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STRUCTURAL DAMAGE IDENTIFICATION IN BEAMS WITH A NEW ADAPTIVE POPULATION-BASED MARKOV CHAIN MONTE CARLO METHOD

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Abstract. *The formulation and solution of the inverse problem of structural damage identification is addressed in the present work, built on the Bayesian approach. A cohesion field, spatially discretized by the finite element method, is considered in order to continuously describe the damage state of the structure. The inverse problem of damage identification is formulated from the Bayesian point of view, whose objective is to estimate the posterior probability densities of the nodal cohesion parameters. The standard Markov Chain Monte Carlo method (MCMC) and a proposed adaptive population-based MCMC method are considered in order to approximate the posterior probability distributions of the parameters of interest. In this approach, prior information on the sought parameters can be used and the uncertainties concerning the measured data can be quantified in the estimation of the cohesion parameters. A simply supported Euler–Bernoulli beam is considered to assess, by means of numerical simulations, the proposed approach.*

Keywords: *Inverse problems, Damage identification, Bayesian inference, Markov Chain Monte Carlo Methods*

1. INTRODUCTION

The development of appropriate techniques aiming at continuously monitoring the integrity of structures and at identifying structural damages in initial stage have been the object of study of the scientific community in recent decades. Degradation, even under normal operating conditions, directly affects the performance of a structure, which can lead to its total interdiction or to its collapse. The structural health monitoring and damage identification, which allows the preventive implementation of efficient maintenance and repair actions, generate direct impacts on the reduction of operating costs, maintain proper functioning and also increase the useful life of the structure in study (Mukhopadhyay and Ihara, 2011).

the damage identification problem is formulated built on changes in the impulse response function of the structure, due to the presence of damage. Structural integrity is continuously described by a cohesion parameter (Stutz *et al.*, 2005; Teixeira *et al.*, 2016). The Finite Element Model (FEM) (Reddy, 1984; Friswell and Mottershead, 1995) parameterized by the nodal cohesion parameters is considered for modeling the structure under concern. The inverse problem of damage identification is formulated in the framework of the Bayesian Inference, employed to sample the *posterior* probability distribution density function of the nodal cohesion parameters of the structure. In this approach, the nodal cohesion parameters are modeled as random variables and probability distribution functions (PDFs) are used to incorporate previous information about them into the damage identification process. Considering the Baye's rules and a sampling method, an estimate for the *posterior* probability distribution function associated with each one of the cohesion parameter are obtained (Brooks *et al.*, 2011).

Although conventional MCMC methods, implemented by the Metropolis-Hastings algorithm, are robust and effective in solving inverse problems, they may present some difficulties in properly exploring the search space of the parameters. It is known that the efficiency of the Metropolis-Hastings algorithm can be improved by an adaptive adjustment of the auxiliary probability distribution. In addition, population-based approaches can also be employed, in conjunction with adaptive techniques, as they use various solutions that exchange information with each other as they explore the *posterior* distribution of the parameters of interest (Brooks *et al.*, 2011; Teixeira, 2018). Therefore, in this work, standard Metropolis-Hastings algorithm (MH) and another proposed adaptive population-based version (AdPop-MH) are used to solve the inverse problem of structural damage identification. To evaluate the effectiveness of the considered approaches,

a simply supported Euler-Bernoulli beam is addressed in the numerical simulations.

The remainder of this work is organized as follows. Section 2 presents the mathematical formulation necessary for the definition of the damage identification problem. Thus, the equation of motion of the structure is presented in this section. Section 3 presents the formulation of the inverse damage identification problem built on the Bayesian inference, where standard Metropolis-Hastings algorithm (MH) and the proposed adaptive population-based version (AdPop-MH) are presented. Section 4 presents the numerical assessment of the proposed damage identification approach applied to a simply supported Euler-Bernoulli beam, for one damage scenario and two levels of noise. Finally, Section 5 presents the concluding remarks.

2. DIRECT PROBLEM FORMULATION

The structure's integrity is considered as being continuously described by a structural parameter called cohesion parameter $\beta(x)$ (Stutz *et al.*, 2005; Teixeira *et al.*, 2016). This parameter is related to the connection between the body's material points and can be interpreted as a measure of the local cohesion state of the material. It is considered that the damage affects only the elastic properties of the structure. This way, for the special case of an Euler-Bernoulli beam, the stiffness matrix of the finite element model of the structure can be written as

$$\mathbf{K}(\beta_h) = \int_{\Omega} \beta(x) E_0 I_0 \mathbf{H}^T(x) \mathbf{H}(x) d\Omega \quad (1)$$

where \mathbf{H} is the discretized differential operator, E_0 e I_0 are, respectively, the nominal young modulus and the nominal area moment of inertia and β represents the cohesion field in the elastic domain Ω of the structure. Defining the vector β , with n_p nodal cohesion parameters

$$\beta = [\beta_1, \beta_2, \dots, \beta_{n_p}]^T \quad (2)$$

where β_i is the cohesion parameter at the i -th node of the spatial discretization of the cohesion field. Consequently, the cohesion spatial field $\beta(x)$, which continuously describes the structural integrity, is approximated by an interpolation from its nodal values $\beta_i, i = 1, \dots, n_p$. In the context of the damage identification problem in beams, considering an arbitrary one-dimensional element, with two extreme nodes and a nodal cohesion parameter per node, the cohesion field within its interior can be approximated by

$$\beta^e(x) = N_1^e(x) \beta_1^e + N_2^e(x) \beta_2^e \quad (3)$$

where $N_1^e(x)$ and $N_2^e(x)$ are linear Lagrangian interpolation functions and β_1^e and β_2^e are the nodal cohesion parameters of the considered element.

Therefore, considering Eqs. (1) and (3), the stiffness matrix of an Euler-Bernoulli beam finite element is given by

$$\mathbf{K}^e = \frac{E_0 I_0}{2L_e} \begin{bmatrix} \frac{12}{L_e^2} (\beta_1^e + \beta_2^e) & \frac{4}{L_e} (2\beta_1^e + \beta_2^e) & -\frac{12}{L_e^2} (\beta_1^e + \beta_2^e) & \frac{4}{L_e} (\beta_1^e + 2\beta_2^e) \\ \frac{4}{L_e} (2\beta_1^e + \beta_2^e) & 2(3\beta_1^e + \beta_2^e) & -\frac{4}{L_e} (2\beta_1^e + \beta_2^e) & 2(\beta_1^e + \beta_2^e) \\ -\frac{12}{L_e^2} (\beta_1^e + \beta_2^e) & -\frac{4}{L_e} (2\beta_1^e + \beta_2^e) & \frac{12}{L_e^2} (\beta_1^e + \beta_2^e) & -\frac{4}{L_e} (\beta_1^e + 2\beta_2^e) \\ \frac{4}{L_e} (\beta_1^e + 2\beta_2^e) & 2(\beta_1^e + \beta_2^e) & -\frac{4}{L_e} (\beta_1^e + 2\beta_2^e) & 2(\beta_1^e + 3\beta_2^e) \end{bmatrix} \quad (4)$$

where L_e is the length of the finite element. Consequently, the equation of motion of a system with n degrees of freedom, obtained via FEM, is given by

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{D}\dot{\mathbf{u}}(t) + \mathbf{K}(\beta)\mathbf{u}(t) = \mathbf{f}(t) \quad (5)$$

where $\mathbf{u}(t)$ is the vector of generalized nodal displacements, \mathbf{M} , \mathbf{D} and $\mathbf{K}(\beta)$ are, respectively, the mass, damping and stiffness matrices of the structure, \mathbf{f} is the vector of external nodal loads.

3. INVERSE PROBLEM FORMULATION

From the Bayesian point of view, the solution of the inverse problem, given the experimental observations \mathbf{D} , is a *posterior* probability density function of β , which, according to the Baye's rule, may be written as

$$p(\beta | \mathbf{D}_{exp}) = \frac{p(\mathbf{D}_{exp} | \beta) p(\beta)}{p(\mathbf{D}_{exp})} \quad (6)$$

where $p(\boldsymbol{\beta})$ is the *prior* probability density of the cohesion parameter, $p(\mathbf{D}_{exp}|\boldsymbol{\beta})$ is the likelihood function and $p(\mathbf{D}_{exp})$ is the marginal density that works as a normalization constant. Assuming that the experimental errors are additive and present a normal probability distribution, the likelihood is given by (Kaipio and Somersalo, 2004)

$$p(\mathbf{D}_{exp}|\boldsymbol{\beta}) = \frac{1}{\sqrt{(2\pi)^{N_d}}} \frac{1}{\sqrt{\det(\mathbf{W})}} \exp \left[-\frac{1}{2}(\mathbf{D}_{exp} - \mathbf{D}(\boldsymbol{\beta}))^T \mathbf{W}^{-1}(\mathbf{D}_{exp} - \mathbf{D}(\boldsymbol{\beta})) \right] \quad (7)$$

where $\mathbf{D}(\boldsymbol{\beta})$ is the response predicted by the FEM, N_d is the number of experimental data, \mathbf{W} is the covariance matrix of the measurement errors and $\det(\mathbf{W})$ is the determinant of \mathbf{W} .

The *posterior* probability density function of the parameters of interest, whose direct assessment is not feasible, can be estimated by sampling techniques, as the Markov Chain Monte Carlo methods (MCMC), for instance. The general idea of MCMC methods is to simulate random samples in the parameter domain $\boldsymbol{\beta}$, so that they converge to the *posterior* distribution probability $p(\boldsymbol{\beta}|\mathbf{D}_{exp})$, using iterative simulation techniques based on Markov chains.

3.1 Adaptive population-based Markov Chain Monte Carlo method

In the present work, the Markov chains are obtained using the Metropolis-Hastings algorithm, which makes use of an auxiliary probability density function $q(\boldsymbol{\beta}^*|\boldsymbol{\beta}^{j-1})$, that denote the probability of occurrence of the candidate $\boldsymbol{\beta}^*$, given the current state of the chain $\boldsymbol{\beta}^{j-1}$.

The Metropolis-Hastings algorithm can be specified by the following steps (Metropolis *et al.*, 1953; Hastings, 1970; Brooks *et al.*, 2011)

Step 1 - Determine an initial state $\boldsymbol{\beta}^0$ for the Markov chain and attribute $j = 1$;

Step 2 - Generate a candidate $\boldsymbol{\beta}^*$ from the current state $\boldsymbol{\beta}^{j-1}$, using the auxiliary density $q(\boldsymbol{\beta}^*|\boldsymbol{\beta}^{j-1})$;

Step 3 - Calculate the acceptance ratio α , i.e. the Hastings ratio given by

$$\alpha = \min \left[1, \frac{p(\boldsymbol{\beta}^*|\mathbf{D}_{exp})q(\boldsymbol{\beta}^{j-1}|\boldsymbol{\beta}^*)}{p(\boldsymbol{\beta}^{j-1}|\mathbf{D}_{exp})q(\boldsymbol{\beta}^*|\boldsymbol{\beta}^{j-1})} \right] \quad (8)$$

Step 4 - Generate a random number U from an uniform distribution in the range $[0, 1]$. If $U \leq \alpha$, then the candidate is accepted and $\boldsymbol{\beta}^j = \boldsymbol{\beta}^*$, otherwise, the candidate is rejected and $\boldsymbol{\beta}^j = \boldsymbol{\beta}^{j-1}$;

Step 5 - Increment j counter to $j + 1$ and go back to step 2, in order to generate the Markov chain $\{\boldsymbol{\beta}^1, \boldsymbol{\beta}^2, \dots, \boldsymbol{\beta}^{N_{mcmc}}\}$, where N_{mcmc} is the length of the chain.

The generated states until the distribution equilibrium is reached are called burn-in samples (Kaipio and Somersalo, 2004), its size will be denoted by $N_{burn-in}$. Such samples are discarded and the remainder of the states of the chain are considered in order to perform the statistical inference of the parameters of interest.

The efficiency of the Metropolis-Hastings algorithm can be improved by a careful adjustment of the auxiliary distribution q (Haario *et al.*, 2001; Marwala *et al.*, 2017; Brooks *et al.*, 2011). In multidimensional estimation problems, as discussed in this paper, manually calibrating the auxiliary distribution support of each parameter of interest may prove to be an impractical or at least costly task. Alternatively, an adaptation of the Metropolis-Hastings Algorithm can be used to automatically adjust the support of the auxiliary distributions. In this work, was considered for each cohesion parameter a uniform auxiliary distribution with the adaptive support given by (Teixeira, 2018)

$$\mathbf{S}_{adp}^j = \gamma_{sd}(\boldsymbol{\beta}^{j-1} - \mathbf{B})^2 \quad (9)$$

where \mathbf{B} is a nominal value, which will be considered to be the values for the intact structure (that is, all parameters equal to 1.0) and γ_{sd} is a real constant. The adaptive process starts after $t_0 < N_{mcmc}$ iterations of the Markov chain, so initially the parameters run through the search space using a fixed and equal support for all parameters and only after t_0 iterations this support will be adapted.

With the purpose at keeping the acceptance rate τ of the method within a pre-established range, it was incorporated an adaptation in the constant γ_{sd} , which initiates at 1 and is adapted from t_0 on as follows (Teixeira, 2018)

$$\gamma_{sd}^j = \begin{cases} \gamma_{sd}^{j-1}(1 - \lambda^j), & \text{if } \tau < \tau_{inf}, \\ \gamma_{sd}^{j-1}(1 + \lambda^j), & \text{if } \tau > \tau_{sup} \end{cases} \quad (10)$$

with $\tau_{inf} = 15\%$ and $\tau_{sup} = 50\%$. It was considered that $\lambda^j = L_i + (L_s - L_i)r^j$, $L_i = 0.05$, $L_s = 0.15$ and r^j is a random number with a uniform distribution between 0 and 1. After the burn-in period, $N_{burn-in}$, both adaptations are no longer performed and then the support remains constant and equal to the support obtained for each parameter at the end of the burn-in.

Even adapting q in the Metropolis-Hastings algorithm, the method may have difficulty with efficiently transiting in the domain of the parameters, among different regions of high probability. Thus, recently, some works have proposed a combination of Genetic Algorithms (GAs) with MCMC, in order to more efficiently sample the *posterior* distribution of the parameters. These methods use various solutions that exchange information among themselves, as they explore the *posterior* distribution of the parameters of interest (Nichols *et al.*, 2011). Similar to the standard MCMC, the population-based MCMC aims to obtain samples of the *posterior* distribution of interest $p(\beta | \mathbf{D}_{exp})$, where a new compound density is obtained as a function of the parameter vectors $\beta^{1:N_{pop}} = (\beta^1, \beta^2, \dots, \beta^{N_{pop}})$ as follows

$$p_c(\beta^{1:N_{pop}} | \mathbf{D}_{exp}) = \prod_{n=1}^{N_{pop}} p_n(\beta^n | \mathbf{D}_{exp}) \quad (11)$$

where N_{pop} is the size of the population. However, it is a necessary condition that one of the distributions that compose the compound density is the true *posterior* distribution of interest, i.e., $p_n(\beta^n | \mathbf{D}_{exp}) = p(\beta | \mathbf{D}_{exp})$ for at least one n . There is a good degree of freedom in the choice of the distributions $p_n(\cdot)$, and also in the selection of different ways to explore the parameters' space. In this work the following distribution is used

$$p_n(\beta^n | \mathbf{D}_{exp}) \approx p(\mathbf{D}_{exp} | \beta^n)^{\zeta_n} p(\beta^n) \quad (12)$$

where $\zeta_n \in (0, 1]$, $\zeta_1 = 1$, $\zeta_{n+1} = \zeta_n - 1/N_{pop}$, $n = 1, \dots, N_{pop} - 1$, with ζ_1 yielding the true posterior distribution of interest. Small values of ζ_n yield smoother versions of the true posterior distribution. The idea is that the smoothed distribution, which still carries information about the high probability regions, are easier to explore. The implementation of the AdPop-MH is made as described ahead

Step 1. Determine an initial state for the chain of each population member $\beta^{0,1}, \dots, \beta^{0,N_{pop}}$ and attribute $j = 1$;

Step 2. Update each population member $\beta^{j,n}$, with $n = 1, \dots, N_{pop}$ via adaptive Metropolis-Hastings algorithm, with probability of acceptance $\alpha_1 = \min(1, \gamma_1)$;

Step 3. Choose two population members randomly, $\beta^{j,u}$ and $\beta^{j,v}$, with $u, v \in \{1, \dots, N_{pop}\}$. At each iteration of the chain, make a choice with 50% probability of performing step 4a or 4b.

Step 4a. Execute a Swap (Swap) move between $\beta^{j,u}$ and $\beta^{j,v}$, with probability of acceptance of the move $\alpha_2 = \min(1, \gamma_2)$. Increment the counter j to $j + 1$ and go back to **Step 2**.

Step 4b. Execute a Differential Evolution (DE) move for $\beta^{j,u}$ and $\beta^{j,v}$, with probability of acceptance of the move $\alpha_3 = \min(1, \gamma_3)$. Increment the counter j to $j + 1$ and go back to **Step 2**;

In this case, $\beta^{*,n}$ is a candidate value, $\beta^{j,u'}$ and $\beta^{j,v'}$ are the trial vectors (given by the DE method) and γ_1, γ_2 and γ_3 are given by (Nichols *et al.*, 2011; Jasra *et al.*, 2007)

$$\begin{aligned} \gamma_1 &= \frac{p^{\zeta_n}(\beta^{*,n} | \mathbf{D}_{exp})q(\beta^{j,n} | \beta^{*,n})}{p^{\zeta_n}(\beta^{j,n} | \mathbf{D}_{exp})q(\beta^{*,n} | \beta^{*,n})}, & \gamma_2 &= \frac{p^{\zeta_u}(\beta^{j,v} | \mathbf{D}_{exp})p^{\zeta_v}(\beta^{j,u} | \mathbf{D}_{exp})}{p^{\zeta_u}(\beta^{j,u} | \mathbf{D}_{exp})p^{\zeta_v}(\beta^{j,v} | \mathbf{D}_{exp})}, \\ \gamma_{3,u} &= \frac{p^{\zeta_u}(\beta^{j,u'} | \mathbf{D}_{exp})}{p^{\zeta_u}(\beta^{j,u} | \mathbf{D}_{exp})}, & \gamma_{3,v} &= \frac{p^{\zeta_v}(\beta^{j,v'} | \mathbf{D}_{exp})}{p^{\zeta_v}(\beta^{j,v} | \mathbf{D}_{exp})}. \end{aligned} \quad (13)$$

4. NUMERICAL RESULTS

In this work, a simply supported Euler-Bernoulli beam considered in the numerical simulations of structural damage identification problems, where the Metropolis-Hastings (MH) and the AdPop-MH (population-based MH combined with the proposed adaptive technique) algorithms were employed. All numerical simulations considered in the present work were performed in Matlab. The beam was spatially discretized into 24 elements, with two degrees of freedom (DOF) and one nodal cohesion parameter per node. The same mesh was considered for approximation of both the cohesion field and the displacement one. Therefore, the FEM of the beam has a total of 48 DOFs and 25 nodal cohesion parameters. The physical and geometrical properties of the beam are given in Table 1.

Table 1. Mechanical and geometrical properties of the beam.

Length	Thickness	Width	Specific Mass	Young modulus
$L = 1.46\text{m}$	$h_0 = 0.008\text{m}$	$w = 0.0762\text{m}$	$\rho = 7.85 \times 10^3 \text{kg/m}^3$	$E = 207\text{GPa}$

The experimental impulse response functions, provided by the FEM of the beam, with prescribed values for the vector of nodal cohesion parameters β_p , were supposed to be measured by a collocated acceleration sensor at $x = 0.2433$ m and with a 1 kHz sampling frequency, considering a time interval of 10s. However, only the data within the time interval of 0.5s was considered in the damage identification process, since, due to damping effects, most of the information about the impulse response of the structure is retained within a short time interval. Therefore, the synthetic experimental data were obtained corrupting the impulse response functions of the structure, with the prescribed damage states, additive noise with a Gaussian distribution, zero mean and variance obtained from the signal to noise ratio (SNR), which is given by

$$SNR = 10 \log \left(\frac{\sigma_s^2}{\sigma_n^2} \right) \quad (14)$$

where σ_s^2 and σ_n^2 stand for the variance of the corrupted signal and the variance of the noise, respectively. In the present work, two noise levels are considered, $SNR = 30$ and 20 dB. For simplicity and without loss of generality, it will be considered that the beam has a rectangular section and that the modulus of elasticity is constant, which means that $E(x) = E_0$. Therefore, the cohesion parameter is given by

$$\beta(x) = \left(\frac{h(x)}{h_0} \right)^3 \quad (15)$$

where h_0 and $h(x)$, are, respectively, the nominal thickness and the thickness of the beam at point x . Table 2 presents the damage scenario adopted in the simulations, with the description of the position of the corresponding FEM node, the relative thickness of the beam, and the corresponding cohesion parameters associated with the deeper point of the assumed V-notch damage.

Table 2. Damage scenario.

Damage position (m)	FEM node	$h(x)/h_0$	$\beta(x)$
0.5475	10	0.9	0.729
1.2775	22	0.9	0.729

In addition, a constant support $\mathbf{S} = [S_1, \dots, S_{n_p}]$, where $S_i = 0.0002$ is considered, for all auxiliary distributions, in the conventional MH algorithm. For the adapted population-based version, AdPop-MH, the process of adaptation of the support starts after $t_0 = 5,000$ states. Markov chains with $N_{mcmc} = 100,000$ states and burn-in = $50,000$ states were used for both methodologies addressed. Besides, due to the extremely low sensitivity of the structure responses with respect to the nodal cohesion parameters at the extreme nodes (β_1 and β_{25}), these parameters were not estimated in the inverse problem. As *a priori* information, $p(\beta)$, a combination of two normal probability distributions was used, both with average 1.0 and standard deviations given by

$$\sigma_i = \begin{cases} 0.3 & \text{if } \beta_i \leq 1 \\ 0.005 & \text{if } \beta_i > 1 \end{cases} \quad (16)$$

The considered test cases, involving the damage scenario described in Table 2 and the two inference inverse methods, are summarized in Table 3.

Table 3. Summary of the test cases investigated.

Method	SNR = 30 dB	SNR = 20 dB
MH	Case 1A	Case 2A
AdPop-MH	Case 1B	Case 2B

The results obtained for the methodologies proposed in this work are presented in Figure 1 for Cases 1A and 1B. This figure presents the exact and estimated damage fields, along with the corresponding 95% credibility interval and the Markov chains for the estimated parameters.

In table 4 the sample statistical properties are presented, estimated for a marginal *posterior* probability distribution of the cohesion parameters β_{10} and β_{22} , the estimated average (μ_{est}), standard deviation (σ_{est}), relative error (E_r) between the estimated mean value and the exact value ($\beta_{exact} = 0.729$) and credibility intervals and also the acceptance ratio (τ) of the corresponding sampling method and the root mean squared error (E_{RMS}), related with all estimated parameters. The relative error and the root mean squared error are defined, respectively, as

$$E_r = \frac{|\mu_{i,estimated} - \beta_{i,exact}|}{|\beta_{i,exact}|}, \quad E_{RMS} = \sqrt{\frac{\sum_{i=2}^{24} (\mu_{i,estimated} - \beta_{i,exact})^2}{23}} \quad (17)$$

Table 4. Estimated statistical properties for Cases 1A and 1B.

Case	E_{RMS}	τ	Parameter	μ_{est}	σ_{est}	E_r	Credibility Int.
1A (MH)	0.0377	14%	β_{10}	0.7722	0.0011	0.0592	[0.7694; 0.7740]
			β_{22}	0.7978	0.0024	0.0943	[0.7947; 0.8030]
1B (AdPop-MH)	0.0017	31%	β_{10}	0.7306	0.0050	0.0021	[0.7228; 0.7434]
			β_{22}	0.7272	0.0026	0.0024	[0.7219; 0.7323]

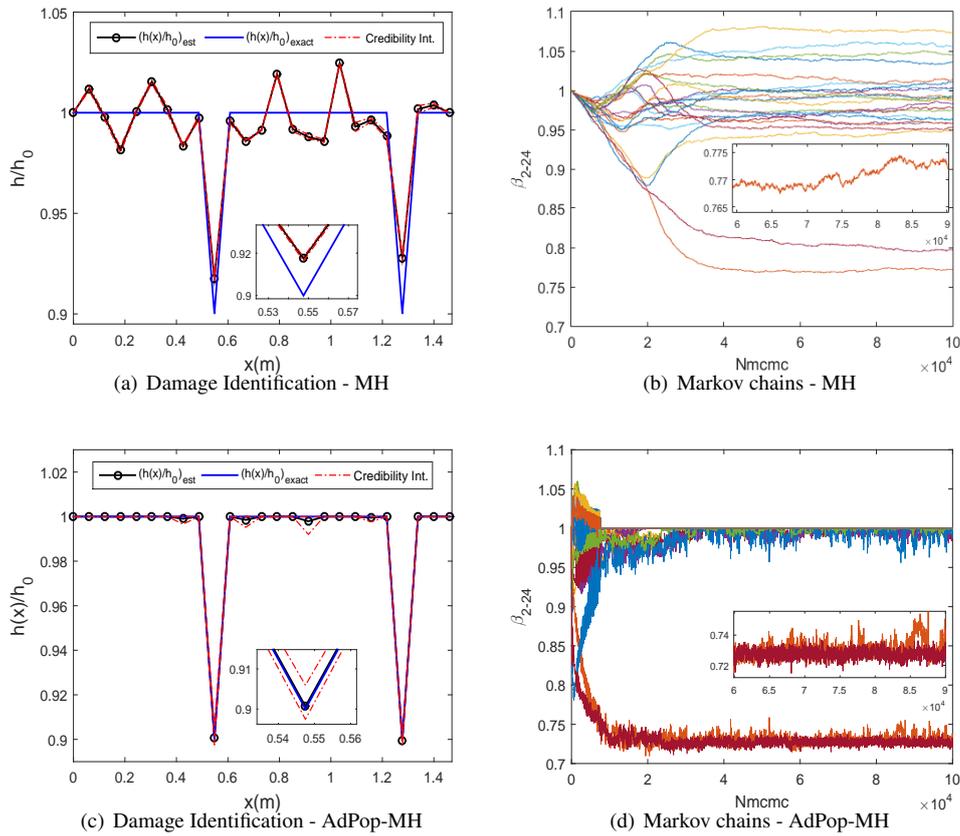


Figure 1. Damage Identification and Markov Chains for Cases 1A and 1B (Table 3)

Figure 2 presents the estimated *posterior* probability density function (PDF) and Autocorrelation Function (ACF) regarding the parameters β_{10} and β_{22} for Cases 1A and 1B.

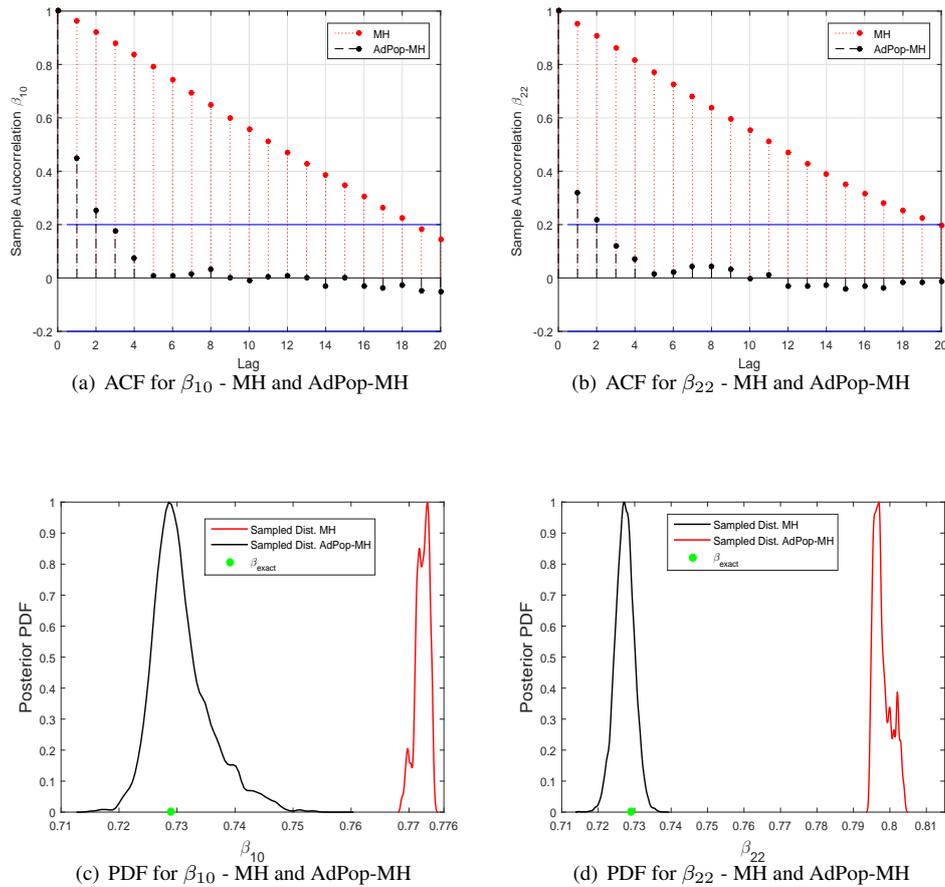


Figure 2. ACFs and *Posterior* PDFs for Cases 1A and 1B (Table 3).

As it can be observed in Table 4, both methods presented acceptance rates within the previously stipulated ranges. From the results presented, it can be observed that the conventional Metropolis-Hastings algorithm (MH), despite having located the existing damages in the structure, also located false damages in undamaged regions of the structure. In addition, it showed Markov chains did not converge, as a consequence, estimated averages farthest from the exact values, and consequently, values of E_{RMS} and E_r of the order of 10^{-2} , as can be seen in Tab. 4. For the adaptive population-based Metropolis-Hastings algorithm (AdPop-MH), the obtained results showed that the method was able to accurately identify the existing damages in the structure, locating only the true damages and estimating with accuracy the intensity of the same. It also presented chains with fast convergence, from about 30,000 states, as well as credibility intervals containing the exact values and values of E_{RMS} and E_r of the order of 10^{-3} , showing that the estimation of the averages are close to the exact values.

In Figure 2 is possible to observe that the autocorrelation functions obtained for Case 1B corroborate the fact of the fast convergence of the respective Markov chains, showing that, approximately, less than 40,000 states were necessary to obtain non-correlated samples, whereas in the Case 1A this behavior doesn't happen, i.e., until final state of chains the samples present correlation, thus showing that more than 100,000 states are necessary to obtain non-correlated values. Besides, the *posterior* probability density functions estimated for Case 1B presented a behavior close to a normal distribution, thus showing that values close to the estimated average present high probability of occurrence. A fact that doesn't happen for case 1A, where the sampled distribution presents multimodal behavior, where distinct values present high frequency of occurrence.

Figure 3 shows the results of the damage identification for Cases 2A and 2B. The sample statistical properties of the marginal *posterior* distribution probability of the cohesion parameters β_{10} and β_{22} for Cases 2A and 2B are presented in Table 5.

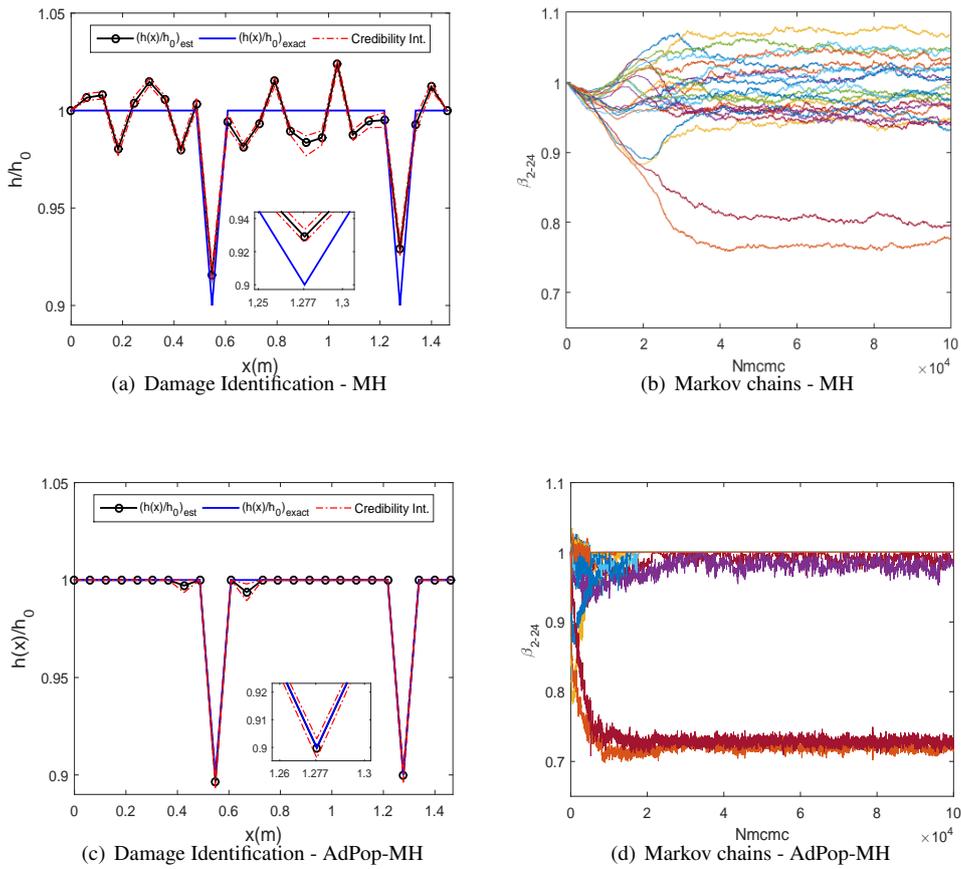
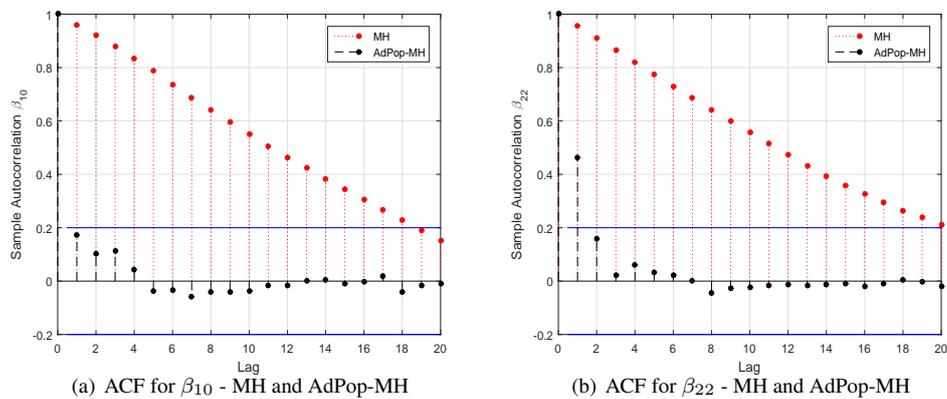


Figure 3. Damage Identification and Markov Chains for Cases 2A and 2B (Table 3)

Table 5. Estimated statistical properties for Cases 2A and 2B.

Case	E_{RMS}	τ	Parameter	μ_{est}	σ_{est}	E_r	Credibility Int.
2A (MH)	0.0402	24%	β_{10}	0.7676	0.0046	0.0529	[0.7612; 0.7781]
			β_{22}	0.8018	0.0061	0.0998	[0.7933; 0.8131]
2B (AdPop-MH)	0.0046	36%	β_{10}	0.7207	0.0038	0.0113	[0.7129; 0.7291]
			β_{22}	0.7283	0.0042	0.0096	[0.7197; 0.7365]

Figure 4 presents the estimated *posterior* probability density function (PDF) and Autocorrelation Function (ACF) regarding the parameters β_{10} and β_{22} for Cases 1A and 1B.



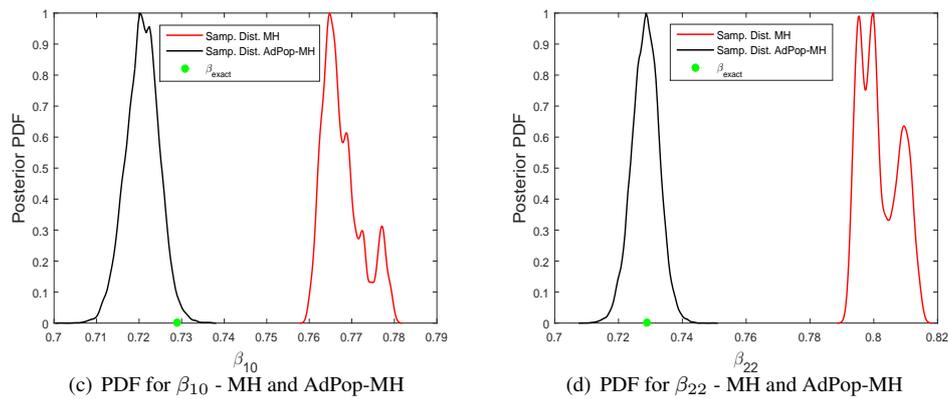


Figure 4. ACfs and *Posterior* PDFs for Cases 2A and 2B (Table 3).

From the results presented, it can be observed that even for a higher noise level (20dB), both methods presented similar behavior to the ones previously presented (Cases 1A and 1B), that is, the conventional Metropolis-Hastings algorithm indicated false damages in non-damaged regions of the structure, Markov chains non-converged and the estimated PDFs presented various modes. Besides, there was a reduction of the accuracy to identify the intensity of both damages. The adaptive population-based Metropolis-Hastings algorithm (AdPop-MH) succeeded in estimating the damage field with accuracy, presenting chains with fast convergence and credibility intervals containing the exact values. These results indicate that the AdPop-MH method is robust with respect to the uncertainties inherent to the experimental data, thus demonstrating that can be applicable to real problems, in which diverse sources of uncertainties are present in the experimental processes.

5. CONCLUSIONS

In the present work, the application of the Bayesian approach in problem of structural damage identification in beams, built on the impulse response of the structure, was considered. The structural damage identification problem was solved as an inverse problem, whose objective was to estimate the *posterior* probability density function of the cohesion parameters, where the standard Markov chain Monte Carlo method, implemented by Metropolis-Hastings (MH) algorithm, and the proposed adaptive population-based version (AdPop-MH) were employed. As a main conclusion, it can be highlighted that the combination of the adaptive technique with the population-based method significantly improved the accuracy of the results obtained, with Markov chains converged and averages close to the exact values, even in the case of a higher noise level, the AdPop-MH succeeded in the damage identification, thus demonstrating its effectiveness.

6. ACKNOWLEDGEMENTS

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