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DEVELOPMENT OF A GRAPHICAL APPLICATION FOR THE DYNAMIC ANALYSIS OF LINEAR ELEMENTS STRUCTURAL MODELS

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Abstract. *This paper presents a tool of simple usage for the dynamic analysis of reticulated structural models, entirely implemented in Matlab® for the LESM (Linear Elements Structure Model) program, an open source structural analysis software with an educational approach and an interactive graphical interface for modeling. The eigenvalue problem of an undamped oscillation system is solved with a Matlab built-in function to find the requested vibration modes. The structural response, admitting a damped system, to dynamic loading and initial conditions is obtained by one of four implemented numerical solvers, including Newmark's and Wilson-*θ *methods, considering the system of differential equations of motion to be either coupled or not. Analytical solutions for free vibration are achieved via an uncoupled modal system. Results of kinematic behavior are provided in time and frequency domain.*

Keywords: *dynamic analysis, vibration, reticulated structures, educational tool, Matlab.*

1. INTRODUCTION

Designing and constructing structures to sustain buildings are some of the main topics that revolve around the field of Civil Engineering. For a long time, structures were generally conceived well within a safety zone as to prevent rupture and ensure static stability, what is evidenced by the robustness of old buildings. It is usual to see grandiose pillars on historical constructions, for example. Modern Engineering aims to reduce resource consumption, making efforts to optimize project parameters such as the cross-section of unidimensional structural elements, commonly resulting on thinner pieces. The increasing slenderness of modern structures has often presented engineers with problems that rarely would occur to the sturdy ancient designs, such as excessive vibration and buckling of beams and columns, thus adding the field of dynamics to structural analysis (Lima and Santos, 2008).

An unforeseen dynamic structural behavior may reach critical levels and deflagrate generalized failure mechanisms, but that is not the only problem, although mishaps with phenomena like mechanical fatigue are an important concern. A common and serious outcome of the lack of dynamic analysis on a structural project is a threat to the performance of the construction and its functionality (Brasil and Silva, 2015). For example, a bridge that vibrates excessively may not necessarily be at risk of a collapse, depending on the amplitude and frequency of the oscillations, but it often may need to be closed, or is likely to cause discomfort to users and be underutilized, thus losing its purpose.

The objective of this paper is to present a user-friendly graphical-interactive computational tool to obtain dynamic structural responses from plane and spatial reticulated models, exposing the formulation of the dynamic problem and the analysis module of the program. All the coding was done on the Matlab environment, as an improvement of the LESM (Linear Elements Structure Model) program, an open source, object-oriented and well documented structural analysis software with an educational background (Martha, 2018) and an interactive graphical interface for modeling in 2D and 3D. The program contemplates five types of analysis models: plane and spatial trusses, plane and spatial frames and grillages. Moreover, for grillage and frame models, the two most used beam bending behavior theories are available: Euler-Bernoulli (also known as Navier) and Timoshenko. The formulation of stiffness and consistent mass matrices uses parameters to unify both theories with the same expressions.

The classical eigenvalue problem to obtain the natural frequencies of the structure's vibration modes is solved with the Matlab built-in function *eig*. Furthermore, four different numerical solvers, including Newmark's (Vaz, 2011) and Wilson-θ (Soriano, 2009) numerical methods, were implemented for the forced vibration problem, admitting a damped system. Free vibration is obtained separately, with the possibility of computing analytical solutions, by considering an uncoupled modal system of differential equations of motion.

Usage of the Object-Oriented Programming (OOP) paradigm granted modularity to the program (Rangel and Martha, 2019), being of great aid on implementing new solvers. New analysis modules could be inserted into the software by the abstraction of the analysis driver class, without compromising or needing to change the previous existent functionalities.

2. DYNAMIC STRUCTURAL ANALYSIS

The dynamic oscillation problem is well known to be analyzed by the differential equations of motion (Tauchert, 1974), depicted mathematically by the expression in Eq. (1).

$$\left[M\right]\left\{\ddot{x}(t)\right\}+\left[C\right]\left\{\dot{x}(t)\right\}+\left[K\right]\left\{x(t)\right\}=\left\{f(t)\right\} \tag{1}$$

Where $[M]$, $[C]$ and $[K]$ correspond to global mass, damping and stiffness matrices, respectively. Vectors $\{x\}$ and $\{f\}$ are related to displacement and external loading of each degree of freedom (DOF).

Equation (1) consists of a coupled matrix system of ordinary differential equations. A common way to solve this problem is making use of a numerical solver. It is also possible to manipulate Eq. (1), as exposed in Subsection 2.2, in order to transform it to an uncoupled system, on the modal domain. This allows the results to be separated as contributions of each vibration mode to the total response.

The LESM program handles both approaches, coupled and uncoupled, so users may choose their preferred solution method and compare results.

2.1 Natural frequencies and vibration modes

It is of great interest to dynamic structural analysis to find the natural frequencies related to the structure's vibration modes, so that any resonance problems may be identified. Considering an undamped system vibrating freely in harmonic movement, it is natural to assume that periodic functions are a solution for $\{x\}$ (Tauchert, 1974). So it is possible to manipulate Eq. (1) to get to Eq. (2), where the term ω^2 corresponds to the squared values of the angular natural frequencies of the studied system. Therefore, the classical eigenvalue problem to obtain the vibration modes becomes clear, as the solution of a null displacement field would not provide any information referring to dynamic oscillations, so it can be disregarded.

$$\left(\left[K\right]-\omega^2\cdot\left[M\right]\right)\left\{x(t)\right\}=0 \tag{2}$$

The eigenvalues, and its eigenvectors, that solve Eq. (2) can be obtained by the usage of the Matlab built-in function *eig*. The eigenvectors are taken as normalized vectors of nodal displacement, depicting the dynamic behavior associated with each vibration mode of the model. By interpolating them with the shape functions of each element, it is possible to capture the modal response of the structure as a whole.

The LESM program always runs this modal analysis prior to solving the system of differential equations, so that the uncoupled system may be assembled.

2.2 Uncoupled semi-analytical solver

A solution of the coupled system of differential equations has its advantages, mostly due to simplicity and efficiency, as it does not require transforming the problem to the modal domain, so fewer operations are needed. However, it fails to capture by itself the separate influence of each vibration mode to the total dynamic response. The evaluation of which natural frequencies are being excited the most is important to engineers, especially to avoid any trouble with resonance.

Once the eigenvectors related to the structure's vibration modes are known, it is possible to transform Eq. (1) to a modal space, setting as variables not the displacement of each DOF, but the contribution of each vibration mode to the oscillations. By assembling a matrix of eigenvectors $[\Phi]$ and pre and post multiplying it to the global matrices of the model, as done for the mass matrix in Eq. (3), it is possible to rewrite Eq. (1) as Eq. (4), where the system of differential equations is uncoupled, meaning there is a solution for each vibration mode (Tauchert, 1974).

$$\left[M_d\right]=\left[\Phi\right]^T\cdot\left[M\right]\cdot\left[\Phi\right] \tag{3}$$

$$\left[M_d\right]\left\{\ddot{x}(t)\right\}+\left[C_d\right]\left\{\dot{x}(t)\right\}+\left[K_d\right]\left\{x(t)\right\}=\left\{f_d(t)\right\} \tag{4}$$

The fact that the system presented in Eq. (4) is uncoupled grants it the possibility to be solved as a list of independent ordinary differential equations. For free vibration, associated with initial conditions, analytical solutions are obtained as expressed in Eq. (5) (Vaz, 2011). Forced vibration requires the aid of a numerical method, due to the multiple DOF nature of the problem and the possibility of various types of dynamic loading acting throughout the analysis interval, what increases the complexity of the solution.

$$x^{(n)}(t)=x_0^{(n)}e^{-\xi_n\omega_n t}\,\mathbf{sin}\left(\left(\sqrt{1-\xi_n{}^2}\right)\omega_n t+\phi_n\right) \tag{5}$$

In Eq. (5), $x^{(n)}$ corresponds to the contribution of vibration mode n to the displacement field of the model, $x_0^{(n)}$ is a modal initial condition, ξ_n, ω_n and ϕ_n consist of the modal critical damping ratio, natural angular frequency associated to mode n, and modal phase angle, respectively.

The LESM program solves the system of ordinary differential equations presented in Eq. (4) and stores the results for each vibration mode, making it possible for users to visualize graphically the influence of each natural frequency on the displacement of each DOF. It is also possible to set a maximum number of vibration modes to be considered on the analysis, to avoid adding processing time on negligible results.

3. LOCAL MASS MATRICES FORMULATION

The formulation of local mass matrices consistent with the displacement field of linear elements was arranged in such a way that each of its components are valid for both Euler-Bernoulli and Timoshenko theories, as it was done by Rangel and Martha (2019) with local stiffness matrices. Timoshenko's dimensionless parameter, exposed in Eq. (6), and a set of descendant auxiliary parameters are used for the expressions to be laid out in a similar fashion to the classical expressions from Euler-Bernoulli's theory. This is justified as a method to simplify the code, and an artifice for the formulation to be familiar to those who are not acquainted with the consideration of shear deformation on beam elements.

$$\Omega=\frac{EI}{GA_s}\cdot\frac{1}{L^2} \tag{6}$$

In the previous equation, E and G are the elastic and shear modulus of the material, respectively, I and A_s are the moment of inertia and effective shear area of the cross-section, and L is the length of the element.

Notice, in Eq. (6), that the assumption of an infinite shear modulus, what means negligible shear deformation, results on a null value for Timoshenko's parameter. In this case, the obtained mass matrix is the same as the one from Euler-Bernoulli's theory.

Alternatively, lumped local mass matrices were implemented as well, so users may compare results of each consideration and analyze models as they see fit. It is also possible to admit customized mass matrices, a combination of consistent and lumped mass with a proportionality factor μ, shown in Eq. (7), as proposed by Felippa (2005).

$$\left[M_{\text{customized}}\right]=\mu\left[M_{\text{consistent}}\right]+(1-\mu)\left[M_{\text{lumped}}\right] \tag{7}$$

As shown by Felippa (2005), different values of $0\leq\mu\leq1$ are advantageous for different types of analysis. For example, for Fourier analysis, in the frequency domain, it is recommended a multiplication factor $\mu=1/2$.

4. DAMPING CONSIDERATION

Viscous damping is commonly considered to be a linear combination of mass and stiffness, as exposed by Eq. (8), being known as Rayleigh damping (Lima and Santos, 2008). This avoids numerical issues when solving the system of ODEs of motion and is a simplification that represents satisfactorily the structural damped vibration behavior.

$$\left[C\right]=\alpha\left[M\right]+\beta\left[K\right] \tag{8}$$

On the LESM program, the Rayleigh damping coefficients may be directly provided, or computed using given critical damping ratios associated to the first and second vibration modes. The relation between critical damping and the natural frequencies of the structure is shown in Figure 1.

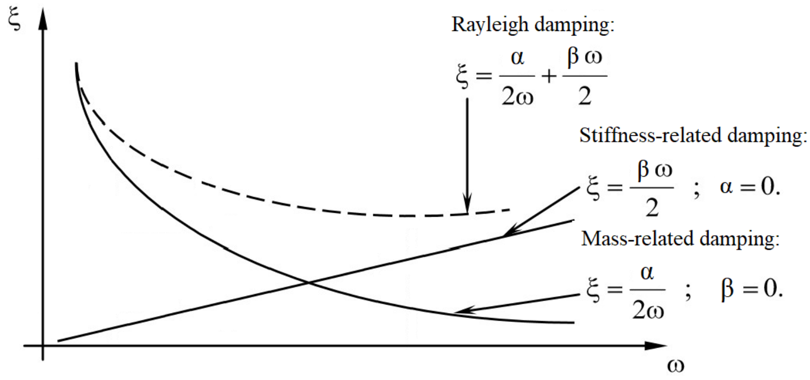

Figure 1. Rayleigh damping coefficients

5. DYNAMIC FORCING CONDITIONS

External load conditions to linear elements are usually considered to be forces or moments concentrated at nodes or distributed along the length of each element. When dealing with static structural analysis, a forcing vector is assembled to characterize the resulting loads acting on each DOF. However, the dynamic analysis problem implies that forcing conditions need to be determined on every time step throughout the analysis interval, therefore, a forcing matrix needs to be defined, where each of its columns represents the forcing vector at a determined instant.

The LESM program deals with concentrated transient loads, described as amplitudes associated to time functions. Periodic, linear and constant functions are implemented, being possible to combine multiple expressions with provided multiplication factors, as to fully depict the forcing oscillation on the time domain. To each node, amplitudes on all of its DOFs and a combination of time functions may be set, as shown in Eq. (9). $\left[f_n\right]$ is the nodal forcing matrix, $\{A\}$ is a column vector of amplitudes on each DOF of the respective node, and $\{F\}$ is a row vector that stores the time function values on every time step of the analysis to be performed.

$$\left[f_n\right] = \{A\}\{F\} \tag{9}$$

6. COMPUTATIONAL MODULE OF A STRUCTURAL VIBRATION ANALYSIS PROBLEM

The process of analyzing the dynamic behavior of reticulated models consists on solving Eq. (1) or Eq. (4) to evaluate the kinematic response of the structure's DOFs. By obtaining nodal displacements and rotations over a time interval, it is possible to compute the deformed configuration of the model on each step of the time domain, using the local shape functions of each element (Martha, 2018). Envelopes of internal forces and bending moment diagrams can also be achieved, on analogous fashion. In addition, once displacement, velocity and acceleration of each DOF are computed, it is possible to obtain their respective spectrums in the frequency domain, by evaluating the Fast Fourier Transform of the transient solution (Quarteroni et al., 2007).

The assembly of the matrix system of ODEs that rule the structural vibration problem depends on information regarding the geometry and physical properties of the model, as well as the external forcing conditions through the interval of analysis and initial conditions of each DOF, as it is an initial value problem. In addition, dynamic analysis parameters need to be defined. Therefore, some modeling steps are required prior to running the analysis, as depicted in Figure 2.

On the LESM program, to solve the system of ODEs, four numerical solvers were implemented. Newmark's method (Vaz, 2011), Wilson-θ method (Soriano, 2009), a fourth order Runge-Kutta method (Chapra, 2012), and a three step Adams-Moulton method (Quarteroni et al., 2007) may be used. According to Soriano (2009), explicit direct integration numerical methods, where the solution on a given step is computed by the equilibrium condition of previous steps, usually require the discretization of the time domain on very small steps in order to obtain a solution representative of the physical problem, thus they are generally not used for vibration analysis. Nevertheless, the Runge-Kutta method is made available for educational purposes. Implicit methods, Newmark and Wilson-θ, specifically, are the most commonly adopted for linear dynamic analysis.

Once model data and preferred solution method are defined, the analysis may be initialized. At this point, the program performs the processing tasks shown in Figure 2 and stores its results to be later displayed, as requested by users.

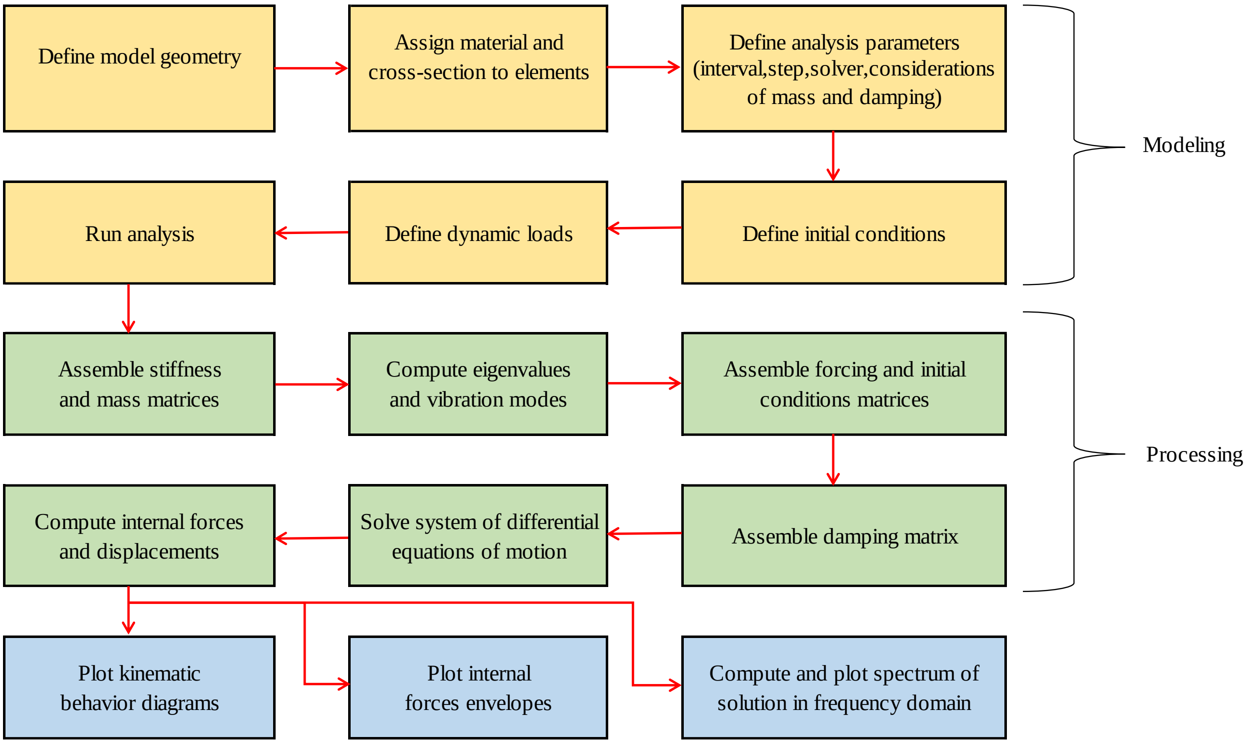

Figure 2. Course of action for modeling and analyzing dynamic problems with LESM

7. RESULTS FROM DYNAMIC ANALYSIS WITH THE LESM PROGRAM

The model exposed in Figure 3 was entirely made with the LESM program to exemplify the functionalities of the dynamic analysis module. Model properties and the dynamic analysis parameters are displayed on Table 1. All elements used are considered to behave accordingly to the Timoshenko formulation.

Table 1. Material and geometric properties, and dynamic analysis parameters of the model depicted in Figure 3.

Elastic Modulus (GPa)	30.0
Poisson Coefficient	0.30
Density (t/m³)	2.50
Cross-Section (cm)	10 × 12
Critical Damping Ratio of 1st and 2nd Modes	0.01
Maximum Number of Vibration Modes	12
Time Interval (s)	5.00
Time Step (s)	0.001

On node 7, a combination of load functions was defined, as portrayed by Figure 4, considering an amplitude of 75 N on the global *z* axis direction. The angular frequency of the periodic time function was chosen as to provoke resonance with the natural frequency of the seventh vibration mode of the structure, which can be seen in Figure 5.

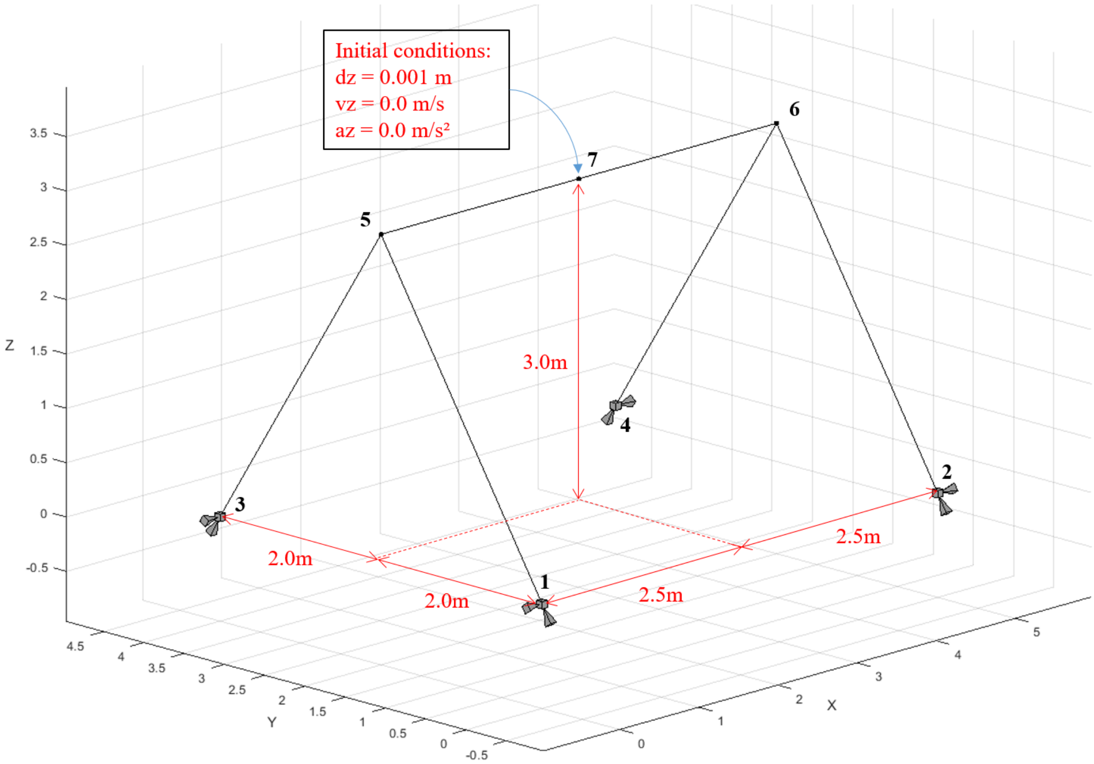

Figure 3. Example of spatial frame model with initial conditions for dynamic analysis on the LESM program

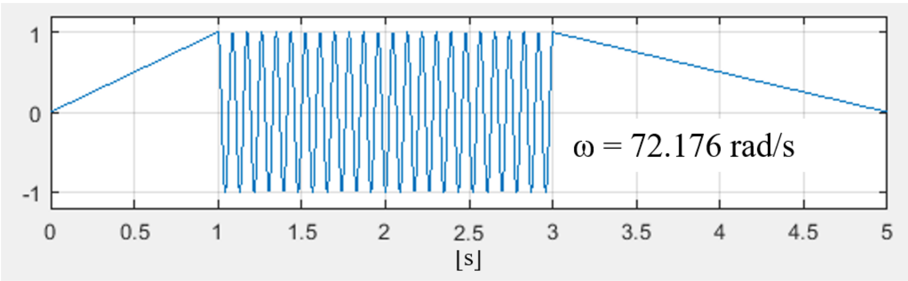

Figure 4. Time function associated to dynamic loading on node 7

M O D A L   A N A L Y S I S

MODE	EIGENVALUE	w [rad/s]	f [Hz]	T [s]
1	5.22037e+02	2.28481e+01	3.63639e+00	2.74998e-01
2	6.57347e+02	2.56388e+01	4.08054e+00	2.45066e-01
3	7.12039e+02	2.66841e+01	4.24690e+00	2.35466e-01
4	2.74099e+03	5.23545e+01	8.33247e+00	1.20012e-01
5	2.74168e+03	5.23610e+01	8.33351e+00	1.19997e-01
6	3.49400e+03	5.91101e+01	9.40766e+00	1.06296e-01
7	5.20933e+03	7.21757e+01	1.14871e+01	8.70540e-02
8	3.16527e+04	1.77912e+02	2.83156e+01	3.53162e-02
9	4.08872e+04	2.02206e+02	3.21821e+01	3.10732e-02
10	6.30066e+04	2.51011e+02	3.99497e+01	2.50315e-02
11	6.70097e+04	2.58862e+02	4.11992e+01	2.42723e-02
12	6.80425e+04	2.60850e+02	4.15155e+01	2.40874e-02

Figure 5. Textual output of the found vibration modes on the LESM program

Figure 5 shows the textual output of the LESM program for the found vibration modes and their natural frequencies, after solving the eigenvalue problem. The following diagrams, displayed in Figures 6, 7 and 8 present the dynamic behavior of node 7, as to its translation on the global *z* axis direction, for free, forced and total vibration, respectively. On those figures, the diagrams on the left are related to displacement, velocity and acceleration found with the Wilson-θ numerical solver, considering the coupled matrix system of ODEs, whilst on the right are depicted the same results for the uncoupled semi-analytical solution, using the Newmark numerical solver for forced vibration.

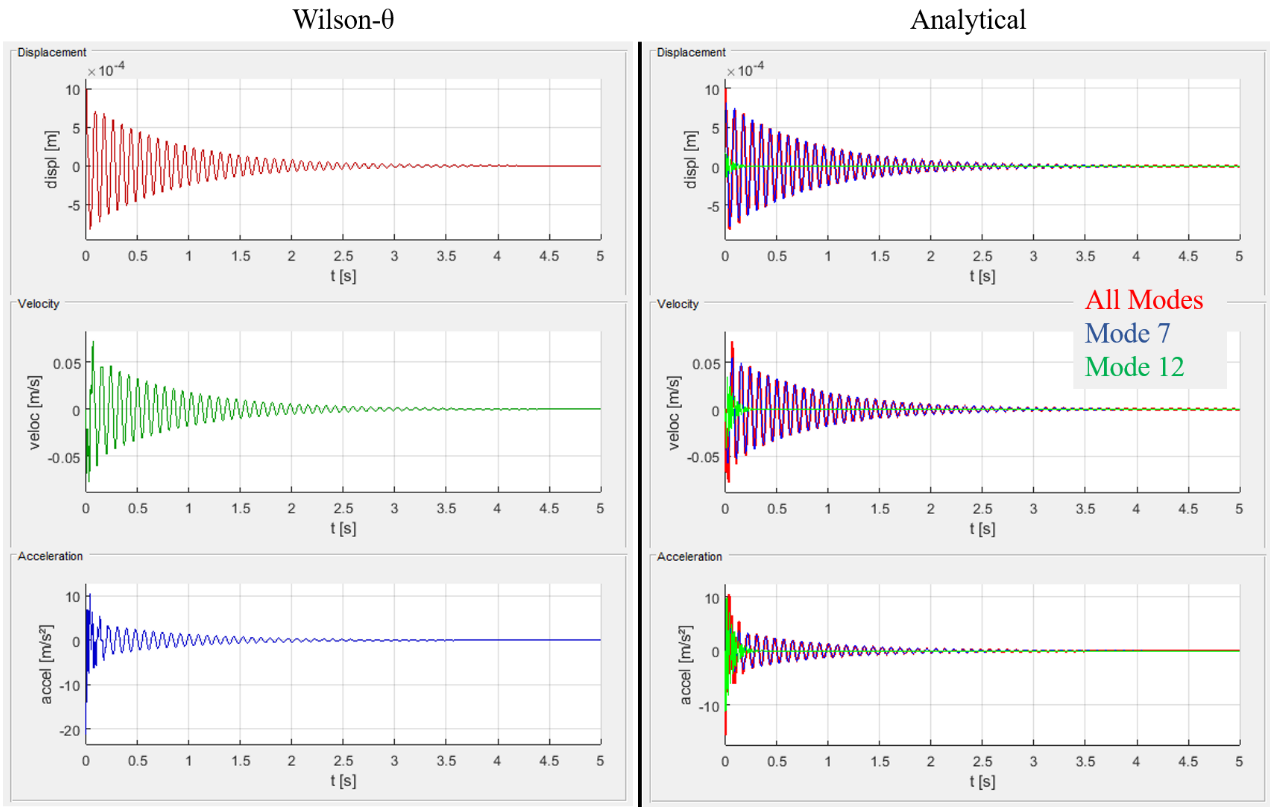

Figure 6. Dynamic response diagrams for free vibration, obtained with the LESM program. Coupled system (left) and uncoupled system (right)

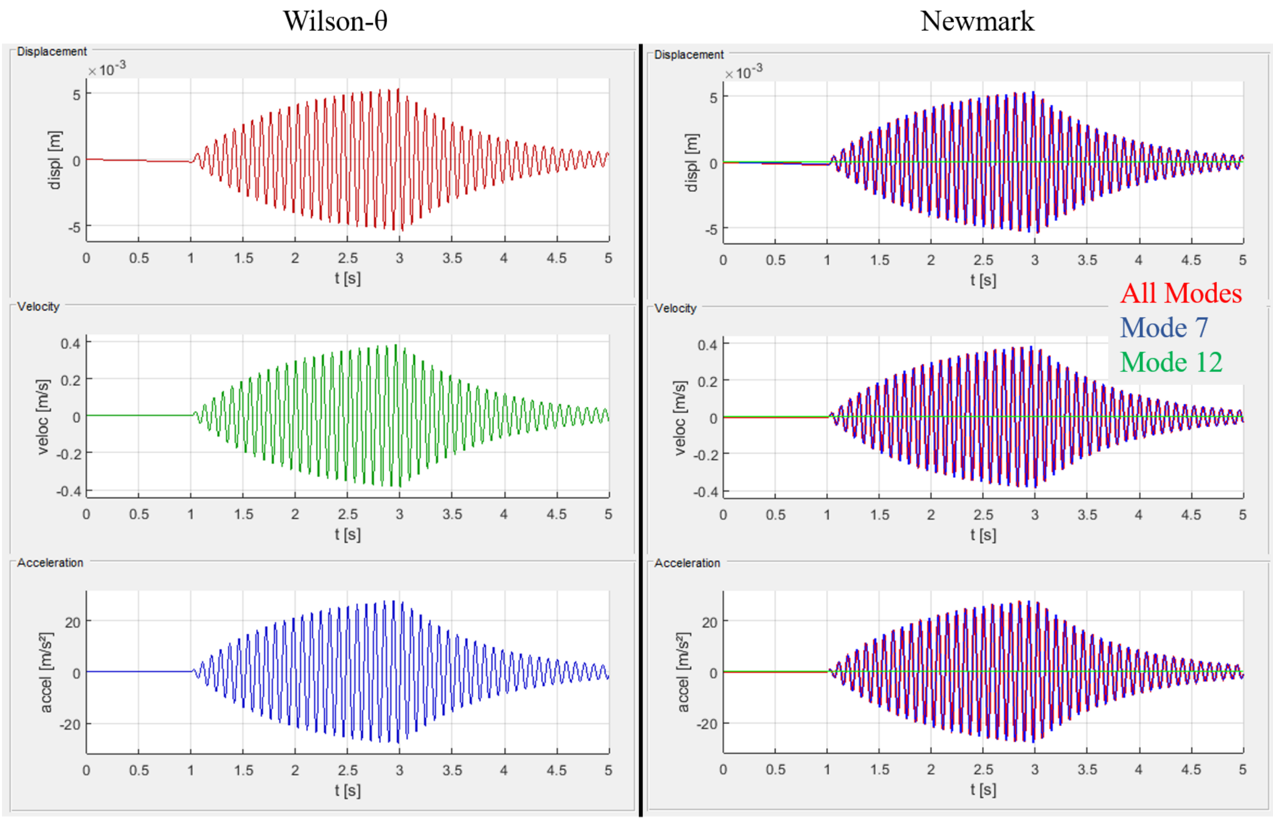

Figure 7. Dynamic response diagrams for forced vibration, obtained with the LESM program. Coupled system (left) and uncoupled system (right)

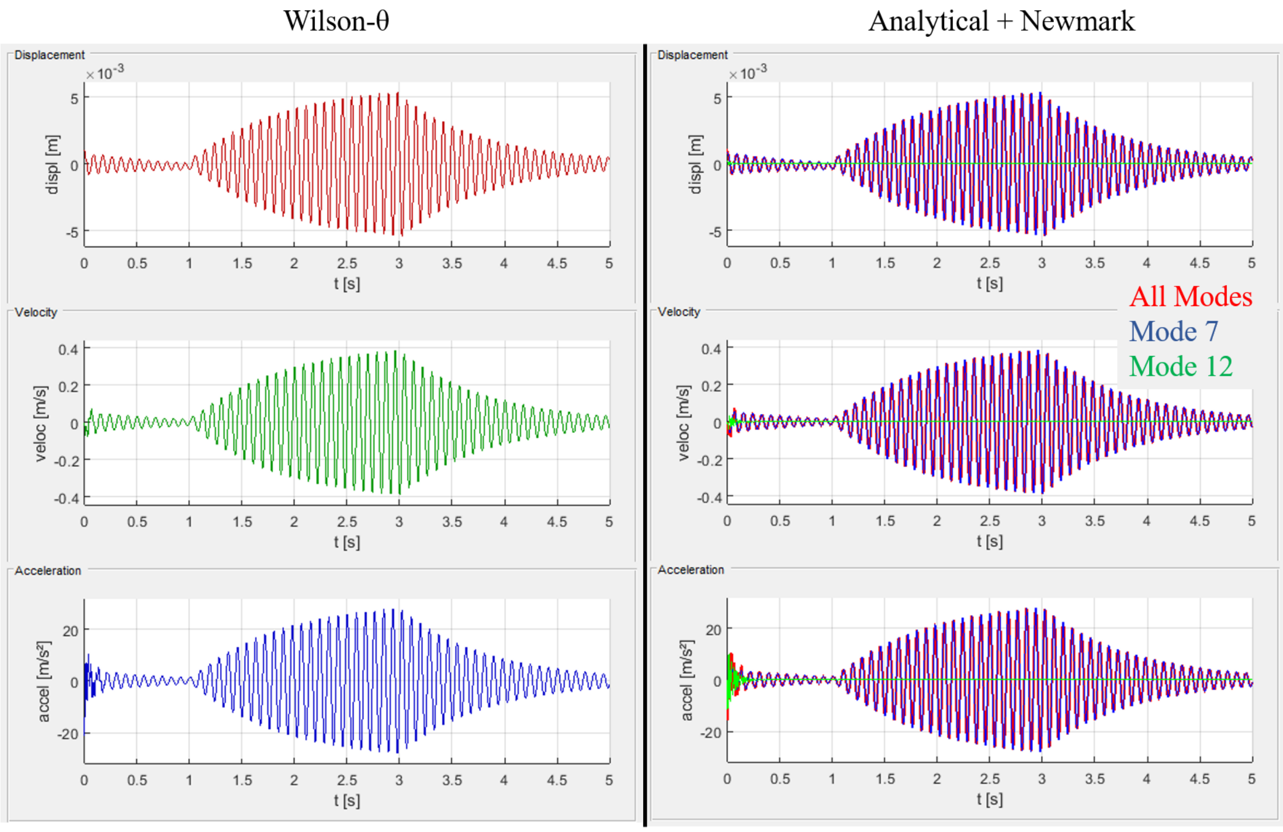

Figure 8. Dynamic response diagrams for total vibration, obtained with the LESM program. Coupled system (left) and uncoupled system (right)

It is also possible to obtain three-dimensional phase portraits of the solution of the system of ODEs. Figure 9 depicts the phase portrait of node 7 behavior, regarding its displacement on the global *z* axis direction, for free vibration.

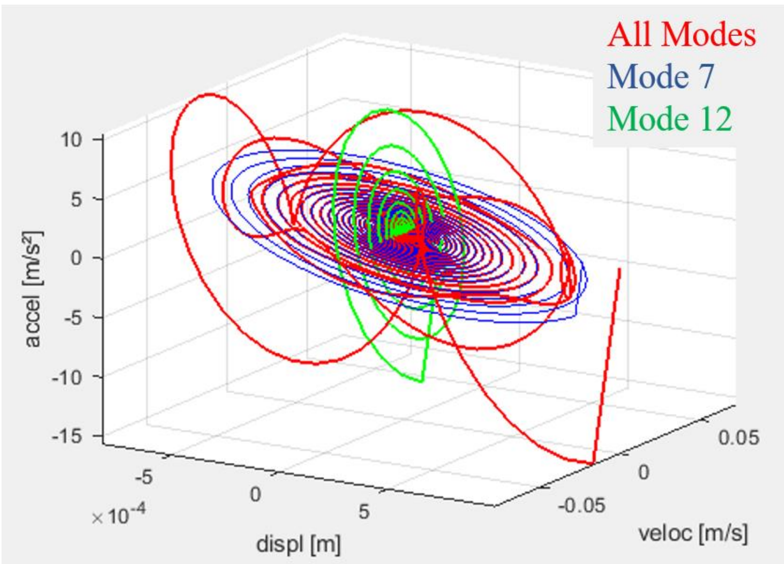

Figure 9. Phase portrait of solution for free vibration, obtained with the LESM program

Another important result of the structural vibration analysis is the spectrum of the solution in the frequency domain. This allows the evaluation of the most solicited natural vibration modes. Figure 10 depicts the acceleration spectrum of node 7, on the global *z* axis direction, for forced vibration.

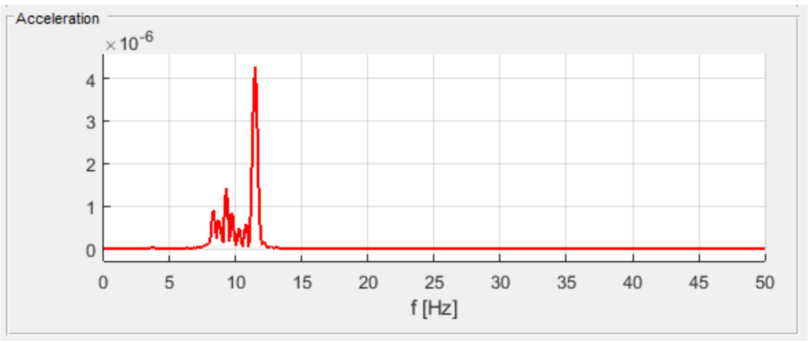

Figure 10. Spectrum of solution in frequency domain for forced vibration, obtained with the LESM program

In addition to the diagrams that depict the solution of the system of ODEs of motion, results such as the dynamic deformed configuration and internal forces and bending moment envelopes are obtained. Figure 11 exposes the normalized deformation related to vibration modes 7 and 12 of the analyzed model, and Figure 12 shows the envelope of bending moment about each element's local *y* axis.

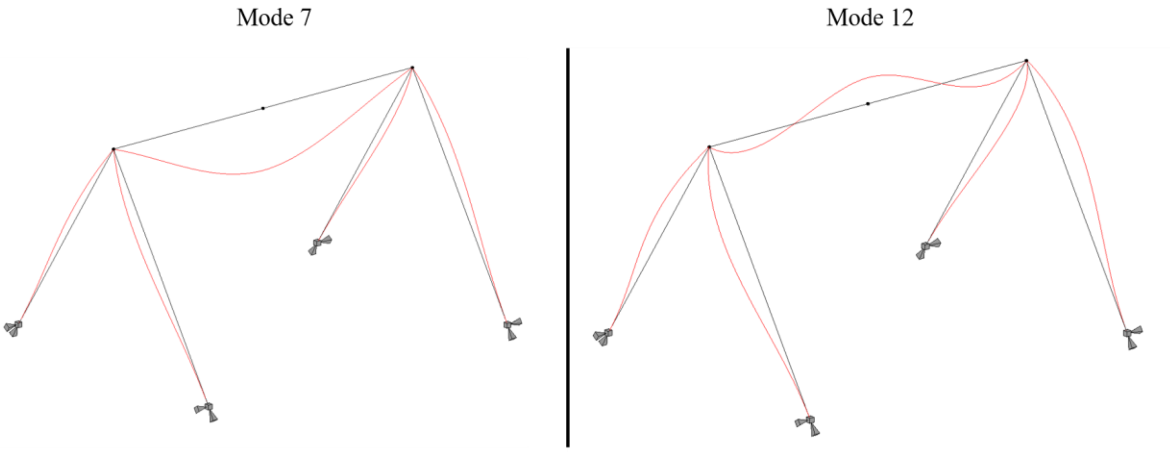

Figure 11. Vibration modes 7 and 12 of the analyzed structure

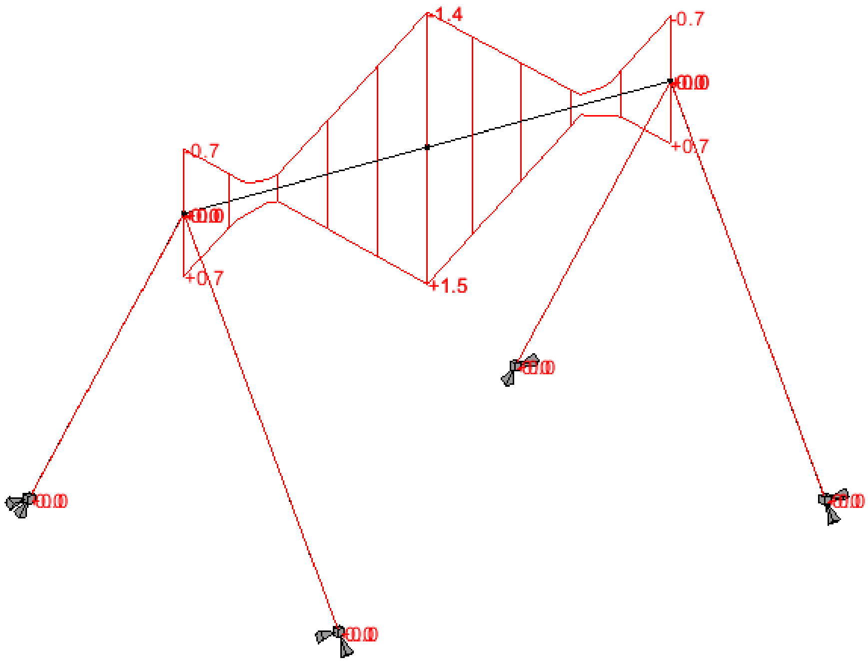

Figure 12. Envelope of bending moment [kNm] about local *y* axis, obtained with the LESM program

8. CONCLUSION

The process of vibration analysis is becoming established as a vital step of modern structural design. An open source computational tool of simple usage, endowed with interactive-graphics functionalities, for studies of this nature is of great utility for engineering practice. Modeling the dynamic behavior of structures on a non-closed software provides users the possibility to gain a deeper understanding of the physical problem and the methodologies adopted to analyze it.

The extension of the LESM program provides a tool of significant educational interest as well. Not only for the teaching of structural dynamics, but also for the usage of numerical methods for solving systems of ODEs and the assimilation of the practice of scientific computation itself.

The Matlab script codes of all classes and methods of the linear elastic static version 2.0 of LESM are available in its website (www.tecgraf.puc-rio.br/lesm). Version 1.0 of LESM is the version associated to the book by Martha (2018). In the site, there is a complete UML documentation of version 1.0. Version 3.0, which is related to this paper, will be available in a near future.

It is the hope of the authors that this tool makes dynamic structural analysis a more accessible process to engineering students and professionals.

9. ACKNOWLEDGEMENTS

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