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STUDY OF MODIFIED FATIGUE LIFE OF CYLINDRICAL ROLLER BEARING UNDER LUBRICATION REGIME

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Abstract. *Rolling bearings are widely used in industry and they are fundamental for the proper functioning of machines to which they are coupled. Thus, it is essential an accurate calculation of their fatigue life. There are several effects that these roller bearings are submitted to, and this situation can make the calculation of fatigue life not sufficiently precise. It is known that most of the failures are due to the wear effects involving the surfaces in contact. Therefore, lubrication has proved to be a good solution for improving bearings operating condition. Using concepts of elastohydrodynamic (EHD) lubrication, oil film thickness can be estimated for proper working conditions of the system, as well as a better choice of viscosity for the lubricating fluid. The fatigue life with lubrication can, then, be calculated considering ABNT NBR ISO 281:2010 standard. The aim of this paper is to show the importance of considering lubrication and its impact in the proper choice of lubricant used in the lubrication process, besides relating the L10 improvement with the lubricant film growth.*

Keywords: *elastohydrodynamic lubrication, viscosity, nonlinear optimization, dynamic of cylindrical roller bearing.*

1. INTRODUCTION

Cylindrical roller bearings are widely used in industry because they are capable to support high loads on the contact area between rolling elements and raceways. A well-designed roller bearing provides a more careful approach about lubrication conditions and dynamic behavior. Lubricated contacts contribute to a more extended life of the rolling bearings because they help on heat dissipation and reduction of friction between contacting parts. According to NSK data of Campanha (2008), about 3% of the rolling bearings do not reach the calculated fatigue life, because the factors that involve calculating the rating life of these elements require attention to the exact working conditions and a more accurate dynamic analysis to list probable sources of failure.

The procedure for calculating fatigue life modified by lubrication presented in this study associates the load distribution on the rolling bearing and the regime of lubrication. The load distribution model is proposed by Harris (1991) and the most common type of lubrication for non-conforming contacts is the elastohydrodynamic lubrication (Dowson and Higginson, 1977). In this work, the lubrication conditions are estimated from analytical solution that allows the calculation of the minimum film thickness in EHD line contacts. Viscosity is another important parameter, once the oil film thickness conditions directly depend on the feasible bands of the ratio between the minimum thickness of the oil film and the average roughness of the surfaces in contact (lubrication factor λ). In 2007 the ISO 281 standard was published to parameterize the calculation of the bearings' life considering the correction by the lubrication factor, taking into account the kinematic apparent viscosity parameters. Theoretically, higher viscosities may result in a higher oil film thickness, but it does not guarantee a direct relationship in the increase of fatigue life, since viscous drag usually were not considered in previous studies. The proper choice of fluid is related to speed, working temperature and distribution load, because viscosity of the oil is strongly affected by these parameters and, consequently, affecting the lubrication conditions. In this study, temperature and rotation speed are considered in the viscous behavior curves of the fluid, referenced from Vogel Equation (Knežević and Savić, 2006)

2. METHODOLOGY

2.1 Mathematical model

Rolling bearings are elements of machines and consist mainly of inner and outer raceways, cage and rolling elements (Fig. 1). The rollers that provide movement between raceways are an important part of the bearing set. They can have

special geometry and manufacture. Many rollers have crowned profile at their ends which, according to studies, contributes significantly to a better fatigue life (Göncz et al., 2015).

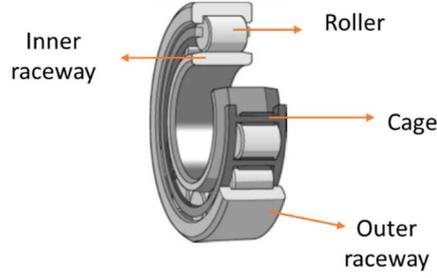


Figure 1. Components of cylindrical roller bearing (adapted from Harris, 2007).

2.1.1 Kinematic analysis

For dynamic analysis, it is necessary to calculate the main speeds involved in the running of the rolling bearing, whereas the inertial force F_c acting on the rolling elements directly interferes on the load distribution in the contacts. Lubrication also depends on speed. Based on the operating conditions of the bearing raceways, the velocities of bearing components change lubrication conditions and, consequently, influence the life of these machine elements.

The speed of the internal (n_i) and external (n_o) raceways, roller (n_r), cage (n_m) and entrainment velocity (u_i) are fundamental for the description of dynamics and lubrication in this study.

Considering the following rotation relation:

$$\omega_m = \frac{2\pi}{60} n_m \quad (1)$$

The main speeds can be defined as:

$$n_m = \frac{1}{2} \left[n_i \left(1 - \frac{D_r}{d_m} \right) + n_o \left(1 + \frac{D_r}{d_m} \right) \right] \quad (2)$$

$$n_r = \frac{1}{2} \frac{d_m}{D_r} \left(1 - \frac{D_r^2}{d_m^2} \right) (n_o - n_i) \quad (3)$$

$$u_i = \frac{d_m}{2} \left[\left(1 - \frac{D_r}{d_m} \right) (\omega_i - \omega_m) + \frac{D_r}{d_m} \omega_r \right] \quad (4)$$

where d_m is the bearing pitch diameter and D_r is the roller diameter.

2.1.2 Hertz contact model

The Hertz contact model is widely used to characterize nonconforming contacts, where elastic deformation occurs due to a small area of contact compared to the radii of curvature of the bodies. In Harris (1991), the Hertz contact model is employed in the distribution of loads on cylindrical roller bearings. In Fig. 2(a), the external radial force F_r applied to the set is distributed on the rolling bearing, where Q_{oj} and Q_{ij} are load on outer and inner raceway, respectively. In Eq. (5) is representing an equilibrium of forces in each rolling element - Fig. 2(b) - at any Azimuth angle position ψ_j . The load-displacement relation is represented by Eq. (6) and K_n is the equivalent dry contact stiffness.

$$Q_{oj} - Q_{ij} + F_c = 0 \quad (5)$$

$$Q = K_n \delta^{10/9} \quad (6)$$

Associating Eq. (5) and Eq. (6), thus

$$K_n \delta_{oj}^{10/9} - K_n \delta_{ij}^{10/9} - F_c = 0 \quad (7)$$

The radial elastic deformation δ_{rj} of each rolling element can be evaluated in function of deformation on inner and outer raceway.

$$\delta_{rj} = \delta_{ij} + \delta_{oj} \quad (8)$$

For any angular position ψ_j , the total radial deformation δ_r of bearing can be determined by Eq. (9) (for null clearance).

$$\delta_r = \frac{\delta_{rj}}{\cos(\psi_j)} \quad (9)$$

Equilibrium of forces on cylindrical roller bearing, Fig. 2(a), is evaluated in the inner raceway in the direction of the applied radial force according to Eq. (10).

$$F_r - \sum_{j=1}^z Q_{ij} \cos(\psi_j) = 0 \quad (10)$$

Replacing Eq. (8) and Eq. (9) in Eq. (7), the force on each loaded roller element is given by:

$$K_n (\delta_r \cos(\psi_j) - \delta_{ij})^{10/9} - K_n \delta_{ij}^{10/9} - F_c = 0 \quad (11)$$

Inertial force F_c acting on each roller is described as:

$$F_c = \frac{\pi^3 \rho D_r^2 L_{eq} d_m n_m^2}{7200} \quad (12)$$

Considering L_{eq} the effective length of the roller, ρ the density of roller material and all dimensions in millimeters.

Equations (10) and (11) represent a nonlinear system with δ_r and δ_{ij} unknowns and the Newton Raphson method is used to solve them. After calculating the displacement, it is possible to calculate the load on each loaded roller.

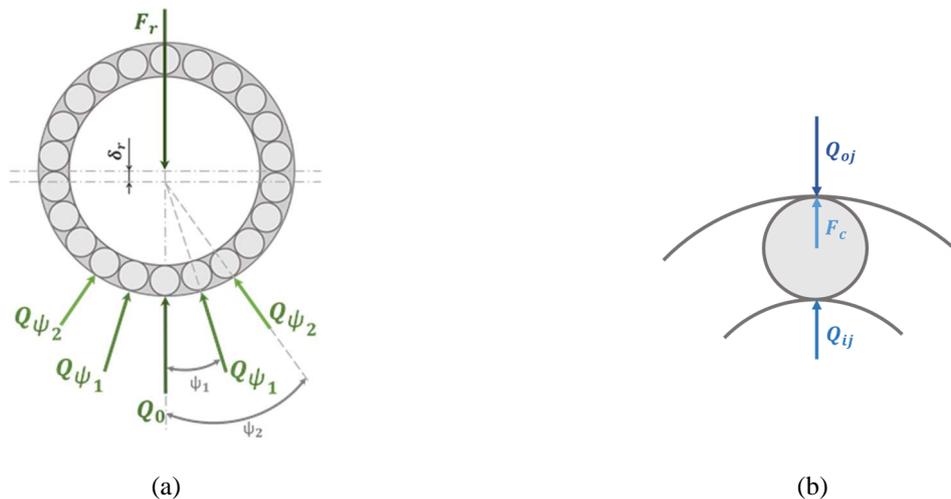


Figure 2. (a) Bearing load distribution diagram (adapted from Tsuha, 2019).
(b) Loads on a rolling element at Azimuth angle position ψ .

2.1.3 Lubrication model

Friction is a physical variable present in the mechanical contacts and, until today, a lot of cases are studied in which the friction can be minimized, thus avoiding losses due to wear and heating. Lubrication of the contacts, made with graphite, grease or mineral oil, enables less aggressive operating conditions, because the friction present in these contacts is minimized and, as a result, better operating conditions of the machines are obtained. The type of lubrication in which the contacts may be subjected is related to the geometric characteristics of the system and operating conditions, such as temperature, load, rotation speed and material. For nonconforming contacts, under high pressure and elastic deformation, the most suitable type of lubrication is the elastohydrodynamic (EHD).

The EHD lubrication had its first studies in the late 19th century with Barus. Since then, several papers have been published to solve and better understand its physical phenomenon (Nonato, 2006). With the development of computing, the numerical solution became feasible and later, it was possible to establish specific equations for the calculation of the central and minimum film thickness, as well as the location of the point of higher pressure and its magnitude. In this study, the important information that lubrication brings is the minimum oil film thickness. For more efficient operation, it is necessary that the contact is minimally lubricated, thus avoiding the dry friction.

Dowson and Higginson (1977), through experimental results, established a dimensionless relation for the calculation of the minimum thickness of oil film:

$$H_{\min} = 2.65 U^{0.7} \cdot G^{0.54} \cdot W^{-0.13} \quad (13)$$

The terms involved in Eq. (13) are written according to the dimensionless groups of load (W), speed (U) and material (G), where Q_{\max} is the force on the most loaded element (from dynamic analysis), E' is the effective elastic modulus, R_{eq} is the effective radius of curvature, α is the pressure-viscosity coefficient, η_0 is the viscosity at atmospheric pressure and R_1 and R_2 are the radii of curvature of bodies in contact.

$$W = \frac{Q_{\max}}{2E'R_{eq}} \quad (14)$$

$$G = \alpha E' \quad (15)$$

$$U = \frac{\eta_0 u_i}{2E'R_{eq}} \quad (16)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad (17)$$

The pressure-viscosity coefficient is empirically defined by Hutchings and Shipway (2017) in Eq. (18), whereas η_0 is in [mPa·s] and α in [Pa⁻¹]:

$$\alpha = (0,6 + 0,965 \log \eta_0) 10^{-8} \quad (18)$$

The oil film thickness is not the only factor associated with good lubrication conditions, but also the average roughness of the contacts. Therefore, the λ ratio was established to classify the lubrication regimes (boundary, mixed and full film), where h_{\min} is the dimensional film thickness and R_a and R_b the average roughness (RMS) of the related surfaces.

$$\lambda = \frac{h_{\min}}{\sqrt{R_a^2 + R_b^2}} \quad (19)$$

According to Göncz et al (2015), the inner raceway has a more severe lubrication condition than the outer raceway, since the inner raceways have smaller contact area. Thus, the analyzes of film thickness and load distribution are directed to this critical region of lubrication. Studies published by He et al. (2017) suggest that the elastohydrodynamic lubrication regime can be considered as accepted in a range of parameter λ between 1.5 and 8.

2.1.4 Modeling of viscosity grade of mineral oils

The most appropriate viscosity for a mechanical system is part of the relation between the operating conditions of the system and the lubrication requirements.

For cases where lubrication oil film is thinner, the conditions of the lubricating fluid become even more sensible, since the increase of the viscosity provides thicker films, but on the other hand, the losses by viscous friction increase. A viscosity curve model for mineral oils under the influence of constant temperature is proposed by Knežević and Savić (2006). The curves are fitted according to Vogel equation – Eq. (20) – from the standardization proposed by ISO 3448:1992. The constants a, b, c were calculated with the aid of commercial software Matlab® by the method of the trust region of Dogleg, developed for solution of nonlinear equations. Figure 3 shows the viscous behavior of some mineral oils under temperature variation.

$$\mu_0 = a e^{\frac{b}{(T-c)}} \quad (20)$$

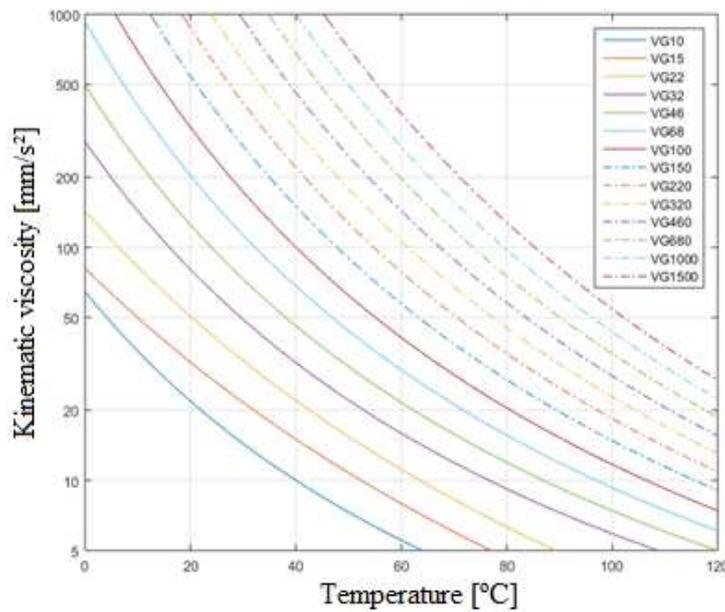


Figure 3. Viscosity grade of mineral oils.

2.1.5 Fatigue life adjusted for lubrication

The rolling bearing fatigue life presented by Lundberg and Palmgren (1947) is given by

$$L_{10} = \left(\frac{C_d}{P}\right)^n \quad (21)$$

Being L_{10} the life calculated for reliability of 90%, C_d the dynamic load capacity for $L_{10} = 1 \times 10^6$ revolutions, P the equivalent load applied to the bearing and n the exponent relative to each kind of contact (for line contact is 10/3). According to Harris (1991), the dynamic load capacity is defined in Eq. (22) and considers geometric and manufacturing aspects of the rolling bearing.

$$C_d = b_m f_c (L_{eq})^{7/9} (Z)^{3/4} D_r^{29/27} \quad (22)$$

With:

$$f_c = 207.9 \lambda_r \nu_r \left(\frac{D_r}{d_m}\right)^{2/9} \frac{(1 - \frac{D_r}{d_m})^{29/27}}{(1 + \frac{D_r}{d_m})^{1/4}} \left[1 + \left\{ 1.04 \left(\frac{1 - \frac{D_r}{d_m}}{1 + \frac{D_r}{d_m}}\right)^{143/108} \right\}^{9/2} \right]^{-2/9} \quad (23)$$

where λ_r is the reduction factor to account for edge loading and non-uniform stress, ν_r is the reduction factor used in conjunction with a load-life exponent $n=10/3$ and b_m is the rating factor for contemporary material.

The modified fatigue life studies started in the 1970s with researches on defects due to failure (Campanha, 2008). More specific circumstances for lubrication condition have been studied and, thus, in 2007 the ISO 281 standard proposes the fatigue life corrected by the lubricant working conditions.

$$L_{10m} = a_1 a_{iso} \left(\frac{C_d}{P}\right)^n \quad (24)$$

The a_{iso} correction is a function of the oil contamination parameter e_c (which is related to the characteristics of the particles and the filter presented in the lubrication system), the basic static load C_{ru} is static load that cause a permanent deformation of 0.01% of diameter of the most loaded rolling element, the nominal load P is applied to the bearing and K is the intrinsic viscosity ratio of each lubricant. The reliability factor a_1 and the contamination factor e_c can be consulted in the standard ISO 281.

$$a_{iso} = f\left(\frac{e_c C_{ru}}{P}, K\right) \quad (25)$$

The variable K , obtained by Eq. (26), considers the viscosity associated with system operation speed and the average diameter of the rolling bearing (v_1). Parameter v_1 can be calculated by Eq. (27) and Eq. (28) and kinematic viscosity (v) is obtained from lubricant data curves (Fig. 3).

$$K = \frac{v}{v_1} \quad (26)$$

$$v_1 = 45000n_i^{-0.83}D_r^{-0.5} \quad \text{for } n_i < 1000 \text{ rpm} \quad (27)$$

$$v_1 = 4500n_i^{-0.83}D_r^{-0.5} \quad \text{for } n_i \geq 1000 \text{ rpm} \quad (28)$$

Fatigue limit load (C_{ru}) is given by Eq. (29) and Eq. (30):

$$C_{ru} = 0.2453 Z Q_{\max}, \quad \text{if } dm \leq 100 \text{ mm} \quad (29)$$

$$C_{ru} = 0.2453 Z Q_{\max} \left(\frac{100}{dm}\right)^{0.3} \quad \text{if } dm > 100 \text{ mm} \quad (30)$$

Once the coefficients of fatigue life correction are known, the term a_{iso} can be obtained:

$$a_{iso} = 0.1 \left[1 - \left(1.5859 - \frac{1.3993}{K^{0.054381}} \right) \left(\frac{e_c C_{ru}}{P} \right)^{0.4} \right]^{-9.185} \quad \text{for } 0.1 \leq K < 0.4 \quad (31)$$

$$a_{iso} = 0.1 \left[1 - \left(1.5859 - \frac{1.2348}{K^{0.19087}} \right) \left(\frac{e_c C_{ru}}{P} \right)^{0.4} \right]^{-9.185} \quad \text{for } 0.4 \leq K < 1 \quad (32)$$

$$a_{iso} = 0.1 \left[1 - \left(1.5859 - \frac{1.3993}{K^{0.071739}} \right) \left(\frac{e_c C_{ru}}{P} \right)^{0.4} \right]^{-9.185} \quad \text{for } 1 \leq K < 4 \quad (33)$$

3. Optimization process of lubricating oil film

Optimization is a mathematical tool that allows finding the best minimization condition of a function, in which there are limits to be evaluated (Luenberger and Ye, 2008). The optimization method used is the nonlinear constraints. The function to be minimized is the H_{min} , Eq. (13) because the minimum oil film thickness is required. Three nonlinear constraints are applied to the system. The restriction g_1 is related with Eq. (19). For the EHD lubrication regime, the parameter λ is recommended to be between 1.5 and 8, according to He et al. (2017). g_2 are the limits of kinematic viscosity (Fig. 3). g_3 is the relation of the operating lubrication regimes for line contact proposed in Harris (1991).

$$\begin{aligned} \text{Minimize} \quad & H_{\min} = 2.65 U^{0.7} \cdot G^{0.54} \cdot W^{-0.13} \\ \text{Subjected to} \quad & g_1(v): 1.5 \leq \lambda \leq 8 \\ & g_2(v): 5 \leq v \leq 1000 \\ & g_3(v): \log_{10} \left[1.5 \cdot 10^6 \left(\frac{G}{5000} \right)^2 \frac{Q_{\max}^3}{U} \right] \geq 1 \end{aligned} \quad (34)$$

The optimization method of this work was developed using the *fmincon* function from Matlab®. This function solves problems of constrained nonlinear programming based on gradient method with three algorithms: interior point, sequential quadratic point (SQN) and trust region reflective.

4. Numerical Solution

The theoretical investigation proposed in this study starts with the Hertz contact analysis to obtain the load distribution on the rolling elements of the bearing in the raceways. According to Schäfer (2016), the condition of minimum oil film occurs in inner raceway, so the calculations involving modified fatigue life are dedicated to this region.

The process of choosing the best lubricant that provides a minimum thickness of oil film is calculated using constrained nonlinear optimization process. The most suitable lubricant is selected, and the fatigue life is adjusted according to standard ISO. In this study, modified fatigue life is based on the guidelines contained in ABNT NBR ISO 281:2010 standard.

The iterative method finishes when all restrictions are fulfilled. Figure 4 shows a computational processing flowchart used to calculate the modified fatigue life.

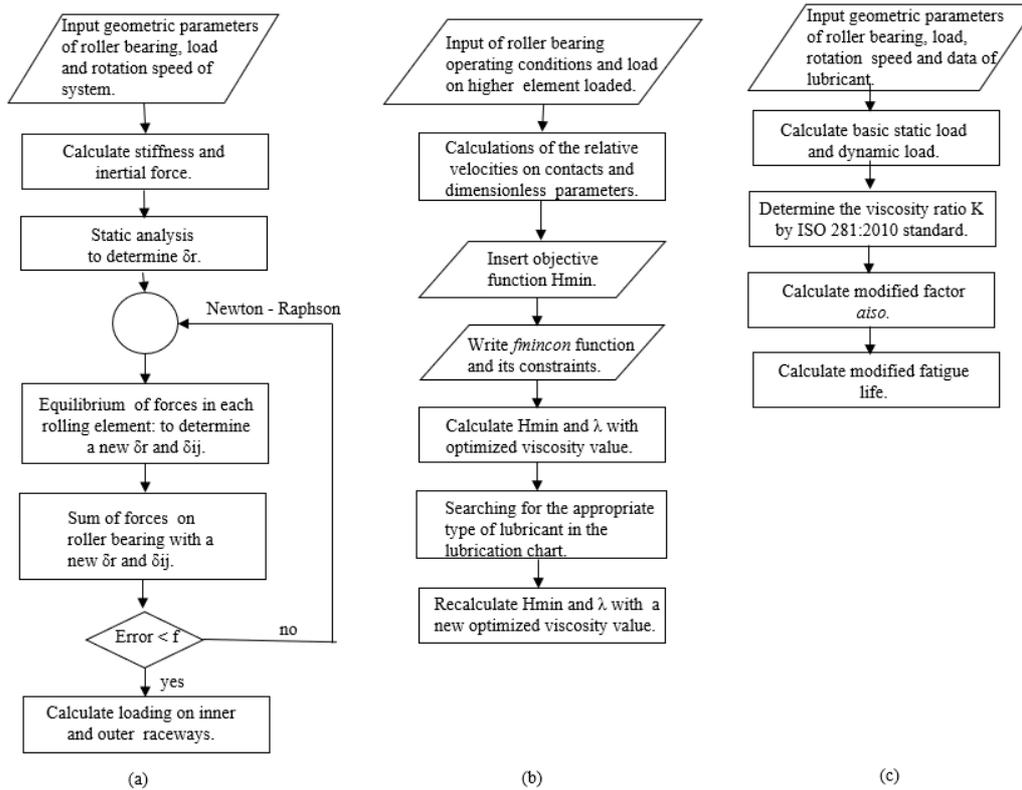


Figure 4. Flowchart of the computational procedure used to calculate the modified fatigue life. (a) Load distribution. (b) Optimized oil film thickness. (c) Modified fatigue life.

5. Results

The data used for the numerical procedure proposed, in Fig. 4, are for a 209 cylindrical roller bearing specified in Tab. 1, with surface roughness (RMS) of 0.1 μm for raceways and rollers (Hamrock et al.,2004) and K_n is $5.869 \times 10^5 \text{ N/mm}^{10/9}$. The viscosity grade used is composed only of mineral oils, ranging from VG 10 to VG 1500.

Table.1: Characteristics of cylindrical roller bearing 209.

Pitch diameter - d_m	65 mm	Contamination factor - e_c	0.8
Number of rollers - Z	14	Dynamic capacity C_d	38799 N
Roller diameter - D_r	10 mm	Nominal load - P	4450 N
Roller effective length - L_{eq}	9.6 mm	Edge Factor - λ_r	0.45
Inner raceway diameter - d_i	54.991 mm	Load-life Factor - ν_r	1.36
Outer raceway diameter - d_o	75.032 mm	Material Factor - b_m	1.2
Diametral clearance - P_d	0	Fatigue life - L_{10}	$1364 \times 10^6 \text{ rev}$

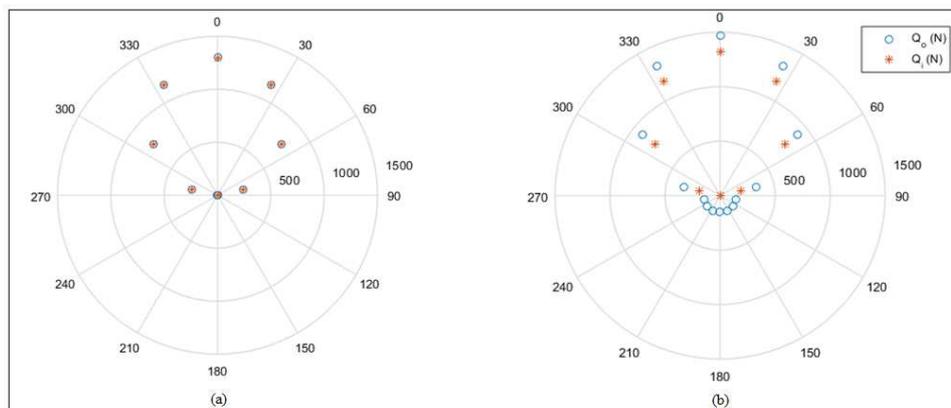


Figure 5. Distribution of external radial load of 4450 N on 209 cylindrical roller bearing. (a) Rotation speed of 1000 rpm. (b) Rotation speed of 20000 rpm.

Figure 5 shows the load distribution under two rotation speed. It is noticed that with increasing rotation speed, the magnitude of the contact forces of the outer raceway also increases, due to the action of inertia force given by Eq. (12). This change causes a rebalancing of contact forces of the inner raceway, thus, changing their magnitudes, as shown by Tab. 2. Assuming the most critical conditions of oil film thickness occurs on inner raceway, the load Q_{max} will be represented by the force on the inner raceway Q_i .

Table 2. Results for optimized condition of the oil film and radial external force of 4450 N.

Cases	Rotation speed [rpm]	Load Q_{max} [N]	Minimum viscosity required (optimized) [mm ² /s]	λ ratio	Minimum thickness [μ m]
1	500	1298.73	96.26	1.500	0.300
2	1000	1298.76	52.11	1.506	0.301
3	4000	1299.71	15.95	1.505	0.301
4	8000	1302.54	11.38	1.840	0.368
5	15000	1311.90	11.41	2.860	0.573
6	20000	1322.06	11.41	3.490	0.699

According to Tab. 2, the minimum viscosity required for a suitable operating condition of a lubricated system under an EHD regime decreases with the increasing of rotation speed. The λ ratio increases to satisfy the system optimization conditions and the minimum oil film thickness also increases. Systems that operate at low speeds (about 5000 rpm) require more attention, because any change in operating conditions can cause changes in the lubrication film.

Table 3: Lubricant type selection for commercial oils.

Cases	Operating Temperature [°C]	Minimum viscosity required by optimization [mm ² /s]	Viscosity [mm ² /s]	Lubricant ISO VG
1	40	96.26	100	100
	60		106	320
	80		97	1000
2	40	52.11	68	68
	60		57	150
	80		57	460
3	40	15.95	22	22
	60		21	46
	80		20	100
4	40	11.38	15	15
	60		16	32
	80		12	46
5	40	11.41	15	15
	60		15.9	32
	80		12	46
6	40	11.41	15	15
	60		15.9	32
	80		12	46

Table 3 shows the type of lubricant ISO VG selection based on the minimum viscosity required by optimization of cases 1 to 6 from Tab. 2. This analysis was evaluated varying the average operating temperature, besides the rotation speed range already considered. When the bearing is under lower rotation speed (cases 1, 2 and 3), the choice of a single lubricant type for the same temperature operation is compromised. Both parameters – speed and temperature – affect the choice of oil. However, when the rotation speed increases (cases 4, 5 and 6), the type of lubricant recommended based on viscosity optimization is the same for the same temperature. In the case studied, lubricant ISO VG 15 is the recommendation for 40°C. Lubricant ISO VG 32 and ISO VG 46 are the recommendations, respectively, for 60°C and 80°C. Analyzing the cases with the same type of lubricant, the influence of rotation speed can be observed in Fig. 6. The

minimum film thickness, as well as factor λ , increase with the increment of rotation speed, which indicates decreasing of friction between the rolling surfaces. In Fig. 7, the modified fatigue life (L_{10m}) is quite representative when compared to the traditional calculation of fatigue life (L_{10}) of 1364×10^6 rev (Tab. 1). Since the traditional fatigue life L_{10} does not account the lubricant and speed conditions, it remains constant for all cases studied. The viscosity of the lubricating fluid is a very sensitive parameter to temperature, which makes this information a relevant data for a more elaborate estimation of bearings' life and interferes with the choice of lubricant type that meets the viscosity requirement for full lubrication. At the same rotation speed, the film thickness and factor λ decreases for the higher temperature, leading to a decrement in the calculation of roller bearing modified fatigue life (Fig. 7).

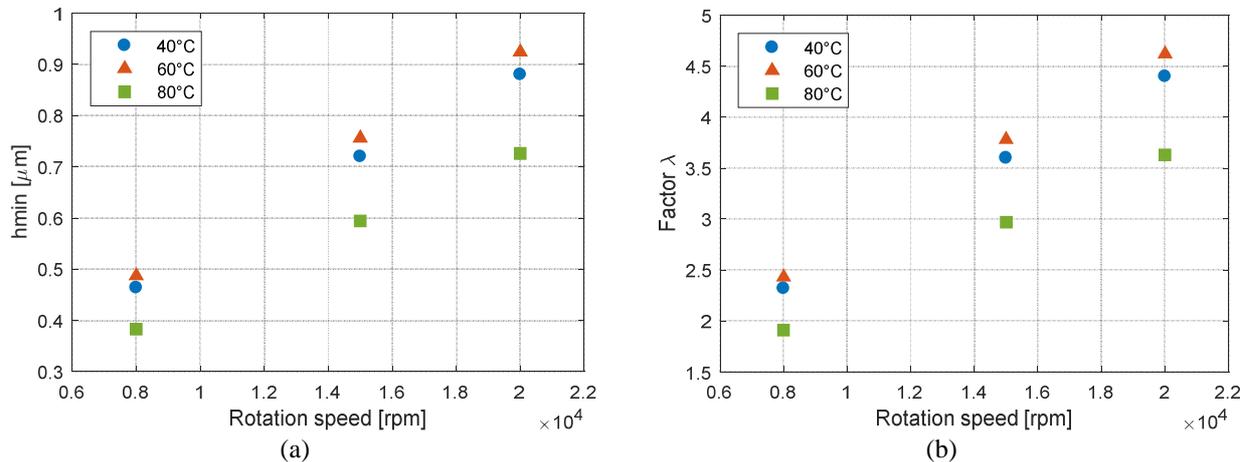


Figure 6. Results of optimization in function of rotation speed and temperature. (a) Minimum film thickness h_{min} . (b) Factor λ .

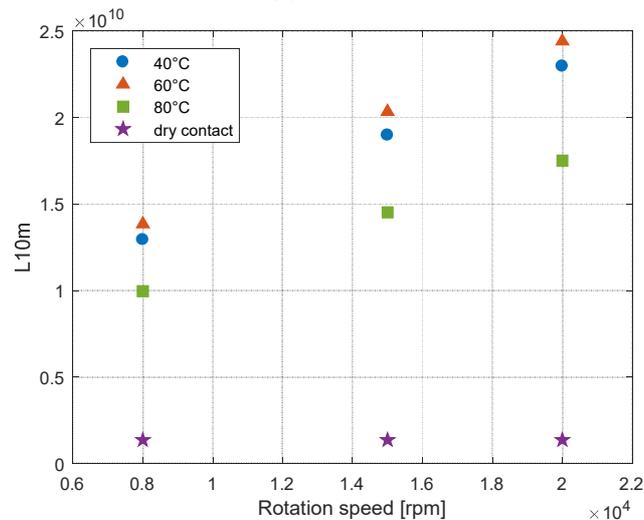


Figure 7. Modified fatigue life L_{10m} in function of rotation speed and temperature.

5. Conclusion

The ABNT NBR ISO 281:2010 standard lists several important aspects for estimating fatigue life, not previously discussed by Lundberg e Palmgren (1947). Furthermore, it also allows the discussion of new analysis about the factors affecting the operation of bearings and the importance of considering the aspects of lubrication on rolling element bearing. In this work, the influence of the lubricant type, as well as the rotation speed and the operation temperature were evaluated in the estimation of the minimum film thickness, factor λ and modified fatigue life L_{10m} . Through the numerical simulations, when roller bearing operating conditions are known, fatigue life can be predicted based on more operating factors not considered in L_{10} , which only considers in its calculations the geometrical design conditions and the external load applied on the roller bearing. The modified fatigue life L_{10m} brings to the life estimation analysis the lubrication analysis with average operating temperature.

The contribution of lubrication can lead to increased fatigue life calculation, if lubricant conditions are optimal. Otherwise, the presence of contaminated lubricant can considerably reduce bearing life according to ABNT NRB ISO 281:2010 .

Lubrication fatigue life correction lists several important aspects for more realistic prediction of bearing life and enables discussion about future L_{10m} corrections, considering the distribution of load and clearance during its operation, as suggested by Campanha (2008).

The fatigue life is important to predict the frequency of replacement of the bearing in operation and it can be a useful tool to indicate adequate maintenance of the mechanical system where the bearing is inserted.

6. ACKNOWLEDGEMENTS

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