25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COB-2019-1459
OSCILLATING MOTIONS IN PULSATING HEAT PIPE

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Abstract. *The objective of this work is to present the oscillating motions of a pulsating heat pipe equilavelente to a mass - spring - damper system with a single element. To do so, we proceed from the analysis of works of Ma et al. (2006), Chiang et al. al (2012) and Rao et al. (2014). Thus, it was observed in the three studies that the oscillatory movement was well characterized as a function of the filling fraction, the hydraulic diameter and the temperature difference between the evaporator and the condenser.*

Keywords: *Oscillating Motions, Mathematical Modelling, Pulsating Heat Pipe.*

1. INTRODUCTION

The pulsating heat pipes or oscilating heat pipe was propesed by Akachi(1990), its heat transfer mechanism is based on the oscillatory or pulsating movement of the vapor plugs and the liquid slugs along the tube. The pulsating heat pipe presents three basic regions: the evaporator, the condenser and the adiabatic region. Fig. 1 shows the typical structure of a pulsating heat pipe.

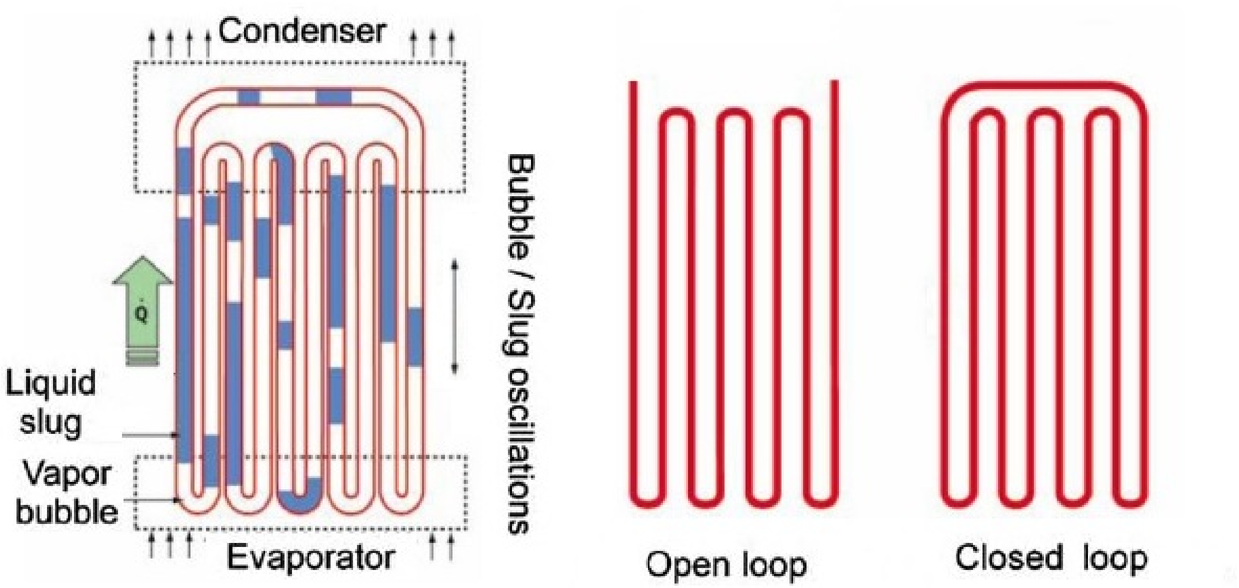


Figure 1. Schematic of a pulsating heat pipe and its design variations, Charoensawan et al. (p.2011, 2003)

According to Groll and Khandekar (2004) the mathematical modeling of pulsating heat pipe you can display models that use the a mass - spring system - damper composed of a single element, the work of Ma et al. (2006), Chiang et al. (2012) and Rao et al. (2014) fall into this case, models mass - spring system - damper consisting of multiple elements are found in the works of Gursel et al. (2015) and Cilleta (2016) . Application of mass, momentum and energy conservation equations to a given control volume of the pulsating heat pipe in Shafii et al. (2001) and Zhang and Faghri(2002).

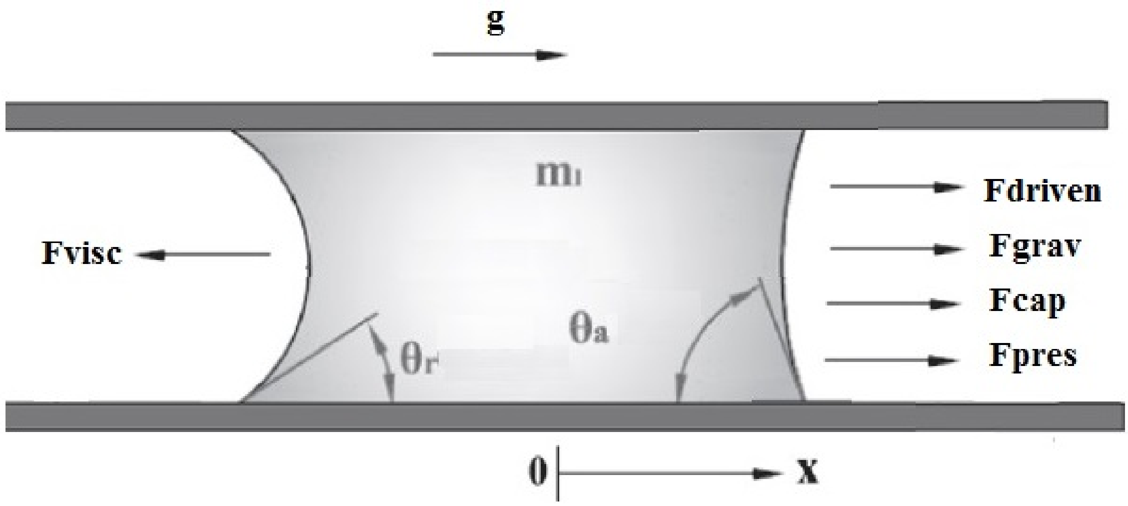
Mathematical analysis using chaos theory was developed in the work of Maezawa et al.(1996) and Maezawa et al.(2000). Adjustment of semi - empirical correlations based on non dimensional number presented in the work of Khandekar et al.(2003). Use of the concept of artificial neural networks in Khandekar et al. (2002).

The objective of this work is to analyze the mathematical models for a mass - spring - damper system composed of a single element from the works of Ma et al. (2006), Chiang et al. (2012) e Rao et al. (2014).

2. OSCILLATING MOTIONS IN PULSATING HEAT PIPE

The figure 2 shows the forces acting on a liquid slug in the evaporator in pulsating heat pipe.

Figure 2. Forces acting on a liquid slug in pulsating heat pipe



Fonte: Chiang et. al (2012, pp. 925)

In evaporating region heat is added that is transferred to the working fluid, so the saturated liquid evaporates the temperature of the evaporator T_{evap} with saturation pressure p_{sat}.The saturation temperature and pressure are described by the Clapeyron - Clausius equation (equation 1).

$$ln\left(\frac{p_{evap}}{p_0}\right)_{sat} \approx \frac{h_{lv}}{R}\left(\frac{1}{T_0} - \frac{1}{T_{evap}}\right)_{sat} \tag{1}$$

According to equation 1 the evaporator pressures p_{evap} and the condenser p_{cond} are equation 2 and equation 3, respectively.

$$p_{evap} = p_0 \exp\left[\frac{h_{lv}}{R}\frac{(T_{evap} - T_0)}{T_{evap}T_0}\right] \tag{2}$$

$$p_{cond} = p_0 \exp\left[\frac{h_{lv}}{R}\frac{(T_{cond} - T_0)}{T_{cond}T_0}\right] \tag{3}$$

The pressure difference Δp between the evaporator and condenser is given by equation 4.

$$p_{evap} - p_{cond} = p_0\left[\exp\left(\frac{h_{lv}}{R}\frac{(T_{evap} - T_{cond})}{T_{evap}T_{cond}} - 1\right)\right] \tag{4}$$

Utilizing a Taylor series introducing gas equation of state and neglecting high order terms the pressure difference can be written as equation 5.

$$\Delta p = p_{evap} - p_{cond} = \left[\frac{h_{lv}\rho_{vap,cond}(T_{evap} - T_{cond})}{T_{evap}} \right] \tag{5}$$

The expansion process occurring in the evaporator due to heat input and the contraction process occurring in the condenser due to heat rejection causes oscillatory movement of the pulsating heat pipe. The difference between the evaporator temperature T_{evap} and temperature the condenser T_{cond} have maximum values ΔT_{max} and minimum ΔT_{min} with oscillation frequency equal to ω_d. This temperature difference ΔT is expressed from equation 6.

$$\Delta T = T_{evap} - T_{cond} = \frac{\Delta T_{max} - \Delta T_{min}}{2}\left[1 + \cos(\omega_d t)\right] \tag{6}$$

The driving force of the oscillatory motion of the pulsating heat pipe is given by equation 7.

$$F_{motriz} = \left(\frac{A h_{l,v}\rho_{v,c}}{T_e} \right) \left(\frac{\Delta T_{max} - \Delta T_{min}}{2} \right) \left[1 + \cos(\omega_d t)\right] \tag{7}$$

Due to the interaction of the liquid pistons and the vapor bubbles with the walls of the tube along the flow the viscous frictional force arises, so the pressure gradient along the tube is given by equation 8.

$$\frac{dp_{visc}}{dx} = -\frac{4\tau_s}{D} \tag{8}$$

where τ_s is the frictional shear stress at the solid - liquid interfacegiven by equation 9.

$$\tau_s = \frac{1}{8} f\rho \left(\frac{dx}{dt} \right)^2 \tag{9}$$

Substituting equation 9 in equation 8 we have equation 10.

$$\frac{dp_{visc}}{dx} = -\frac{4}{D}\frac{f\rho}{8}\left(\frac{dx}{dt} \right)^2 = \frac{f\rho}{2D}\left(\frac{dx}{dt} \right)^2 \tag{10}$$

The viscous friction force F_{visc} is given by equation 11.

$$F_{visc} = A\left[(f_l.Re_l)\left(\frac{\mu_l L_l}{2D_h^2} \right) + (f_v.Re_v)\left(\frac{\mu_l L_v}{2D_h^2} \right) \right] \frac{dx}{dt} \tag{11}$$

According Incropera e Dewitt (2016) the friction factor f can be calculated by equation12.

$$\begin{aligned} f &= \frac{64}{Re} \qquad \text{para} \qquad Re < 2.10^3 \\ f &= 0.316 Re^{-1/4} \qquad \text{para} \qquad 2.10^3 < Re < 2.10^4 \\ f &= (0.79\ln(Re) - 1.64)^{-2} \qquad \text{para} \qquad 2.10^4 < Re < 5.10^6 \end{aligned} \tag{12}$$

Where Re is number of the Reynolds equation 13.

$$Re = \frac{GD}{\sigma} \tag{13}$$

The pressure of the vapor bubbles acting on the liquid piston consists of pressures acting at the liquid - vapor interface delimited by the cross section of the tube. At the instant of time t the vapor pressure is given by equation 14.

$$p_{vap,t} = \frac{m_{vap}RT}{L_{vap}A_{trans}} \tag{14}$$

For the time interval $t + \Delta t$ assuming that heat has been added in the evaporator an increase in pressure occurs so there is a decrease in the volume of the vapor given by $-xA_{trans}$. The pressure in the time interval $t+\Delta t$ is given by equation 15.

$$p_{vap,t+\Delta t} = \frac{m_{vap}RT}{(L_{vap} - x)A_{trans}} \tag{15}$$

Assuming $x << L_{vap}$ the pressure variation Δp_{vap} can be approximated by the equation 16.

(16)

$$\Delta p_{vap} = \frac{\rho_{vap}RT}{L_{vap}}x \tag{17}$$

The force due to vapor pressure F_{pres} is given by equation 18.

$$F_{pres} = \frac{A\rho_v RT}{L_v}x \tag{18}$$

According to Cilleta (2016) the capillary force arises as the effect of the pressure difference at the ends of the liquid piston because of the different angles of contact that change as the liquid piston moves. The capillary force F_{cap} in a liquid piston is given by equation19.

$$F_{cap} = \frac{2\sigma}{D}(\cos(\theta_a) - \cos(\theta_r)) \tag{19}$$

Where θ_a e θ_r are dynamic contact angle of advancing and receding. The force of gravity F_{grav} for a liquid piston is described by equation 20.

$$F_{grav} = m_{liq}g\sin(\alpha) \tag{20}$$

Where α is the inclination angle of the pulsating heat pipe.

2.1 MATHEMATICAL MODELING FOR SINGLE SPRING - MASS - DAMPER SYSTEM.

In the work of Ma et al. (2006) the oscillatory motion of the working fluid in the pulsating heat pipe is governed by equation 21 similar to the equation of the forced damped harmonic oscillator.

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t) \tag{21}$$

Where

$$\begin{aligned}
m &= A(L_l\rho_l + L_v\rho_v) \\
c &= A\left[(f_l Re_l)\left(\frac{\mu_l L_l}{2D_h^2}\right) + (f_v Re_v)\left(\frac{\mu_l L_v}{2D_h^2}\right)\right] \\
k &= \frac{A\rho_v RT}{L_v} \\
B &= \left(\frac{Ah_{l,v}\rho_{v,c}}{T_e}\right)\left(\frac{\Delta T_{max} - \Delta T_{min}}{2}\right) \\
f(t) &= B\left[1 + \cos(\omega_d t)\right]
\end{aligned} \tag{22}$$

The natural frequency of oscillation ω_n, o damping factor ζ and the damping frequency ω_d are given by equation 23, 24, 25.

$$\omega_n = \sqrt{\frac{k}{m}} \tag{23}$$

$$\zeta = \frac{c}{2m\omega_0} \tag{24}$$

$$\omega_d = \omega_n\sqrt{(1-\zeta^2)} \tag{25}$$

The initial conditions for the equation 21 are equation 26.

$$\begin{aligned}
x(0) &= 0 \\
x^{'}(0) &= 0
\end{aligned} \tag{26}$$

The equation 21 is non-homogeneous second order EDO with exact solution obtained from the Laplace transform (equation 27).

$$\begin{aligned}
x(t) &= \frac{B}{m}\left[\frac{(\sqrt{\zeta^2-1})\sin(\omega_d t) - e^{-\zeta\omega_d t}\sinh\left[\omega_d(\sqrt{\zeta^2-1})t\right]}{2\zeta\omega_d^2\sqrt{\zeta^2-1}}\right] \\
&+ \frac{1 - e^{-\zeta\omega_d t}\left[\cosh((\sqrt{\zeta^2-1})\omega_d t)\right] + \frac{\zeta\sinh((\sqrt{\zeta^2-1})\omega_d t)}{\sqrt{\zeta^2-1}}}{\omega_d^2}
\end{aligned} \tag{27}$$

In the work of Chiang et al. (2012) the performance of a horizontally closed pulsating heat tube with asymmetry including the capillary effects in the oscillatory movement was analyzed. A governing equation of the oscilatory motion is equation 28.

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2 x = \frac{G}{m}(1 + \cos(\omega_d t)) + \frac{S}{m} \tag{28}$$

Where

$$m = V_t[\phi\rho_{liq} - (1-\phi)\rho_{vap}] \tag{29}$$

e ω_n, ζ e ω_d are equations (23, 24 e 25).

The initial conditions for the equation 28 are equation 26. The exact solution of equation 28 is the equation 30.

$$\begin{aligned} x(t) &= \exp^{-\zeta\omega_n t}\left\{\left[-\frac{G}{m}(a+b)-\frac{S}{m}a\right]\cos(\omega_d t)\right. \\ &+ \left.\left[\frac{G}{m}\frac{-\gamma c-\zeta(a+b)}{\sqrt{(1-\zeta^2)}}-\frac{S}{m}\frac{\zeta}{\omega_n^2\sqrt{(1-\zeta^2)}}\right]\sin(\omega_d t)\right\} \\ &+ \frac{G}{m}\left[a+b\cos(\omega_d t)+c\sin(\omega_d t)\right]+a\frac{S}{m} \end{aligned} \tag{30}$$

Where

$$a = \frac{1}{\omega_n^2} \tag{31}$$

$$b = \frac{\omega_n^2-\omega_d^2}{(\omega_n^2-\omega_d^2)^2+(2\zeta\omega_n\omega_d)^2} \tag{32}$$

$$c = \frac{2\zeta\omega_n\omega_d}{(\omega_n^2-\omega_d^2)^2+(2\zeta\omega_n\omega_d)^2} \tag{33}$$

$$\gamma = \frac{\omega_d}{\omega_n} \tag{34}$$

In Rao et al. (2014) capillary effects were also included. From Newton's law the governing equation for fluid flow in pulsating heat pipe is equation 35.

$$\begin{aligned} A(L_l\rho_l+L_v\rho_v)\frac{d^2x}{dt^2} &+ 32\left[\left(\frac{\mu_l L_l+\mu_v L_v}{2D_h^2}\right)\right]A\frac{dx}{dt}+\frac{A\rho_v RT}{L_v}x \\ &+ \frac{N\left[2\sigma(\cos(\theta)_r-\cos(\theta)_a\right]A}{R} \\ &= \left(\frac{Ah_{l,v}\rho_{v,c}}{T_e}\right)\left(\frac{\Delta T_{max}-\Delta T_{min}}{2}\right)\left[1+\cos(\omega_d t)\right] \end{aligned} \tag{35}$$

The initial conditions for the equation 35 are equation 26.

3. RESULTS

For the comparison of the results of the works Ma et al. (2006), Chiang et al. (2012) and Rao et al. (2014) the data that were used are found in the table 3.

Table 1. Parameters for the analysis of positions and velocity as a function of time

L_{tube}	Working fluid	FR	ΔT_{max}	ΔT_{min}	ΔT	T_{ope}	D_i	T_{cond}	T_{evap}	θ_a	θ_r
0.3048 m	Water	70%	$50°C$	$55°C$	$5°C$	$60°C$	1.65 mm	$40°C$	$80°C$	84°	33°

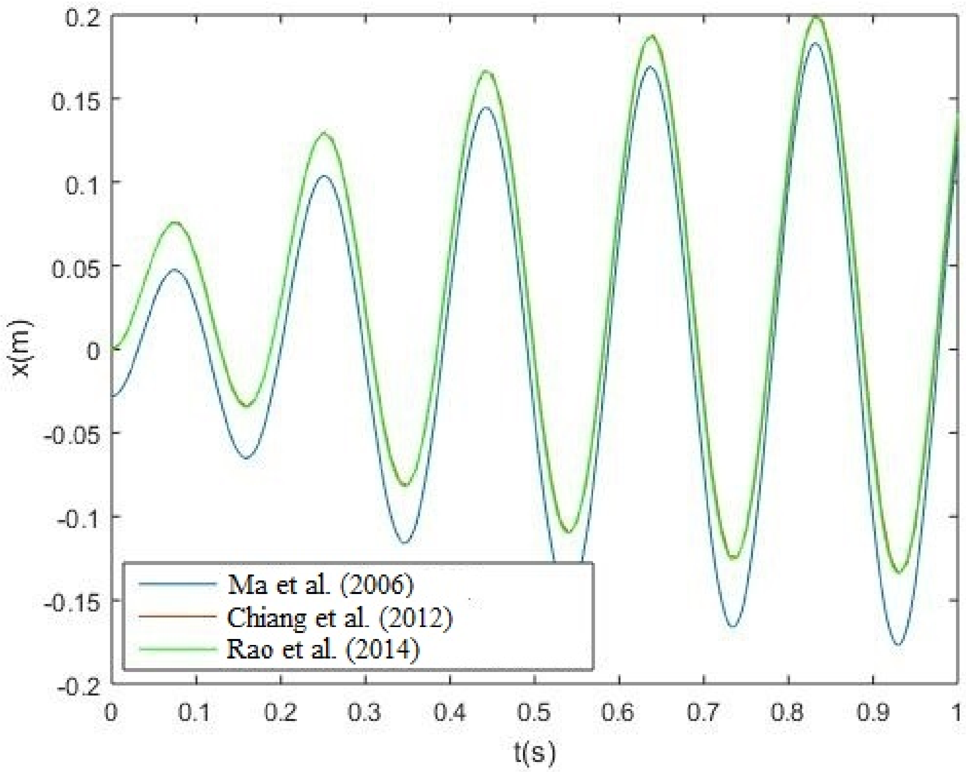


Figure 3. Displacement response

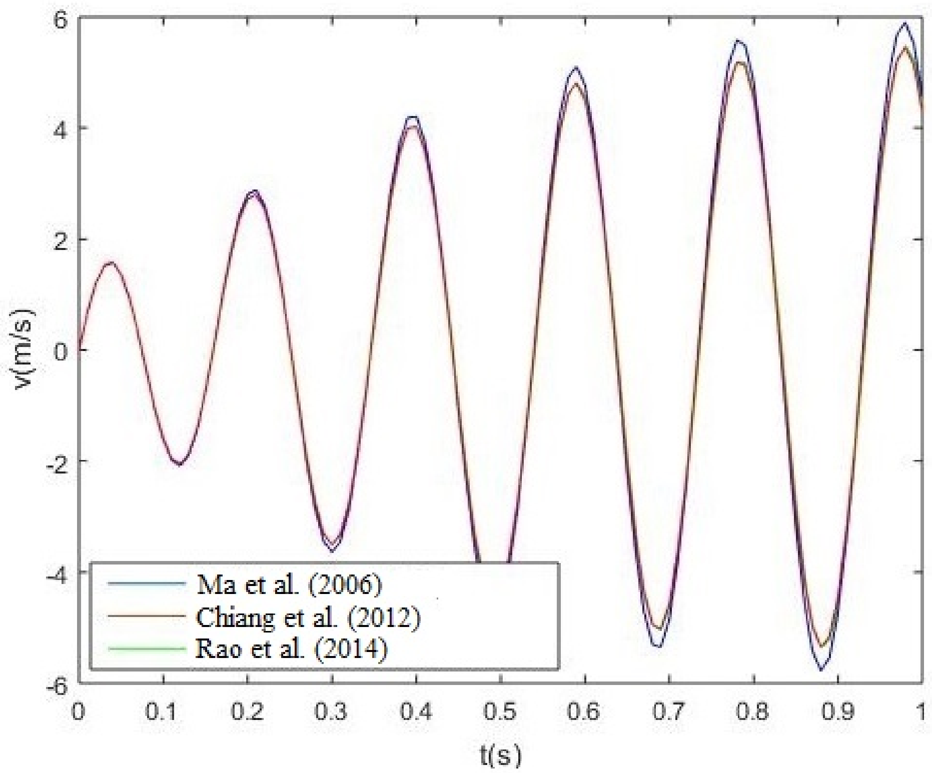


Figure 4. Velocity response

As we can see in figure 4 a) the works of Chiang et al. (2012) and Rao, Gupta; Ramanarasimha (2014) present very close results when compared to work Ma; Hanlon; Chen(2006) present higher oscillation amplitude but the same oscillation frequency. This difference occurs due to the inclusion of capillary forces in the results. A Velocity response figure 4 b) is very similar in the works Ma; Hanlon; Chen(2006) and Chiang et al. (2012).

4. CONCLUSIONS

Based on the analysis of the works can be concluded that the different approaches of the works present results well results for the displacement and the velocity of a liquid piston along the pulsating heat pipe.

5. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support given by CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior).

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