2. METHODOLOGY

2.1 Brock's Experiment

Brock (1969) developed his experiment using clean water in a channel 39.6 m in length and 11.47 m in width. The experiments were performed for natural roll waves, which are waves generated without the control of a frequency of disturbance and also for roll waves generated from a disturbance imposed to the system, with intensity of 0.5% of the normal depth of the flow, making possible the generation of periodic roll waves.

Table 1 shows the conditions of the Brock experiment for roll waves generated with an imposed perturbation. The same conditions were used in the research presented here to validate the numerical simulations.

Table 1: Parameters used by Brock (1969) during the periodic roll wave experiment.

$\tan\theta$	Channel	h_0[mm]	$\bar{u}$ [m/s]	Re	Fr	T [s]
0.05011	Smooth	7.98	1.5	10500	3.71	1.218

Here, θ is the slope of the channel; h_0 is the normal depth of the flow; Fr is Froude's number and T is the period of the imposed disturbance, $\bar{u}$ is the mean flow velocity, and *Re* is Reynolds' number.

2.2 Numerical Modeling

The process of numerical modeling can be split into three distinct parts: Preprocessing, in which the meshing and initial conditions are defined; Solvers, which are the numerical, mathematical and computational techniques that will solve the equation system; and postprocessing, in which are analyzed the solutions obtained by the software.

The mesh adopted was rectangular (375000 elements) and consisted in a 2D channel 40 m in length and constant transversal section, with higher refinement levels at the start and from the bottom of the channel to a height equal to 2 h_0. The channel was filled with water until the height of the water level was h_0.

The following boundary and initial conditions were assumed for the numerical simulations of roll waves in turbulent flow:

-Bidimensional flow across a rectangular channel with 40 m length;

-Wall condition (zero flow) at the bottom of the channel and at the top of the channel's inlet;

-Pressure outlet equal to atmospheric pressure on the outlet and at the top of the channel;

-Velocity inlet at the channel's inlet.

Figure 1 shows the boundary conditions used for the flow.

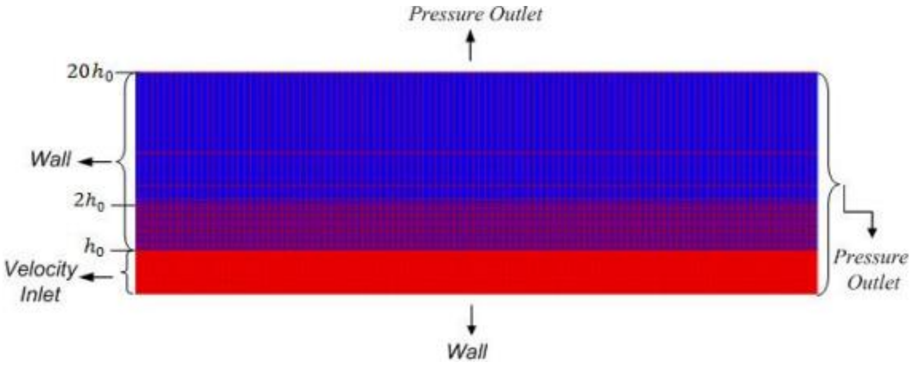


Figure 1. Boundary conditions for the flow.

The disturbance was done by using a sinusoidal function for the entrance velocity, as shown in Eq. (1)

$$V=\bar{u}\left[1+B\sin\left(\frac{2\pi t}{T}\right)\right] \tag{1}$$

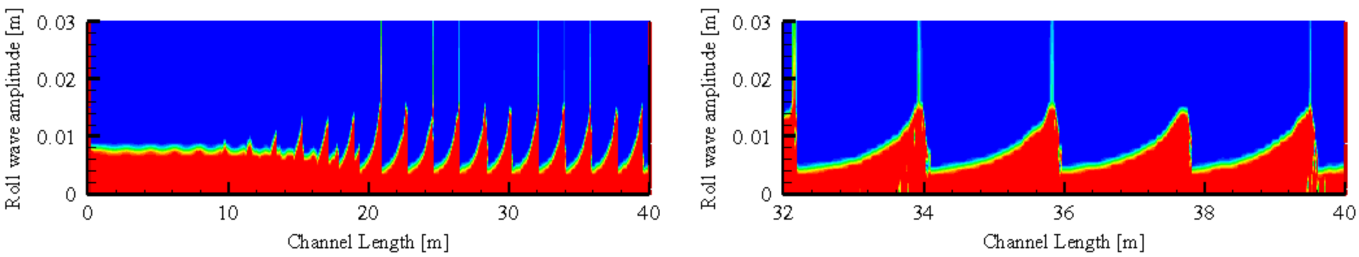
Here, V is the entrance velocity; $\bar{u}$ is the mean flow velocity; B is the intensity of the disturbance, T is the disturbance's period, and t is time.

The Fluent software (version 14.5.7) offers multiple solvers and discretizing options, containing variables that can affect the convergence and desired solution. The multiphase model used was VoF (Volume of Fluid), the viscosity models used were the standard k-ε and standard k-ω, for pressure and velocity coupling Simplec was used, Presto for pressure, the transport equation was solved by Third Order MUSCL (Monotonic Upwind Scheme for Conservation Laws), and the volume fraction was obtained by Modified HRIC (High Resolution Interface Capturing).

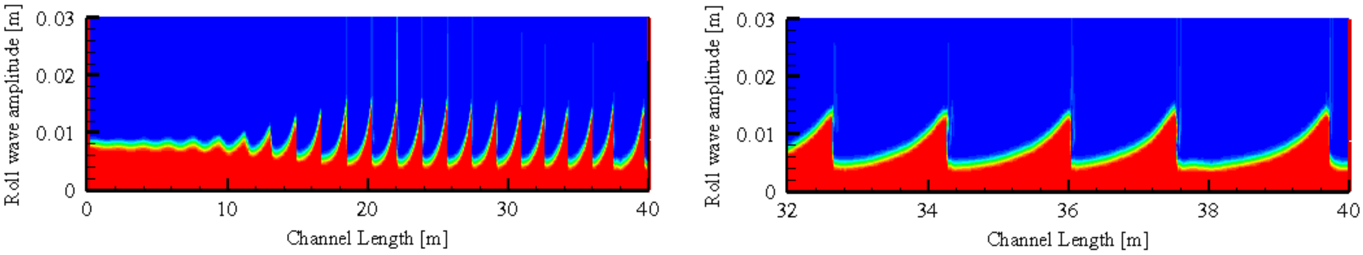
3. RESULTS

Being a long channel, about 30 seconds of simulation time was used to obtain permanent roll waves, and the stabilization region was located after 32 m of the channel. Figure 2 shows the propagation of roll waves across the channel for the two different turbulence models, $k-\varepsilon$ and $k-\omega$. With this simulation, the frequency, length, amplitude and propagation velocity of the roll waves were set as shown in Tab. 2.

Figure 3 shows a dimensionless comparison between the results obtained in the different simulations, Brock's (1969) experimental results, those obtained by the modified k-ε model, proposed by Cao et al. (2014) and the 1D model using the shock condition developed by Dressler (1949). Also in Figure 3, λ represents the wavelength.



(a)



(b)

Figure 2. Propagation of roll waves across the channel. (a) $k-\varepsilon$ Model. (b) $k-\omega$ Model.

Table 2. Properties of the roll waves for Q = 9.72 10^{-4} m^3s^{-1}, Fr = 3.71, θ = 2.87°, smooth channel.

Results	Wavelength [m]	Wave propagating velocity [m/s]	Frequency [s^{-1}]	Amplitude [m]
$k-\omega$ model	1.73	1.44	0.82	0.0136
$k-\varepsilon$ model	1.87	1.55	0.82	0.0147
Brock (1969)	1.81	1.50	0.83	0.0130

By using the data shown in Fig. 2 and Tab. 2, it is possible to see that the stabilized roll waves are long waves and propagate with the same frequency as the imposed disturbance, as seen in the literature (Needham and Merkin, 1987; Maciel, 2001; Zanuttigh and Lamberti, 2002; Ferreira, 2013)

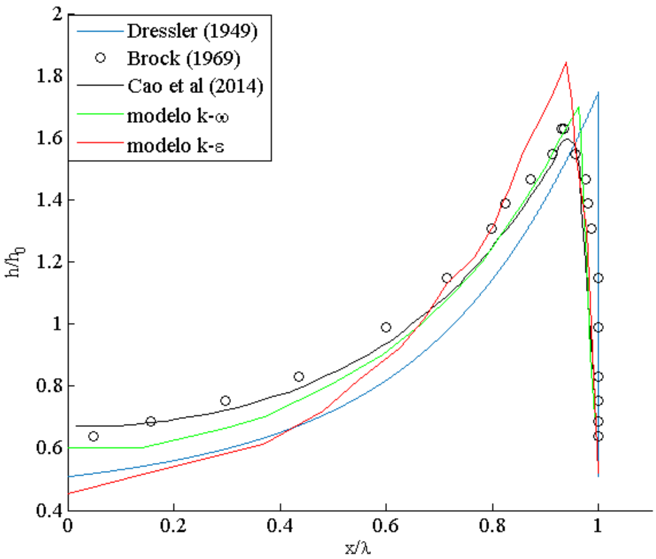


Figure 3. Comparison between the turbulence models used and the literature.

Analyzing Fig. 3, we see that the profiles of the roll waves obtained by the numerical simulation are similar to the profile shown in Dressler's 1D model. Also, it is possible to see that the k-ω model is the one that is most similar to the results obtained by Brock (1969) and Cao et al. (2014), with an amplitude error of 4.42%. The k-ε model showed an amplitude error of 13.20%, however it was more accurate regarding the spot (x/λ at the peak of the wave. Table 3 shows the percentage differences between the simulations and Brock's results (for different locations on the wave).

From Table 3 it can be seen that for the $k-\omega$ model the percentage differences vary from 11.99% to 0.41%, the highest difference being at the start of the wave, and the lowest difference being after its peak. It is important to note that there are points with low errors near the peaks of the waves peak (0.41%). For the $k-\varepsilon$ model the minimum difference is 1.01% and maximum is 25.68%, the maximum occurring also at the start of the wave, and the minimum at a medium point in the wave.

Table 3 – Comparison of waves between tested turbulence models and Brock (1969).

x/λ	$k-\varepsilon$ model	$k-\omega$ model
0.05	25.68%	6.26%
0.15	24.41%	11.81%
0.3	22.83%	11.99%
0.44	18.88%	9.60%
0.6	11.22%	9.09%
0.72	2.88%	7.00%
0.8	1.01%	6.24%
0.83	1.67%	6.68%
0.87	8.82%	2.66%
0.91	10.84%	0.71%
0.93	10.55%	1.89%
0.96	1.09%	7.74%
0.97	1.13%	0.41%
0.98	1.44%	1.44%

Figure 4 shows the flow's velocity field in the presence of stabilized roll waves for the $k-\varepsilon$ turbulence model, along with the air-water interface for VoF values from 0.5 to 0.55. The shear stress at the bottom of the channel is also shown. The velocity field and shear stress for the k-ω model are shown in Figure 5.

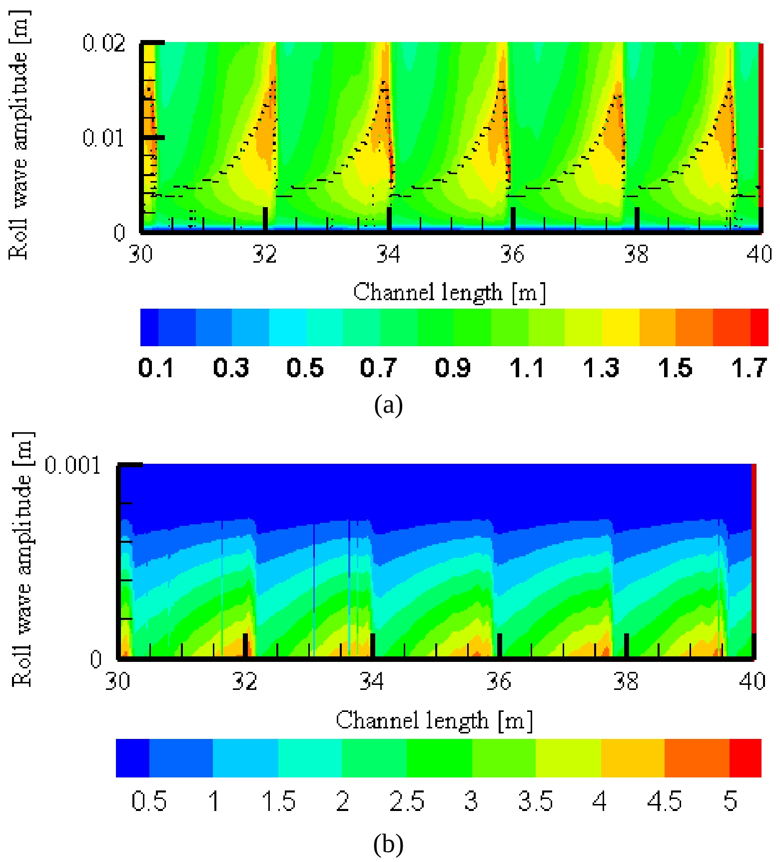


Figure 4. $k-\varepsilon$ Model. (a) Velocity field [m/s] (b) Shear stress at the bottom of the channel [Pa].

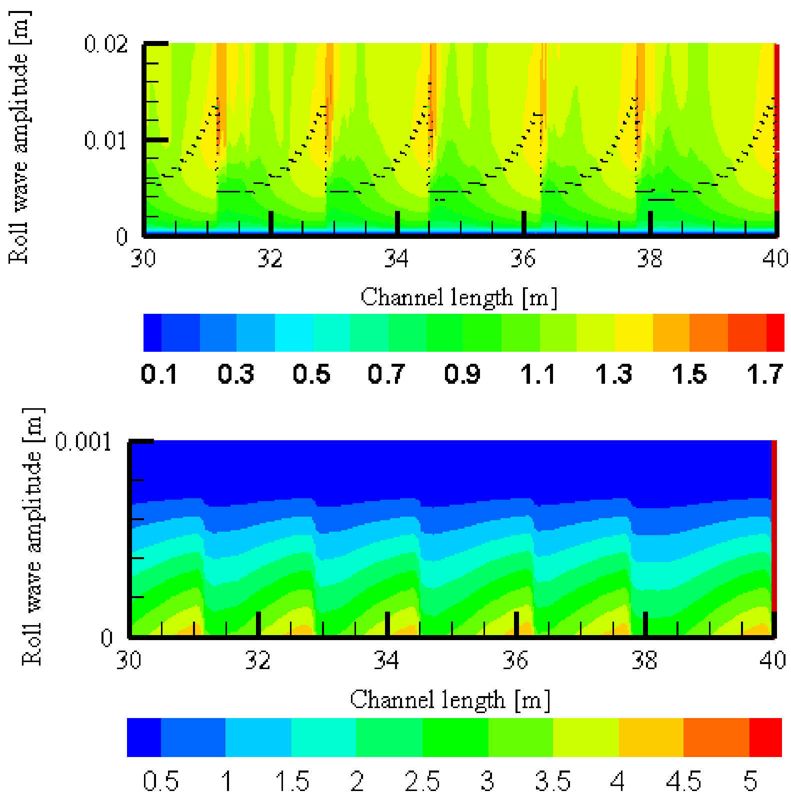


Figure 5. $k - \omega$ Model. (a) Velocity field [m/s] (b) Shear stress at the bottom of the channel [Pa].

It can be seen that both the velocity and the shear stress in the bottom have their minima at the beginning of the wave and increase with the growth of the wave, reaching a maximum in the region that precedes the shock. In relation to the velocity, it can be seen that the peak of the wave reaches a velocity 13% greater than the average velocity of the uniform flow.

For the $k - \varepsilon$ model, this maximum velocity is near the shock, but covers a significant part of the wave, which may justify the differences seen in relation to amplitude, length and wave propagation velocity from the k-ω model.

For this case study, the results point to waves with typical shockwave profiles, in accordance with the results in the literature (Dressler, 1949). In addition, the $k - \omega$ model of turbulence seems to be more effective than the $k - \varepsilon$ model, as its result was closer to Brock's (1969) experiment and Cao's (2014) numerical simulation.

4. CONCLUSION

For this case study, the results show waves with a shockwave profile, as seen in the literature (Dressler, 1949). In addition, the $k - \omega$ turbulence model seems more effective than the $k - \varepsilon$ model, since its results were closer to the experimental results of Brock (1969) and the numerical results of Cao (2015), who simulated the same case study, and using shallow water equations with turbulence effects.

In relation to the properties of the roll waves, it was observed that the fundamental frequency of the waves is identical to that of the disturbance imposed on the flow, which confirms theoretical and numerical results from the literature (Kranenberg, 1992; Needham and Merkin, 1987; Maciel, 2001). This concordance occurred for both tested turbulence models ($k - \varepsilon$ and $k - \omega$). As for wave length and amplitude, the $k - \omega$ model was shown to be more effective.

The numerical model used is a good tool for roll wave simulation in turbulent flows, and allows simulations in different scenarios (such as different Froude and Reynolds numbers). However, for case studies like Brock (1969) involving a long channel with low depth (film flows), the processing time is high (over 30 days).

In spite of the computing time, the advantage of doing simulations based on the complete equations of Navier-Stokes is that important flow properties for engineering projects can be analyzed in space and time, such as the stress at the channel bed and the velocity field in the presence of roll waves. In relation to the amplitude and length of the roll waves, simpler models, based on the shallow water equations, have been shown to be effective, as seen in Dressler's (1949) model.

5. ACKNOWLEDGMENTS

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