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HYDRAULIC POWER OPTIMIZATION OF THE ARCHIMEDES TURBINE USING NUMERICAL SIMULATION

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Abstract. *The objective of this study is to determine the parameters that optimize the hydraulic efficiency of the Archimedes turbine. The OpenFOAM software is used to numerically simulate the flow in an Archimedes turbine. The Nelder-Mead Simplex optimization method controls the parameters used in the numerical simulations. The numerically calculated mechanical power is corrected by an efficiency accounting for the leakage in the gaps between the screw and trough. The water flow and the head are kept fixed and the variables are the rotation, the internal radius, the helical pitch and the inclination of the screw. The maximum hydraulic efficiency calculated was greater than 86%. It was observed that the reduction of inclination and internal radius raises the efficiency of the three-bladed Archimedes turbine. The optimal inclination calculated in the present work is close to the results presented in numerical and experimental reference works. The results of this work show that the Archimedes turbine with the highest efficiency presents low overfilling leakage and constant hydrostatic height variation between each blade. It is concluded that the optimization together with numerical simulation is a viable tool for the Archimedes turbine design.*

Keywords: *Archimedes turbine, micro hydro generation, multiphase flow, optimization, OpenFOAM.*

1. INTRODUCTION

According to data from the Agência Nacional de Energia Elétrica (ANEEL), approximately 60% of electricity generation in Brazil is hydroelectric and about 95% of this energy is produced by Hydroelectric Power Plants (HPP) (ANEEL, 2018). To be classified as a HPP in Brazil, a hydroelectric plant must have a capacity greater than 30 MW. Most of the HPP in Brazil require large reservoirs, which flood regions formerly covered by forests and emit a significant amount of methane, which is a gas with a high potential for global warming (Williams and Simpson, 2009). The construction of large reservoirs can also cause social impacts, due to the displacement of the communities that inhabit the region to be flooded.

In Brazil, microgeneration is characterized by a power plant with an installed power less than or equal to 75 kW (ANEEL, 2018). Most of the technologies used for hydraulic microgeneration are run-of-the-river, that is, they need little or no water storage (Williams and Simpson, 2009), which reduces the environmental impact associated with installation and operation. There are several technologies available for hydraulic microgeneration. Among these, the Archimedes screw generator (ASG) will be studied in the present work. In Figure 1, an ASG with three flights and its main parameters are represented.

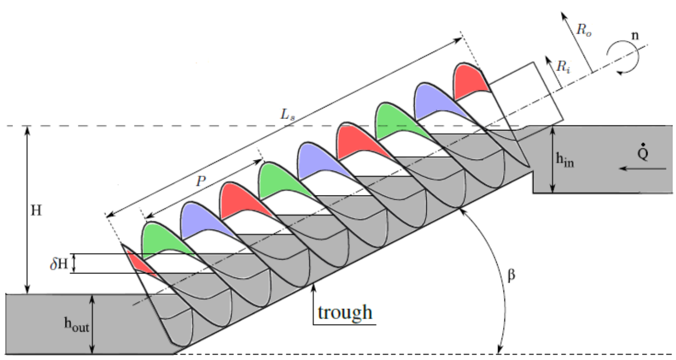


Figure 1. Representation of an Archimedes turbine. Source: adapted from Dellinger *et al.* (2018).

The ASG is composed of helical blades arranged on a cylindrical shaft, due to this geometry this equipment is also known as Archimedes screw. The screw is concentrically positioned to a cylindrical tube with the top surface open or closed, called the trough. The flow has downward direction and it is forced to pass through the turbine. The inclination of the Archimedes screw and the confinement of water between each of its blades results in hydrostatic forces. The turbine shaft can be coupled to an electric generator to produce electric power. The parameters shown in Fig. 1 are: volumetric water flow ($\dot{Q}$), head (H), hydrostatic height variation between each blade (δH), turbine rotation (n), water level at the turbine inlet (h_{in}), and water level at the outlet of the turbine (h_{out}). The main geometrical parameters of the screw are: the helical pitch (P), the outer radius (R_o), the internal radius (R_i), the angle of inclination (β), the length of the screw (L_s) and the number of blades (N). Each volume separated by two subsequent flights are called bucket.

Comparing several types of hydraulic turbines, Williamson *et al.* (2014) verified that the Archimedes turbine has operational advantages in applications with the low head (less than 5 m) and remains efficient until close to 0 m. In experimental verifications, Pálffy *et al.* (1998) reported hydraulic efficiencies greater than 80% for ASG and showed that the flow can vary approximately $\pm$ 20% without a significant impact on its efficiency. The ASG is considered a fish-friendly technology because it causes low impact on aquatic species. This turbine operates in low rotation and has large entrances, which allows the passage of aquatic animals with very low damage and mortality rates (Kozyn and Lubitz, 2017).

The first applications of the ASG were for the pumping of fluids. Only after the 90's, the Archimedes screw was being used as a hydraulic turbine, and to date, most of the methodologies developed for Archimedes turbine design were based on results obtained for pumping fluids. For this reason, further studies related to the ASG design are needed. In this study, an optimization software is used to determine the R_i, β, P and n that maximize the power produced by the Archimedes turbine on a laboratory scale at determined $\dot{Q}$ and H values. The simulation of the Archimedes turbine and calculation of hydraulic power are done using numerical modeling.

2. LITERATURE REVIEW

The invention of the screw used for irrigation and drainage is attributed to the Greek Archimedes (287-212 a.C.) (Rorres, 2000). More than 2,000 years later, in the 90s, the German engineer Karl-August Radlik noted that the Archimedes screw can operate as a hydraulic turbine to produce mechanical power, which can be converted into electricity (Nuernbergk, 2017). Since then, this technology has been developing and there are currently more than 400 ASG installed worldwide (Lashofer *et al.*, 2012).

Rorres (2000) presented an analytical method for the Archimedes screw design, based on the maximization of the volume of water transported in each revolution. Adimentional parameters relating the external radius and the internal radius of the screw, the helical pitch and the volume filled by liquid were determined through the dimensional analysis of the problem. The optimum values for these parameters were presented as a function of the number of turbine blades.

Müller and Senior (2009) used a simplified model for the ASG that idealizes turbine flights as mobile dams, in which the torque produced on the screw shaft occurs due to the hydrostatic force acting on each side of the blade.

Nuernbergk and Rorres (2012) derived an analytical model for the calculation of the water input conditions in the ASG. Analytical equations that model the leakage through the gap between the flights and the trough and by the liquid overflow were incorporated in the calculation of the hydraulic efficiency. The results obtained using the analytical model were close to experimental data.

A mathematical model and an experimental model of the Archimedes turbine were presented by Rohmer *et al.* (2016). In that work, the methodology developed by Nuernbergk and Rorres (2012) is used together with an empirical model for the calculation of friction losses and hydraulic efficiency. An experimental hydraulic bench was built and tested. The theoretical results were very close to the experimental results. Mechanical efficiency and torque were related to screw rotation and water flow. Mechanical efficiencies greater than 80% were obtained.

Dellinger *et al.* (2016) and Dellinger *et al.* (2018) numerically simulated the ASG. The OpenFOAM software was used to solve the average equations of Reynolds (RANS - *Reynolds Averaged Navier-Stokes*) with the closure model k - ω SST in three-dimensional space. The Volume of Fluid method (VOF) (Hirt and Nichols, 1981) was applied to determine the interface between the water and air phases. Those authors also performed experiments with the ASG to validate the numerical solution. The torque and hydraulic efficiency curves obtained numerically and experimentally presented good agreement. The numerical and experimental models reached hydraulic efficiencies above 80%. The experimental results showed a strong influence of the water outlet height on the mechanical efficiency of the equipment.

Reis and Carvalho (2018) also used the OpenFOAM software to simulate the flow in an ASG. The numerically calculated mechanical power is corrected by a factor accounting for the leakage in the gap between the screw and the trough. The main advantage of analytically calculating the gap leakage losses is to reduce the refinement needed to simulate the Archimedes screw. The results obtained by those authors were compared with experimental and numerical data in the literature and errors were near 2% at a flow rate of $\pm$20% of nominal.

Dellinger *et al.* (2019) presented a numerical and experimental study to determine the relation of slope and number of blades with power produced by the ASG. The results of this work showed that the maximum hydraulic efficiency obtained with the 3, 4 and 5 blade screws occurs with slopes of 15.5^o, 20^o, and 24.5^o, respectively.

3. MATHEMATICAL MODEL

The interFoam solver, present in OpenFOAM 6.0, is a program for numerical modeling of biphasic and incompressible flows, in which the volume of fluid method (VOF) (Hirt and Nichols, 1981) is implemented with the proposed modifications for Rusche (2003). In this methodology, only one equation is solved for the momentum balance and one equation for the mass balance. The fluid properties are calculated as weighted averages the volume fraction of each phase.

The Equations (1) e (2) represent the mass and momentum balance for an incompressible and transient flow of a Newtonian fluid with density and viscosity variation.

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{U}) = 0 \tag{1}$$

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \bullet (\rho \mathbf{U}\mathbf{U}) = \nabla \bullet \left[\mu_{eff} \left(\nabla \mathbf{U}\right)\right] + \nabla \mathbf{U} \bullet \nabla \mu_{eff} - \nabla p_d - \mathbf{g} \bullet \mathbf{x} \nabla \rho + \mathbf{F}_s \tag{2}$$

where: t the time, $\mathbf{U}$ the effective velocity vector, ρ the density, p_d the dynamic pressure, μ_{eff} the effective absolute viscosity, $\mathbf{g}$ the acceleration of gravity and $\mathbf{F}_s$ the force due to surface tension. The volumetric fractions of the liquid and the gas are represented by α_l and α_g. As the flow is biphasic, it is considered that: $\alpha_l + \alpha_g = 1$.

The rate of change of the volumetric fraction of liquid phase (α_l) is modeled according to Eqs. (3).

$$\frac{\partial \alpha_l}{\partial t} + \nabla \bullet (\alpha_l \mathbf{U}) + \nabla \bullet [\alpha_l (1 - \alpha_l) \mathbf{U}_r] = 0 \tag{3}$$

where $\mathbf{U}_r$ is defined as: $\mathbf{U}_r = \mathbf{U}_l - \mathbf{U}_g$. The direction of $\mathbf{U}_r$ is always normal to the interface. $\mathbf{U}_r$ is calculated by the Eq. (4).

$$\mathbf{U}_r = min \left(c_\gamma \left|\mathbf{U}\right|, max \left[\left|\mathbf{U}\right|\right]\right) \left(\frac{\nabla \alpha_l}{\left|\nabla \alpha_l\right|}\right) \tag{4}$$

where: c_γ is a constant. It is considered that $c_\gamma = 1$ in this work.

In interFoam, the force is calculated through the continuum surface force (CSF) (Brackbill *et al.*, 1992).

$$\mathbf{F}_s = -\sigma \nabla \bullet \left(\frac{\nabla \alpha_l}{\left|\nabla \alpha_l\right|}\right) \nabla \alpha_l \tag{5}$$

where σ is the surface tension coefficient and $\nabla \alpha_l$ is the vector normal to the interface.

The effective viscosity is the sum of the absolute viscosity (μ) of the fluid with the turbulent flow viscosity (μ_t).

$$\mu_{eff} = \mu + \mu_t \tag{6}$$

In the present work the closure model of two equations $k - \omega\ SST$ is used for turbulence modeling. This model of turbulence was selected because it is widely used in the literature especially for numerical simulation of turbines (Versteeg and Malalasekera, 2007), besides having great range of applicability, simplicity, accuracy and being of relatively low computational cost (Dellinger *et al.*, 2018). The turbulent viscosity is calculated as:

$$\mu_t = \frac{\rho k}{\omega} \frac{1}{max\left(\frac{1}{a^*}, \frac{F_2 S}{\omega a_1}\right)} \tag{7}$$

with k the turbulence intensity, ω the turbulence dissipation rate, F_2 a blending function, a^* a damping coefficient of turbulent viscosity, S the strain rate magnitude and a_1 the model constant that is considered equal to 0.31 (Versteeg and Malalasekera, 2007).

3.1 Leakage between the flights and the trough

The leakage between the screw blades and the trough ($\dot{Q}_G$) is called gap leakage in this work and occurs due to the head difference between buckets. In most cases studied hydraulic efficiency loss due to leakage gap is more than 5% (Rohmer *et al.*, 2016).

From the Torricelli equation, Muysken (1932) developed Eq. (8) to calculate the gap leakage in Archimedes pumps. The Eq. (8) considers that the variation of head between buckets (δH) is constant ($\delta H = (P/N)\, sin\,(\beta)$).

$$\dot{Q}_G = \mu_A s_{sp} R_o \left(1 + \frac{s_{sp}}{2R_o}\right) \sqrt{1 + \left(\frac{P}{2\pi R_o}\right)^2} \left(\frac{2}{3}\alpha_3 + \alpha_4 + \frac{2}{3}\alpha_5\right) \sqrt{2g\delta H} \tag{8}$$

The parameter (μ_A) is the contraction discharge coefficient and lies in the range 0.65–1.00, depending on the shape of the edge of the blade (Nuernbergk and Rorres, 2012). In the present work it is assumed that $\mu_A = 0.825$. An analytical model for the calculation of the angles α_3, α_4 e α_5 was presented by Rorres (2000). This model is used in the present work.

The main advantage of calculating the gap leakage analytically is the solution converges with far fewer cells in the mesh. The numerical modeling of the gap leakage requires a high level of mesh refinement in the region between the blades and the trough, since the ratio of the gap (s_{sp}) to the outer radius (R_o) is a very small value. The maximum gap is usually estimated by the empirical equation ($s_{sp} = 0.0045\sqrt{2R_o}$ m) for most screws (Nuernbergk and Rorres, 2012). The gap leakage between the blades and the trough was modeled numerically by Dellinger *et al.* (2019), these authors concluded that the Torricelli equation is a good predictor for gap leakage, and only slightly under-predicts the simulated flow through the gap.

3.2 TURBINE EFFICIENCY

An ASG converts the energy of a fluid flow into mechanical energy thanks to the rotation caused by fluid pressure on screw blades (Dellinger *et al.*, 2018). The hydraulic power ($\dot{P}_{hydro}$) available in a flow with head H and volumetric flow $\dot{Q}$ can be calculated by Eq. (9).

$$\dot{P}_{hydro} = \rho g \dot{Q} H \tag{9}$$

The hydraulic efficiency of Archimedes turbine (η_H) can be calculated as:

$$\eta_H = \frac{\dot{P}_{mec}}{\rho g \dot{Q} H} \tag{10}$$

with ρ the density of the water and g the acceleration of gravity.

The shaft power ($\dot{P}_{mec}$) of a hydraulic turbine can be calculated from the Eq. (11).

$$\dot{P}_{mec} = \dot{\Omega}\tau \tag{11}$$

where: τ is the torque resulting from the action of the pressure and viscous forces on the screw and $\dot{\Omega}$ is the angular velocity. The OpenFOAM software provides a function for calculating the torque produced by pressure and viscous forces on surfaces.

The gap between the blades and the trough is not considered in the numerical model of the present work. The numerically calculated hydraulic efficiency (η_H) is adjusted using the loss of efficiency due to leakage through the gap ($L_G = \dot{Q}_G/\dot{Q}$) (Nagel and Radlik, 1988). Then, the efficiency due to gap leakage losses (η_G) is calculated by $\eta_G = 1 - L_G$. Thus, the total efficiency (η) is calculated from the Eq. (12).

$$\eta = \eta_H \eta_G \tag{12}$$

4. NUMERICAL MODEL

The partial differential equations governing fluid dynamics are solved numerically by finite volume method (FVM), using the software OpenFOAM. The Pressure Implicit with Splitting of Operators method (PISO) is used to solve the pressure-velocity coupling. Spatial discretization is centered and has second-order precision. The temporal integration is done through the first-order Euler method. The integration time step (Δt) is limited by the maximum Courant number.

To simulate the rotation of the Archimedes screw, dynamic mesh resources are required. The dynamic mesh methodology implemented in OpenFOAM 6.0 is based on the algorithm developed by Farrell and Maddison (2011), in which the domain is divided into a fixed and a rotating part. The fixed and mobile regions are separated by AMI-type boundaries (*Arbitrary Mesh Interface*), which act as boundary conditions in the software.

The meshes are built with the SALOME software. The cells are tetrahedral. The degree of refining is controlled by the length of the edges of the cells (L_{cel}).

4.1 Boundary Conditions

The types of boundaries and their denominations are shown in Fig. 2.

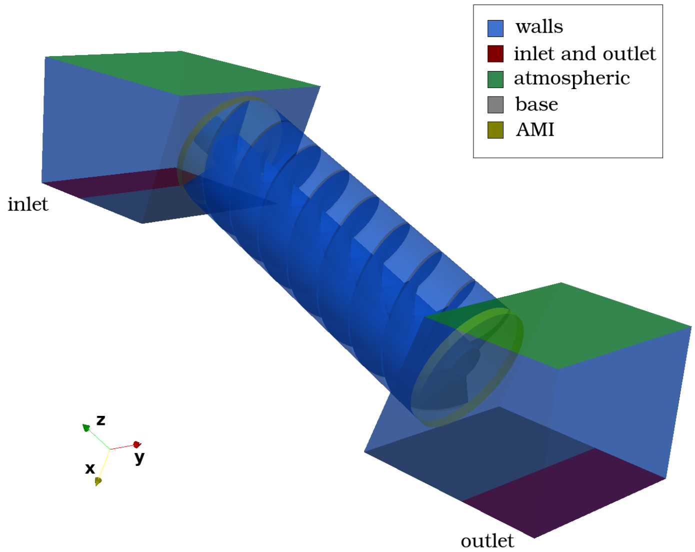


Figure 2. Types of boundaries and their denominations.

At the inlet and outlet boundaries, the water flow is imposed. Atmospheric boundaries allow both inlet and outlet. The total pressures at these boundaries are imposed and calculated as a function of the geometric head (H_{geo}). The non-slip boundary condition is applied to the walls. The base borders are treated as walls. The average pressures on these surfaces are used to calculate the water level at the inlet (h_{inlet}) and outlet (h_{outlet}) of the turbine. For each simulation, the H is calculated as $H = H_{geo} + h_{inlet} - h_{outlet}$.

5. OPTIMIZATION METHOD

The nlopt library (Johnson, 2008) is used, it provides several optimization methods. Optimization is performed using the Nelder-Mead Simplex method (Box, 1965). This method is a direct search. The Nelder-Mead Simplex method is used to optimize non-linear problems for which the derivatives can not be found.

The goal of optimization is to maximize the efficiency of the turbine. For each iteration, OpenFOAM simulates the case and returns the average torque during the last rotation of the screw. The average torque and rotation are used to calculate the mechanical power and efficiency of the turbine. Using the turbine efficiency the optimization method calculates the parameters for the next iteration. The stopping criterion for the optimization is the relative variation of the objective function.

6. RESULTS AND DISCUSSION

The numerical and mathematical models presented in this study were validated by Reis and Carvalho (2018) using the experimental and numerical data obtained by Dellinger *et al.* (2018). The results obtained by these authors were compared with experimental and numerical data in the literature and errors were near 2% at a flow rate of ±20% of nominal. The greatest difference between the total efficiency calculated by Reis and Carvalho (2018) and the experimental one was 5% and occurred for the case with low level of screw filling.

6.1 Optimization

The objective of the optimization is to determine the parameters that maximize the efficiency of a ASG. The $\dot{Q}$ and the H_{geo} are considered natural conditions of the site of installation of the turbine and, therefore, are constant of the problem. The R_o is also constant due to manufacturing limitations of the acrylic tube that will be used as a trough. Only turbines with three blades are considered in this optimization.

The invariable parameters during optimization are shown in the Tab. 1.

Table 1. Invariable parameters during optimization.

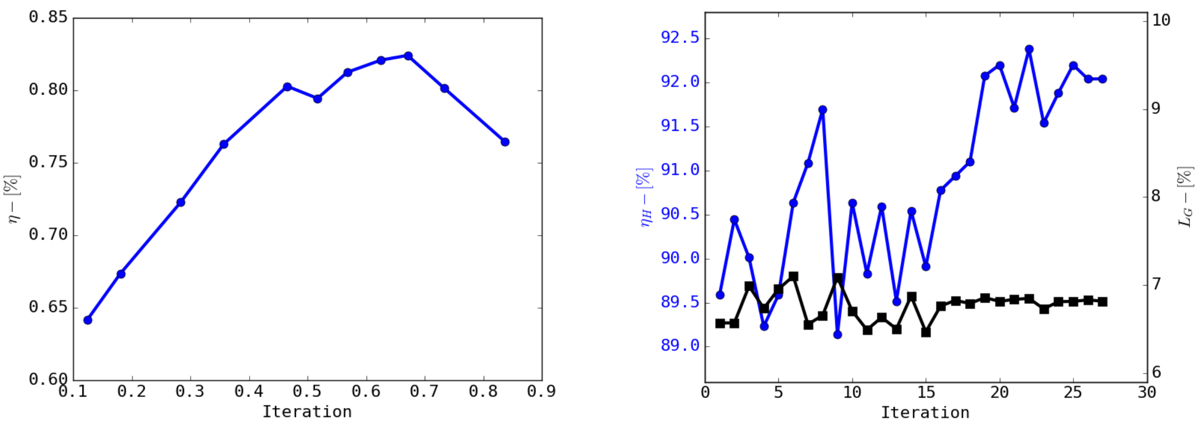
Parameter	Value	Unit
Outer radius - (R_o)	0.0850	m
Geometric Head (H_{geo})	0.1627	m
Flow rate ($\dot{Q}$)	1.899	L/s
Gap between the screw and the trough (S_{sp})	0.0008	m

The variables of the problem are: n, β, P and R_i. The minimum radius ratio (R_i/R_o) found in ASG installed in Europe is 0.3 (Lashofer *et al.*, 2012). In the present work, a minimum radius ratio equal to 0.18 was defined to evaluate this parameter in a wide range. The starting conditions and the limits are shown in the Tab. 2.

Table 2. Starting conditions and the maximum and minimum limits of the variables.

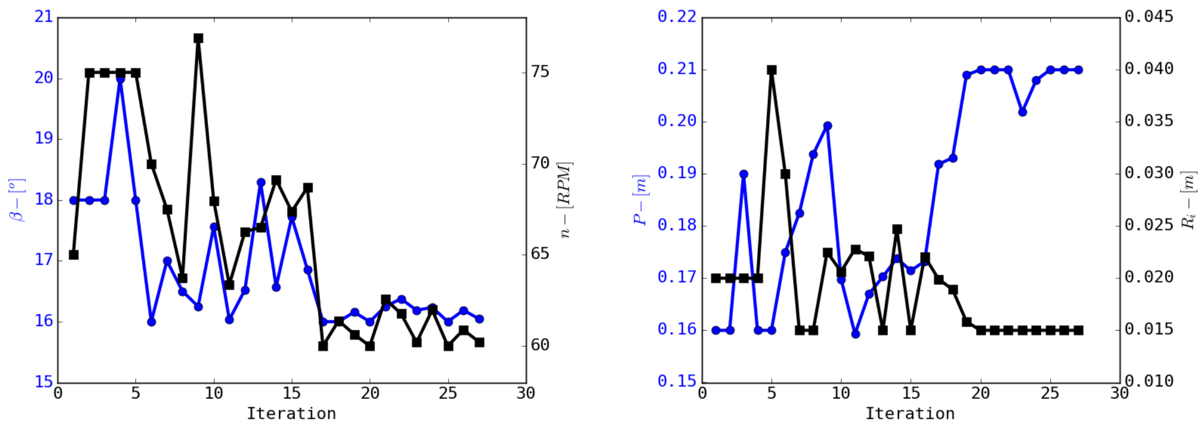
Variable	Starting condition	Minimum	Maximum
n (RPM)	65.0	60.0	80.0
β (^{o})	18.0	16.0	26.0
P (m)	0.16	0.15	0.21
R_i (m)	0.02	0.015	0.05

The $L_{cel} = 0.006$ m in the screw region and in its vicinity. The L_{cel} expands to 0.012m as the distance from the screw increases. The simulations occur until $t = 13$ s. The stopping criterion for the optimization is the relative variation of the objective function equal to 0.0025. Results from the optimization are presented in the Figs. 3 and 4.



(a) (b)

Figure 3. Efficiencies as a function of the iteration. a)η. b)η_H and L_G.



(a) (b)

Figure 4. Optimization variables as a function of the iteration. a)β and n. b)P and R_i.

As can be seen in Fig. 3, 27 iterations were performed and the maximum efficiency reached was close to 86%, which occurred at the iteration 22.. The gain in η was close to 3%, which was mainly influenced by the η_H. The results presented in the Fig. 3(b) show that the loss of efficiency due to leakage through the gaps is higher than 6% in all cases studied. Other authors also observed power losses higher than 5% (Rohmer *et al.*, 2016; Dellinger *et al.*, 2018). Therefore, these leaks should be considered in the model.

The Figure 4 shows the optimization variables as a function of the iteration. The maximum η were obtained with smaller n, β and R_i. The optimal P was close to 0.21 m, which is the maximum limit.

The results of Dellinger *et al.* (2019) showed that the maximum efficiency of the three-bladed ASG occurs when the slope is equal to 15.5^o. This result is very close to that obtained in this work, which was 16.4^o. It should be noted that turbines with lower β are longer, which increases your construction costs. A long screw also suffers higher deflection, so it needs to be built with thicker walls. Maximum efficiency occurs with a small internal radius. Smaller R_i also require larger sheet thicknesses to prevent screw deflection. These results show that a more comprehensive optimization should also take into account the ASG construction costs.

In Figure 5, α_l obtained in iteration 22 is presented.

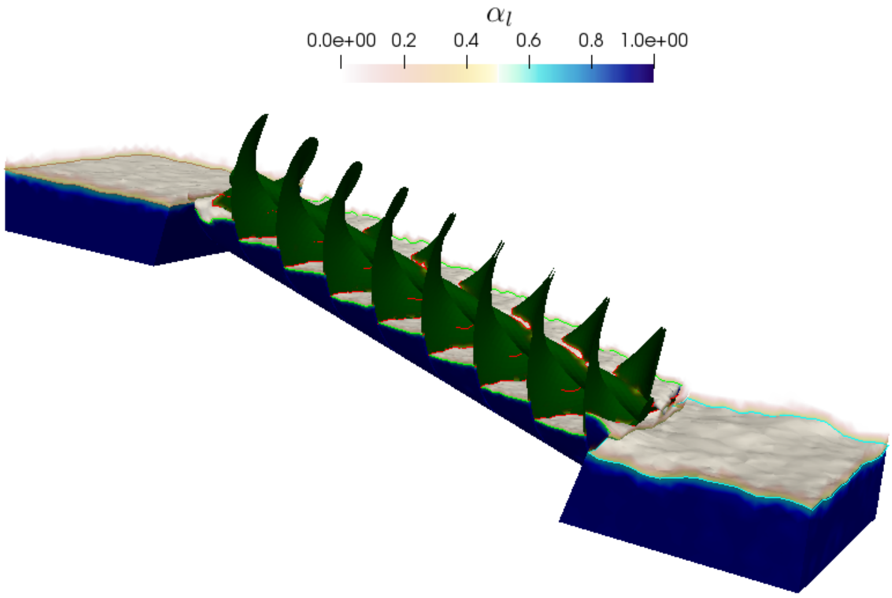


Figure 5. α_l obtained in iteration 22, for *t* = 13 s. A filter was applied to α_l, so that the air phase is transparent.

As can be seen in Fig. 5, the optimized turbine has negligible leakage due to overfilling and uniformity in hydrostatic height variation between each blade along the screw. These conditions are ideal for increasing the efficiency of ASG.

7. CONCLUSIONS

In this work, a methodology for simulation and optimization of ASG was presented. The shaft power is calculated using the results of the numerical simulation. An analytical correction factor is used to include efficiency reduction by leakage through the gaps between the screw and the trough. The turbine efficiency loss due to leakage through the gaps was higher than 6% in all cases studied, so it should be considered in the model.

With the application of the optimization method, ASG with an efficiency of up to 86% were obtained. It was observed that the reduction of rotation, inclination and internal radius together with the increase of the helical pitch raises the efficiency of the three-bladed ASG. The results of this work show that the ASG with the highest efficiency presents low overfilling leakage and constant hydrostatic height variation between each blade.

Although the reduction of the internal radius and the inclination of the Archimedes screw have proved to be beneficial to its efficiency, these alterations must also be studied from the economic and structural points of view, since they increase the constructive cost of the equipment.

Analyzing the results obtained in this study, it is concluded that the application of the optimization together the numerical simulation is a viable tool for Archimedes turbine design.

8. ACKNOWLEDGEMENTS

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