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INFLUENCE OF RADIAL DISTRIBUTION OF ANGULAR MOMENTUM ON GEOMETRY OF ROTOR BLADES OF A MINI HYDRAULIC BULB-TURBINE

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Abstract. *The geometry of the rotor blades of an axial turbine are usually designed from the specification of velocity diagrams on the inlet and outlet sections of the rotor. The author has used a methodology with a constant radial, spanwise, distribution of angular momentum at inlet and outlet sections, with a prescribed non-zero value at inlet section of the rotor and zero at the outlet. The three-dimensional viscous flow through the rotor is replaced by the quasi-three-dimensional approach. The flow on radial and axial surface, S_2 surface as described is replaced by an axisymmetric flow, and the flow on circumferential and axial surface, S_1 surface, is approximated by a two-dimensional flow through a linear cascade of profiles. In this paper the influence of the angular momentum distribution along the spanwise at inlet section is analysed. The axisymmetric flow is computed using a streamline curvature method. Nine different laws of radial distribution of angular momentum, (constant, linear, quadratic, quadratic positive and negative, inverse, quadratic inverse, quadratic inverse modified and modified inverse) are compared. Spanwise profiles of velocity components as well as velocity angles, pressure and velocity coefficients at inlet at outlet sections of guide vanes and rotor are presented and compared with a constant spanwise distribution.*

Keywords: *rotor, axial-flow turbine, angular momentum, streamline curvature method, panel method.*

1. INTRODUCTION

The methodology used on the design of the guide vanes or rotor blades of any turbomachine must be easy to apply and cannot use too much computing resources (CPU time). Full three-dimensional techniques that solve Navier-Stokes equations with a turbulence model cannot be used on the blade geometry design of a small turbine. The quasi-three dimensional approach, as described by Wu (1951) solves flow equations in two stream surfaces families. One of these families, named by Wu as S_1 surface, includes the inner and the outer casing surfaces and is geometry is approximately a revolution surface. The other family includes vanes and rotor blades, as well as, other surfaces connecting hub and shroud surfaces and is named by Wu as S_1 surface. A review of application of quasi-three dimensional methods is presents by Japikse (1976) and Adler (1980). Wu method is iterative and too much CPU time and usually produces blades with double curvature. Wu method can be simplified by the through-flow analysis approach. The three-dimensional flow is then replaced by the quasi-three-dimensional approach that solves an axisymmetric flow on S_2 surface, meridional flow, and the blade-to-blade flows on revolution surfaces S_1 placed at different radial distances.

The flow on meridional surface can be solved by streamline curvature, finite difference or finite element method. The streamline curvature method is a low consuming CPU time method, friendly to be used and has been used successfully applied for several geometries, as referred by Katsanis (1964), Novak (1967), Frost (1972) and Denton (1978).

A design method for the guide vanes of a small tubular turbine is presented at by Ferro *et al*. (2010). The same authors present a design methodology of for the rotor blades of that tubular turbine at (Ferro *et al.*, 2011). For the project of that turbine a constant spanwise distribution of angular momentum rV_θ was prescribed at inlet, leading edge of the blade, and outlet section, trailing edge, of the rotor, with the maximum value at leading edge and zero at trailing edge.

Whitney and Stewart (1973) refer that a constant distribution of angular momentum may have same problems close to the hub. The tangential component of absolute velocity increases from the tip to the hub, where its value is maximum. The deflection of relative flow also has is maximum value close to the hub. Ferro *et al*. (2010) show for that the flow around the aerofoil close to the hub presents separation of the boundary layer at the boundary of the trailing edge. The deflection of the relative flow, ranges from 4.59º at the tip of the rotor to 39.4º for the rotor geometry computed at Ferro *et al*. (2010). On account of this variation of deflection on radial direction the blade is too twisted particularly close to the hub.

In this paper the influence of the angular momentum law distribution along the spanwise at inlet section of the rotor is analysed. The meridional flow, assumes as axisymmetric is solved using a streamline curvature method as described by Denton. Results obtained with linear, quadratic, inverse, quadratic inverse as well as small variations of that laws are tested and compared with constant distribution

2. METHODOLOGY

The application of through-flow methodology on the design of the blades of a rotor of a mini hydraulic tubular turbine is described by Ferro (2009). The meridional flow field at S_2 surface was computed using a streamline curvature method with quasi-orthogonal lines as described by Denton (1978). The quasi-orthogonal lines are straight lines that connect the boundaries of the domain, the inner casing and outer casing of the turbine, being crossed by the streamlines of the flow.

The momentum equation in quasi-orthogonal direction q, is (Denton, 1978),

$$\frac{1}{2}\frac{d\left(V_m^2\right)}{dq}=\frac{dE}{dq}-F_q-\frac{1}{2r^2}\frac{\mathrm{d}\left(r^2V_\theta^2\right)}{dq}+\frac{V_m^2}{r_c}\sin\delta'+V_m\frac{dV_m}{dm}\cos\delta' \,, \tag{1}$$

where V_m is the meridional velocity component, r the radial direction, θ the circumferential direction, δ´ the angle between the streamline and quasi-orthogonal lines and E the specific total energy defined by:

$$E=\frac{p}{\rho}+\frac{V^2}{2}+gz \,, \tag{2}$$

where z is the vertical cartesian coordinate, p the static pressure, ρ the absolute density of the fluid and g the acceleration due to gravity. The first three terms of Eq. (1) are related to radial equilibrium and the last two to streamline curvature. Velocity field is computed by integration of Eq. (1). The distribution of specific angular momentum rV_θ, for a design problem, is prescribed as input of the program. The derivative of specific total energy E with meridional direction m, is zero for the flow on tubular region and through the guide vanes regions, and for the flow through the rotor is

$$\frac{dE}{dm}=\Omega\frac{d(rV_\theta)}{dm} \,, \tag{3}$$

where Ω is the rotational speed of the rotor.

The component of the force at quasi-orthogonal direction q at Eq. (1), for a design problem, is specified using angular momentum distribution rV_θ through the quasi-orthogonal line. The constant of integration of Eq. (1) is calculated by imposing the volumetric flow rate constancy, Q, through the quasi-orthogonal lines,

$$Q=\int_A^B 2\pi r V_m. \tag{4}$$

A and B at Eq. (4) are the extremes of quasi-orthogonal line at inner and outer casing, respectively.

Equation (1) is not linear and only can be solved by an iterative method. The initial velocity field is obtained prescribing a uniform velocity distribution at each quasi-orthogonal line. The streamline slope and curvature and the meridional derivative of V_m are computed from this initial velocity field. The iterative procedure can be instable and a relation factor must be used to have convergence. The velocity field at iteration $k+1$ is computed from the velocity at iteration k by Eq. 5

$$V_m^{(k+1)}=V_m^{(k)}\left(1+\gamma_1\frac{E_r}{N_c\text{-}1}\right)+\gamma_2\left(V_m^{(\mathrm{new})}\text{-}V_m^{(k)}\right). \tag{5}$$

On Eq. 5 $V_m^{(new)}$ is the new computed value of V_m at iteration k and γ_1 and γ_2 the relation factors. The global error E_r at each iteration is

$$E_r=\sum_{i=1}^{N_s}\frac{V_m^{(\mathrm{new})}\text{-}V_m^{(k)}}{V_m^{(k)}} \tag{6}$$

The streamline curvature method and the methodology presented are used on the computation of the meridional flow through a tubular hydraulic turbine. The guide vanes system is conical with six blades and the rotor is axial with four blades radially set between two coaxial cylindrical wall surfaces. The tip diameter of the rotor is D=0.5 m and the hub diameter D_H = 0.214 m. At spanwise direction forty-one streamlines were prescribed, including those of inner and outer casing. Thirty four quasi-orthogonal lines are considered on streamwise direction. The guide vanes system has six quasi-orthogonal lines and the rotor two quasi-orthogonal lines, one at the leading edge and other at the trailing edge of the blade. The domain of computation is represented at Fig. 1.

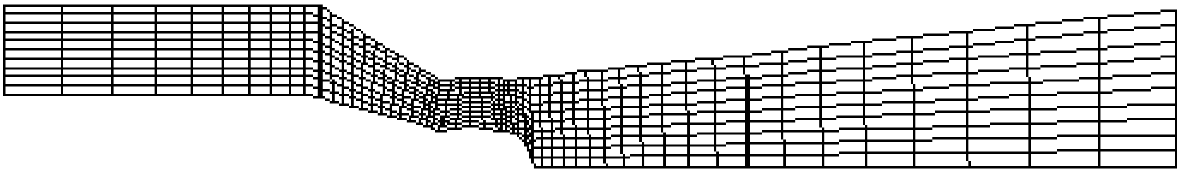

Figure 1. Mesh and computational domain for meridional showing quasi-orthogonal lines.

Eight different spanwise distributions of specific angular momentum, rV_θ, are compared. The rV_θ, distributions considered through the quasi-orthogonal line at the trailing of edge of guide vanes are:

Constant distribution:

$$rV_\theta = C = 0.747\ \mathrm{m^2/s}\ . \quad (7a)$$

Linear distribution:

$$rV_\theta = (rV_\theta)_{\min} + \frac{(rV_\theta)_{\max} - (rV_\theta)_{\min}}{r_0 - r_i} \quad \text{with } (rV_\theta)_{\max} = 0.777\,\mathrm{m^2/s} \text{ and } (rV_\theta)_{\min} = 0.717\,\mathrm{m^2/s}\,. \quad (7b)$$

Quadratic distribution:

$$rV_\theta = ar^2 \text{ with } a = 17.341\ \mathrm{s^{-1}}. \quad (7c)$$

Quadratic distribution with a>0:

$$rV_\theta = ar^2 + br \text{ with } a = 2.593\ \mathrm{s^{-1}} \text{ and } b = 3.123\,\mathrm{m/s}\,. \quad (7d)$$

Quadratic distribution with a<0:

$$rV_\theta = ar^2 + br \text{ with } a = -8.765\ \mathrm{s^{-1}} \text{ and } b = 5.527\ \mathrm{m/s}. \quad (7e)$$

Inverse function

$$rV_\theta = \frac{a}{r} \text{ with } a = 0.1453\ \ \mathrm{m^3/s}\,. \quad (7f)$$

Modified inverse function:

$$rV_\theta = \frac{a}{b - r} \text{ with } a = 0.5186\ \ \mathrm{m^3/s} \text{ and } b = 0.9\ \mathrm{m}. \quad (7g)$$

Quadratic inverse function:

$$rV_\theta = \frac{a}{r^2} \text{ with } a = 0.0269\ \ \mathrm{m^4/s}\,. \quad (7h)$$

Quadratic modified inverse function

$$rV_\theta = \frac{a}{(b - r)^2} \text{ with } a = 0.359\ \frac{\mathrm{m^4}}{\mathrm{s}} \text{ and } b = 0.9\ \mathrm{m^2} \quad (7i)$$

Figure 2 represents all the spanwise distributions of specific angular momentum rV_θ, at the outlet section of the guide vanes.

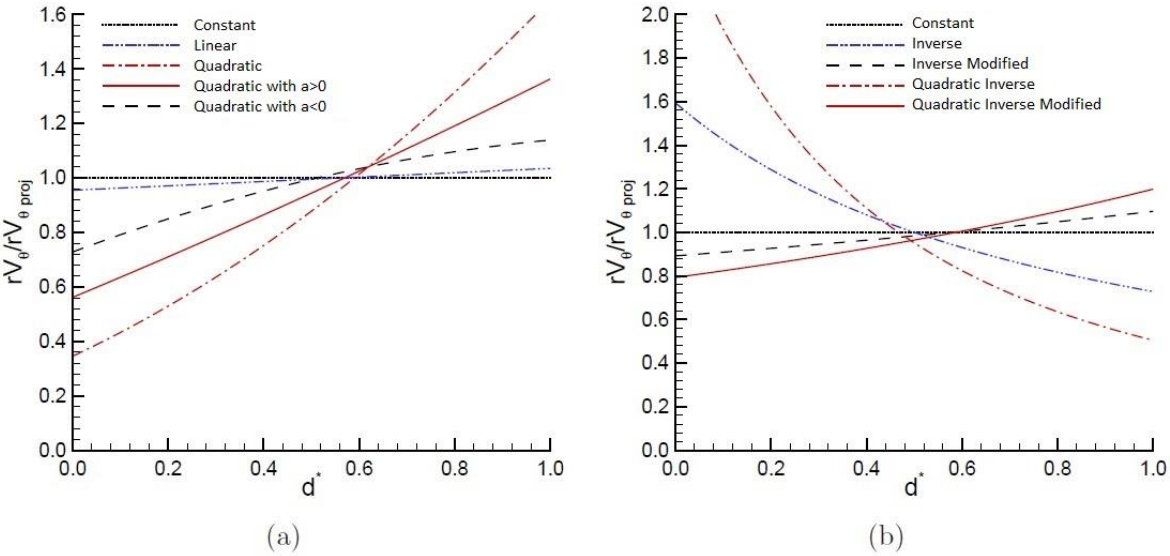

Figure 2. Spanwise distributions of angular momentum; (a) Constant, linear and parabolic distributions; (b) Inverse and quadratic inverse distributions.

3. RESULTS

The methodology described in previous section is applied on a tubular axial hydraulic turbine. The main nominal characteristics and dimensions of this turbine are flow rate Q=1.0 m^3/s, net head H=6.8 m, rotational speed N=750 r.p.m., power output P=59.2 kW, rotor tip diameter D=0.5 m and hub diameter D_H=0.214 m.

The velocity is nondimensionalized by the mean axial velocity component at the rotor inlet section

$$V_{ref} = \frac{4Q}{\pi(D^2 - D_H^2)} \tag{8}$$

and the angular momentum rV_θ by $V_{ref}\,D/2$.

The deflection of the absolute flow for these distributions is presented at table 1. The angle α at Tab. 1 is defined by Eq. 9, where z and θ are the axial and tangential directions, respectively.

$$\tan\alpha = \frac{V_\theta}{V_z}. \tag{9}$$

Table 1. Comparison of velocity absolute angle α, at inlet section of the rotor, for different angular momentum spanwise distributions.

Spanwise distribution	α at inlet section at the hub - α_H (°)	α at inlet section at the tip - α_T (°)(°)	α_T (°) - α_H (°)
Constant	51.41	23.26	28.15
Linear	49.18	24.43	24.75
Quadratic	18.69	49.47	-30.78
Quadratic a>0	29.85	36.92	-7.073
Quadratic a<0	37.90	28.51	9.40
Inverse	81.23	13.97	67.26
Inverse Mod.	46.00	26.65	19.34
Inverse Qua.	83.25	8.24	75.01

The spanwise distributions of angle α computed for constant, linear, quadratic with a<0 and inverse modified angular momentum spanwise distributions are compared at Fig. 3. Results presented at Tab. 1 show that some of the laws of angular momentum distribution may not be used as the absolute flow angle difference between hub and tip sections are too big. The smallest deflection is at quadratic distribution with *a* negative (9.40°) and the biggest one for the quadratic inverse distribution (75.01°). Quadratic, quadratic with a>0, inverse and inverse quadratic spanwise distributions are not going to be studied because they have too big α_T (°) - α_H (°) and will generate very twisted blades. Spanwise distributions of all the components of the velocity, V_z, V_r, V_θ and V_m, where *m* and *r* are the meridional and radial directions as well as velocity angles α and β were computed and compared. Some of these results are presented in this paper.

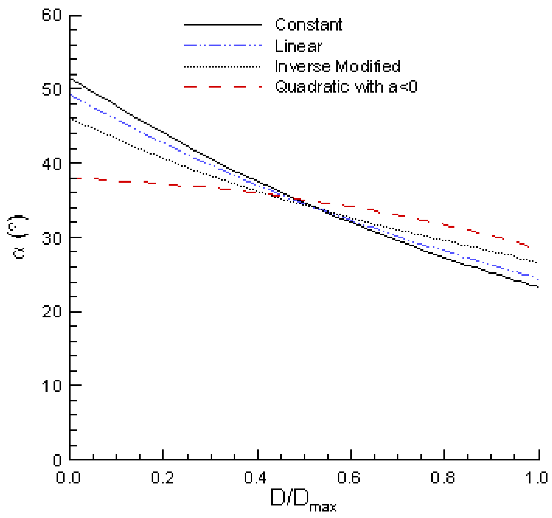

Figure 3. Spanwise distributions of velocity α angle at inlet section of the rotor.

The design of rotor blades must be done with a low CPU timer consumer and friendly program. Ferro *et al.* (2010) have used a panel method with boundary layer to design rotor blades. This method only can be applied when the magnitude of radial components of velocity is small and streamlines of the flow are located at cylindrical surfaces that are coaxial with turbine axe. Spanwise distributions of angle β between radial and axial velocity component, defined as

$$\tan\beta = \frac{V_r}{V_z} \tag{10}$$

are shown at Fig. 4 as well as radial velocity component distribution at Fig 5

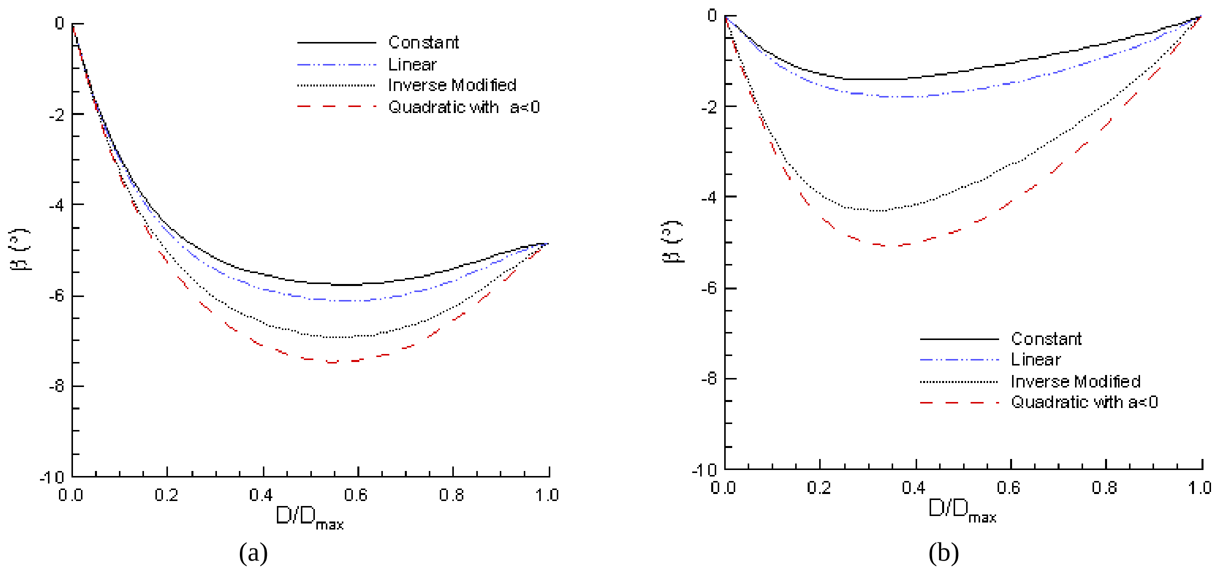

Figure 4. Spanwise distributions of velocity angle β on the rotor: (a) at inlet section and (b) outlet section.

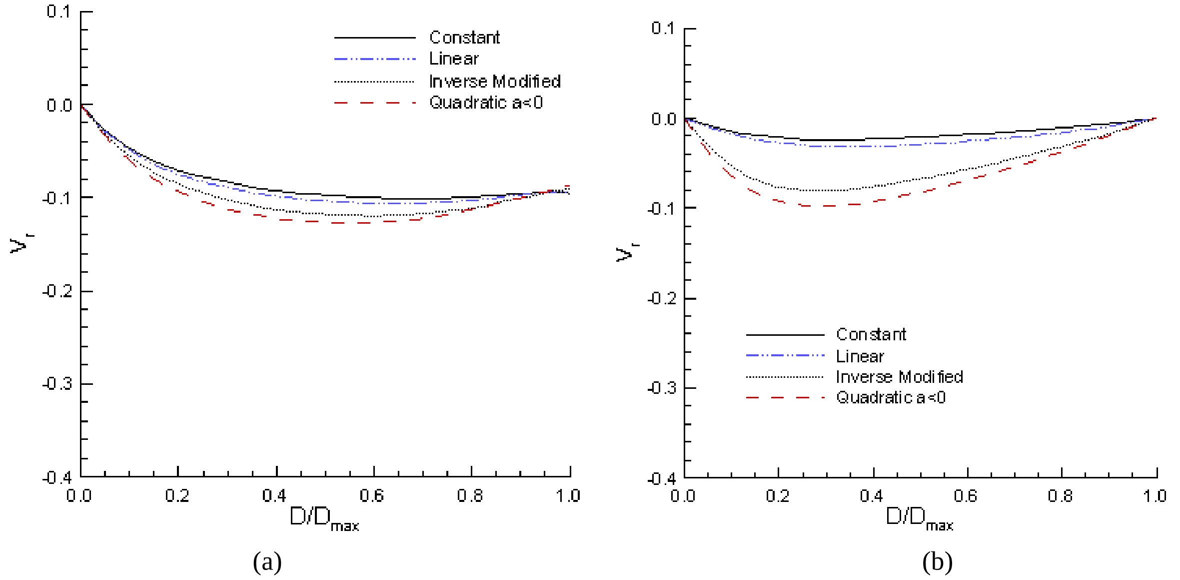

Figure 5. Spanwise distributions of radial velocity component V_r on the rotor (a) at inlet section and (b) at outlet section.

Results presented at Fig. 4 and 5 shows data radial component is small when compared with axial velocity component V_z. For all the angular momentum distributions the rotor geometry may be designed using the approximation of two dimensional flow on a cascade of profiles (Ferro *et al.*, 2011).

Velocities profiles of axial velocity component at inlet and outlet sections are compared at Fig. 6 and show that axial velocity spanwise variation is small. The greatest variation is for the quadratic spanwise distribution with an axial variation of almost 50% of the reference velocity V_{ref}. All the other variations are smaller less than 20%.

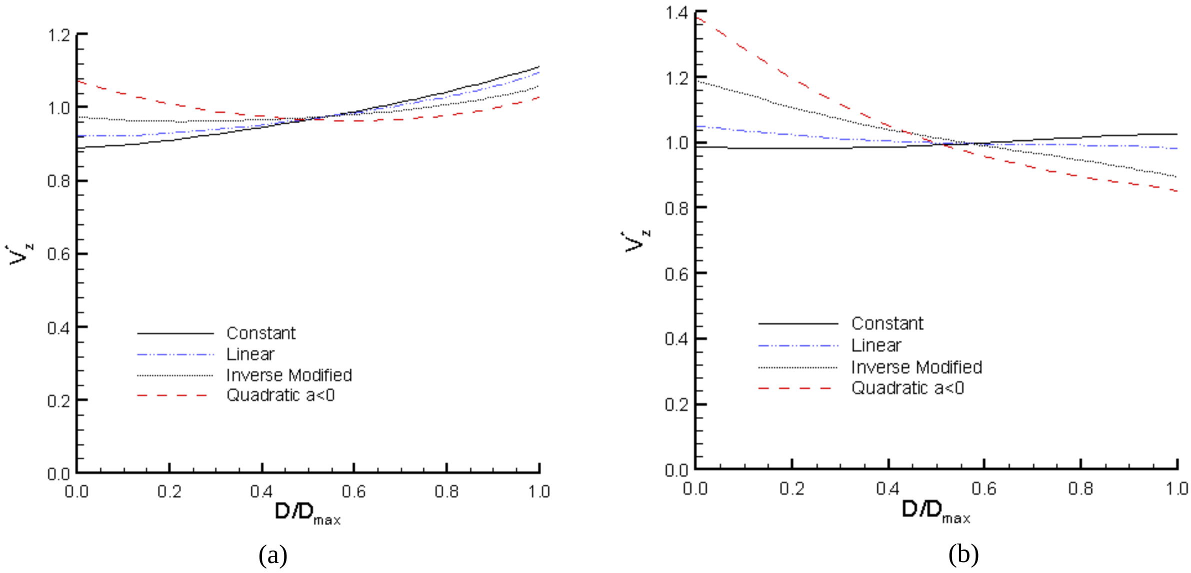

Figure 6. Comparison of spanwise distributions of dimensionless axial component of velocity at the rotor: (a) inlet section and (b) outlet section

Figure 7 shows the distribution of total pressure coefficient C_o, defined as

$$C_0 = \frac{p_{tot}}{\frac{1}{2}\rho V_{ref}^2}. \qquad (11)$$

The result shows that the new distributions present more kinetic energy close to the hub than constant distribution. As referred at (Ferro *et al*, 2009) this can be an advantage, as the Reynolds number becomes greater and the possibility of boundary layer separation will be lower.

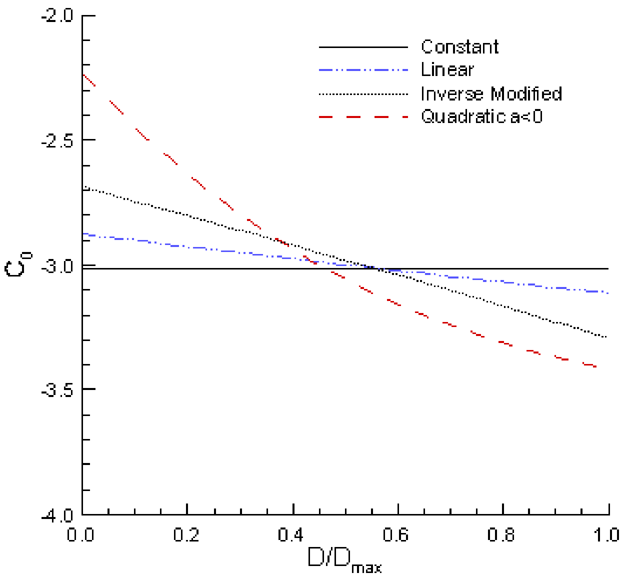

Figure 7. Comparison of spanwise distributions of total pressure coefficient C_O at outlet section of the rotor

4. CONCLUSION

A comparison between different spanwise distributions of angular momentum distribution on the design of the blades of small axial hydraulic turbine is presented. The flow, assumed as axisymmetric and inviscid, was computed using the streamline curvature method. Some of the computed distributions, inverse and inverse quadratic cannot be used, as produced very twisted blades and is used is not acceptable for the design of the rotor. Constant, linear, quadratic with $a<0$ and inverse modified distributions are acceptable. Linear and constant distributions have almost the same radial velocity components distributions, but linear distribution has a small inlet angle. The radial velocity component for all the referred distributions is low. A two-dimensional panel method for aerofoil cascade with boundary layer can be applied on the design of the rotor. Velocity spanwise profiles of axial and radial velocity components, as well of velocity angles α and β are shown. It is expected that the new distributions of angular momentum, will be better than constant distribution, with a less twisted rotor blade geometry, with a better flow behavior close to the hub.

5. ACKNOWLEDGEMENTS

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