STABILIZED FINITE ELEMENT APPROXIMATIONS FOR TEMPERATURE-DEPENDENT NON-NEWTONIAN FLUIDS

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Abstract. *Non-Newtonian fluids are common in many industrial applications such as polymer extrusion and oil well drilling. Viscoplastic functions are commonly employed to model such non-Newtonian behavior in numerical simulations usually neglect the thermodependency of rheological properties. In the present study, the aim was to investigate the flow and heat transfer of a thermodependent Herschel-Bulkley fluid employing a Galerkin Least Squares (GLS) Finite Element Method (FEM). In order perform a sensivity analises of thermodependency parameters, the cross-flow and heat transfer over a confined circular cylinder was investigated. The Herschel-Bulkley model was regularized using Papanastasiou's method, and rheological properties (yield stress and consistency index) were assumed to depend on temperature exponentially, as suggested by the literature. The model was implemented in the context of a self-developed FEM code. The GLS stabilization method was employed in order to guarantee the compatibilization of finite element subspaces and to allow the use of equal-order pressure-velocity elements. The implementation was tested concerning the flow of a temperature-dependent power-law shear-thinning fluid. The results showed that thermal dependency of rheological properties enhance heat transfer. For a viscoplastic fluid, a factorial design was employed to evaluate the effects of rheological and thermodependency parameters on heat transfer, fluid dynamics and pressure drop. The ranges of parameter employed were Reynolds number from 1 to 20, Prandtl number from 1 to 10, flow index from 0.4 to 1, Herschel-Bulkley number from 0.1 to 10 and thermodependency parameters for yield stress and consistency index from 0 to 3. It was found that the temperature dependency parameter that most affect heat transfer is the one that controls the change in the consistency index. Besides, it was found that when employing thermodependency parameters, heat transfer enhancement is predicted.*

Keywords*: Thermodependent fluids, non-Newtonian, GLS, finite elements in fluids.*

1. INTRODUCTION

Cross-flow of Newtonian and non-Newtonian fluids over cylinders of circular and non-circular cross-sections have been extensively studied in the past years. There is a great amount of literature dedicated to unconfined infinitely long cylinders subjected to cross-flow of Newtonian fluids, concerning flow and heat transfer (e.g. Zukauskas, 1987). There are also many published works on non-Newtonian fluids subjected to that type of flow, where the roles of shear-thinning, shear-thickening and viscoplasticity are investigated (Bharti et al., 2007a; Bharti et al., 2007b; Bharti et al., 2008 and references therein). The effect of the wall confinement, i.e., flow over confined cylinders, on drag and heat transfer, has been studied more recently, concerning mainly the effects of channel blockage and cylinder position asymmetry on the flow behavior (Rehimi et al., 2008; Singha and Sinhamahapatra, 2010). Results regarding flow of non-Newtonian fluids in this type of configuration have also been the focus of many articles (Bharti et al., 2007a; Dhiman et al., 2005). These studies focused on the development of correlations for the drag coefficient and Nusselt number as well as for the understanding of how flow and heat transfer are affected by non-Newtonian parameters, especially because of the known phenomenon of heat transfer enhancement (Chhabra and Richardson 2008). However, the dependency of physical properties on temperature and the effect of such phenomenon on drag and heat transfer have been the focus of only a few numerical works (Soares et al., 2010, Zhu et al., 2007), which have concluded the importance of assuming temperature dependence of flow properties in order to correctly predict flow enhancement.

Concerning shear-thinning and viscoplastic flows in general, several authors have approached the effects of temperature dependence of rheological properties on flow and heat transfer (see, for instance Nouar et al., 2005 and references therein). Recently, the results of (Soares et al., 2010) confirmed that shear-thinning fluids increase heat transfer while shear-thickening fluids reduce heat transfer, and that the temperature-dependence mechanisms which facilitate heat transfer are obviously those which promote higher viscosity decrease with temperature.

Concerning the finite element method for non-Newtonian computational fluid dynamics, the classical Galerkin method for incompressible fluids suffers from the need to satisfy the Babuška-Brezzi condition (Franca, Frey, & Hughes, 1992) in matching velocity and pressure subspaces. In addition, there is the inherent instability of central difference schemes in the approximation of advective dominated flows. The Galerkin Least-Squares (GLS) method, introduced for Stokes equations in (Franca & Stenberg, 1991) has been developed to enhance the stability of the original

Galerkin method, by adding mesh-dependent terms to the Galerkin formulation. The GLS methodology has been extended to advection-diffusion equations (Franca et al., 1992) as well as for incompressible Navier-Stokes equations (Franca & Frey, 1992). In the context of non-Newtonian flows, some works have been published concerning the flows of purely viscous as well as viscoelastic and thixotropic liquids (Zinani and Frey, 2006; Zinani and Frey, 2008; Foseca et al., 2013). The coupling between flow and thermal problems, which mathematically represents the coupling between the incompressible flow equations and the advective-diffusion equations, in the context of the GLS method, has been achieved in (Franceschini and Frey 2005). However, the role of temperature dependent rheological properties has not been investigated in the context of stabilized methods.

In this work, numerical simulations of cross-flows of a temperature dependent shear thinning and viscoplastic fluids over a confined circular cylinder were performed. The numerical method employed was the GLS Finite Element Method. An algorithm was developed to accommodate the dependency of rheological parameters on temperature, coupling flow and thermal problems. These effects on heat transfer and pressure drop were investigated.

2. METHODOLOGY

2.1 Mathematical modeling

For flows of thermodependent non-Newtonian fluids, the mathematical modeling employed in this work consisted of the mass balance equation (Eq. (1)), the momentum balance equation for a generalized Newtonian liquid (GNL) (Eq. (2)), and the energy equation in terms of temperature (Bird, Armstrong, & Hassager, 1987) (Eq. (3)):

$$\operatorname{div}\mathbf{u} = 0\,, \tag{1}$$

$$\rho[\operatorname{grad}\mathbf{u}]\mathbf{u} = -\operatorname{grad} p + \operatorname{div}\boldsymbol{\tau} + \mathbf{f}\,, \tag{2}$$

$$\operatorname{grad} T \cdot \mathbf{u} = \operatorname{div}(\alpha \operatorname{grad} T) + S\,. \tag{3}$$

where $\mathbf{u}$ is the velocity field, ρ is the mass density, p is the pressure, $\boldsymbol{\tau}$ is the extra-stress tensor, $\mathbf{f}$ is the body force vector, T is the temperature field, α is the thermal diffusivity and S is the thermal energy source term. The extra-stress tensor is modeled for a generalized Newtonian fluid as

$$\boldsymbol{\tau} = \eta(\dot{\gamma}, T)\mathbf{D}(\mathbf{u})\,. \tag{4}$$

In order to predict non-Newtonian behavior, boht a shear-thinning and a viscoplastic models were employed. For shear-thinning the power-law model with temperature dependent consistency index was used:

$$\eta(\dot{\gamma}, T) = K_0 \exp[-b(T - T_0)]\dot{\gamma}^{n-1}\,, \tag{5}$$

where K_0 is the consistency constant, b is the temperature thinning parameter, n is the power-law index, $\dot{\gamma} = \left(2\operatorname{tr}\mathbf{D}^2\right)^{1/2}$ is the magnitude of the strain rate tensor, $\mathbf{D}(\mathbf{u})$, T_0 is the reference temperature.

For viscoplasticity, a modified temperature dependent Papanastasiou-Herschel-Bulkley model was employed. The original Herschel-Bulkley model was regularized using Papanastasiou's method (Mistoulis, 2007; Papanastasiou, 1990),

$$\eta(\dot{\gamma}, T) = K(T)\,\dot{\gamma}^{\,n-1} + \frac{\tau_0(T)}{\dot{\gamma}}\left[1 - \exp(-m\dot{\gamma})\right], \tag{6}$$

where $K(T)$ is the temperature-dependent consistency, $\tau_0(T)$ is the temperature-dependent yield stress, n the power-law index and m is the Papanastasiou coefficient, which must be high enough in order to mimic the original Herschel-Bulkley model The rheological parameters were assumed as functions of the temperature as follows:

$$\tau_0 = \tau_{0,ref} \exp[-a(T - T_{ref})]\,, \tag{7}$$

$$K = K_{ref} \exp[-b(T - T_{ref})]. \tag{8}$$

In Eqs. (7) and (8) the parameters a and b control the temperature dependence of K and τ_0 from their reference values K_{ref} and $\tau_{0,ref}$, at temperature T_{ref}.

2.2 Stabilized finite element formulation

Based on the above mathematical modeling, a boundary value problem for a thermodependent non-Newtonian fluid fluid was stated as

$$\operatorname{div}\mathbf{u} = 0 \quad \text{in} \quad \Omega\,, \tag{9}$$

$$\rho[\operatorname{grad}\mathbf{u}]\mathbf{u} = -\operatorname{grad} p + \operatorname{div}\boldsymbol{\tau} + \mathbf{f} \quad \text{in} \quad \Omega\,, \tag{10}$$

$$\operatorname{grad} T \cdot \mathbf{u} = \operatorname{div}(\alpha \operatorname{grad} T) + S \quad \text{in} \quad \Omega\,, \tag{11}$$

$$\mathbf{u} = \mathbf{u}_g \quad \text{in} \quad \Gamma_g\,, \tag{12}$$

$$(\boldsymbol{\tau} - p\mathbf{I})\cdot\mathbf{n} = \mathbf{t}_h \quad \text{in} \quad \Gamma_h\,, \tag{13}$$

$$T = T_g \quad \text{in} \quad \Gamma_g\,, \tag{14}$$

$$\mathbf{q}\cdot\mathbf{n} = q_h \quad \text{in} \quad \Gamma_h\,, \tag{15}$$

where $\mathbf{u}$ is the admissible velocity field, T is the admissible temperature field, $\Omega \subset \Re^{N=2,3}$ is a bounded domain with regular boundary Γ with outward unit normal $\mathbf{n}$, Γ_g is the portion of Γ where Dirichlet conditions are imposed ($\mathbf{u}_g$ and T_g), Γ_h is the portion of Γ subjected no Neumann boundary conditions ($\mathbf{t}_h$ and q_h).

The variational formulation of the above boundary value problem was approximated by a finite element method using a stabilization scheme based on Galerkin Least-Squares (GLS) formulations developed in (Hughes et al., 1986; Franca and Frey, 1992 and Zinani and Frey, 2006). In order to approximate velocity, pressure and temperature fields, the usual spaces of functions in fluid dynamics (Ciarlet, 1978) were employed:

A GLS formulation for Eqs. (9)-(15) was stated a two-way coupled problem:

Find the pair $(\mathbf{u}^h, p^h) \in \mathbf{V}_g^h \times P^h$ *such as:*

$$B_{\mathbf{v}}(\mathbf{u}^h, p^h; \mathbf{v}^h, q^h) = F_{\mathbf{v}}(\mathbf{v}^h, q^h) \quad \forall(\mathbf{v}^h, q^h) \in \mathbf{V}^h \times P^h\,, \tag{16}$$

where

$$\begin{aligned} B_{\mathbf{v}}(\mathbf{u}, p; \mathbf{v}, q) &= \left(\rho[\operatorname{grad}\mathbf{u}]\mathbf{u}, \mathbf{v}\right) + (2\eta(\dot\gamma, T)\mathbf{D}(\mathbf{u}), \mathbf{D}(\mathbf{v})) - (\operatorname{div}\mathbf{v}, p) - (\operatorname{div}\mathbf{u}, q) \\ &+ \sum_{K\in\Omega^h}\left(\rho[\operatorname{grad}\mathbf{u}]\mathbf{u} + \operatorname{grad} p - 2\operatorname{div}(\eta(\dot\gamma, T)\mathbf{D}(\mathbf{u})), \tau_{\mathbf{v}}(\mathrm{Re}_K)(\rho[\operatorname{grad}\mathbf{v}]\mathbf{u} + \operatorname{grad} q - 2\operatorname{div}(\eta(\dot\gamma, T)\mathbf{D}(\mathbf{v})))\right)_{\Omega_K} \end{aligned} \tag{17}$$

and

$$F_{\mathbf{v}}(\mathbf{v}, q) = (\mathbf{f}, \mathbf{v}) + (\mathbf{t}_h, \mathbf{v})_\Gamma + \sum_{K\in\Omega^h}\left(\mathbf{f}, \tau_{\mathbf{v}}(\mathrm{Re}_K)(\rho[\operatorname{grad}\mathbf{v}]\mathbf{u} + \operatorname{grad} q - 2\operatorname{div}(\eta(\dot\gamma, T)\mathbf{D}(\mathbf{v})))\right)_{\Omega_K}\;; \tag{18}$$

and find $T^h \in W_g^h$ *such as:*

$$B_w(T^h; w^h) = F_w(w^h) \quad \forall(w^h) \in W^h\,, \tag{19}$$

where

$$\begin{aligned} B_w(T; w) &= (\operatorname{grad} T\cdot\mathbf{u}, w) + (\alpha \operatorname{grad} T, \operatorname{grad} w) + \\ &+ \sum_{K\in\Omega^h}\left(\operatorname{grad} T\cdot\mathbf{u} - \operatorname{div}(\alpha \operatorname{grad} T), \tau_w(\mathrm{Pe}_K)(\operatorname{grad} w\cdot\mathbf{u} - \operatorname{div}(\alpha \operatorname{grad} w))\right)_{\Omega_K} \end{aligned} \tag{20}$$

and

$$F_w(w) = (S, w) + (q_h, w)_\Gamma + \sum_{K\in\Omega^h}\left(S, \tau_w(\mathrm{Pe}_K)(\operatorname{grad} w\cdot\mathbf{u} - \operatorname{div}(\alpha \operatorname{grad} w))\right)_{\Omega_K}\,, \tag{21}$$

with $(\cdot , \cdot)$ and $(\cdot , \cdot)_K$ representing the L^2-inner product in the domains Ω and Ω_K, respectively. The stability parameters $\tau_v(\mathrm{Re}_K)$ and $\tau_w(\mathrm{Pe}_K)$ – which are functions of the element Reynolds number, Re_K, and the element Péclet number, Pe_K – were defined based on the works of Franceschini and Frey (2005) and Zinani and Frey (2006).

2.3 Solution strategy

The formulation stated above is two-way coupled, because the velocity field influences the temperature field and vice versa. The discretization of the GLS formulation defined by Eqs. (16)-(21) was obtained through the expansion of the finite element approximations for ($\mathbf{u}^h$, p^h, T^h) and ($\mathbf{v}^h$, q^h, w^h) as a combination of their respective shape functions and degrees of freedom. The non-linear system resulting from Eq. (16) was solved via a quasi-Newton method, which was described in detail in Zinani and Frey (2006). The linear problem resulting from the discretization of Eq. (19) was solved employing a linear decomposition as described in Franca and Frey (1992). The complete methodology was implemented in the in-house code NNFEM, which is developed using FORTRAN language and capability (Franca and Frey, 1992).

2.4 Problem domain and boundary conditions

The problems considered in the present work were steady state flows over a cylinder confined between two parallel walls. This geometry was modeled in two-dimensions, and only its upper-half was taken as the problem domain, as shown in Fig. 1. The cylinder diameter was equal to D and the distance between channel walls was equal to H. At the inlet, the mean velocity and temperature were u_0 and T_0, and these are taken as reference velocity and length for the definition of dimensionless parameters governing this problem. The cylinder wall was kept at T_w. The channel walls were assumed impermeable, no-slip and adiabatic. The outflow condition was applied as a traction free boundary condition and zero heat flux boundary condition. The dimensionless parameters which govern this problem are the Reynolds number, the Prandtl number, the Herschel-Bulkley number (also known as Bingham number), the power-law index, and the dimensionless versions of parameters a and b in Eqs. (8)-(9):

$$\mathrm{Re} = \frac{\rho u_0^{2-n} D^n}{K_{ref}}; \quad \mathrm{Pr} = \frac{c K_{ref}}{k}\left(\frac{u_0}{D}\right)^{n-1}; \quad \mathrm{Hb} = \frac{\rho D^n}{K_{ref} u_0^{n-2}}; \quad n; \quad a^* = \frac{a}{T_w - T_0}; \quad b^* = \frac{b}{T_w - T_0} \tag{22}$$

where the fluid properties c and k are the specific heat and thermal conductivity.

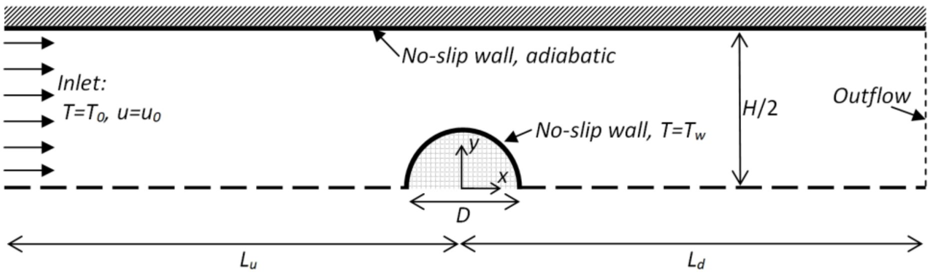


Figure 1 - Problem statement.

The dimensionless temperature field is calculated as

$$\theta = \frac{T - T_w}{T - T_w}. \tag{23}$$

The main problem consists in determining the Nusselt number,

$$\mathrm{Nu} = \frac{hD}{k}, \tag{24}$$

with h the convection heat transfer coefficient defined after Newton's cooling law:

$$q_w = hA(T_w - T_0) \tag{25}$$

and q_w the heat transfer from the cylinder surface; and the drag coefficient,

$$C_d = \frac{F}{A_f \tau_{0,ref}} \tag{26}$$

with F the drag force and A_f the cylinder frontal area.

2.5 Mesh quality and code validation

A careful mesh quality study was performed in order to achieve enough refinement so that the primal variable fields and global results were not affected by discretization. After this study, a mesh consisting of 14400 elements of type $Q_1/Q_1/Q_1$ (four node quadrilateral elements, Franca and Frey, 1992) for velocity, pressure and temperature fields was adopted. A difference between the results from the mesh to be adopted and a more refined with elements 30% was required to be less than 2%. Besides mesh quality a validation study was performed comparing the results of the present model with results of flow and heat transfer around a cylinder of the work of Bharti et al. (2007). Good agreement was achieved. A verification study concerning the achieving of analytic velocity profile inside a tube for a viscoplastic fluid was also conducted. The numerical result was capable to reproduce the analytic velocity profile with good accuracy.

3. RESULTS AND DISCUSSION

3.1 Thermodependent shear-thinnning fluid

For the shear-thinning fluid, some preliminary results are presented for the combination of parameters Re = 1, Pr = 10, n = 0.6 and the temperature thinning parameter, b^*, equal to 0, 1 and 3. Figure 2 presents the dimensionless temperature field, $\theta = (T - T_0)/(T_w - T_0)$, for the b^*' s tested. It is possible to observe the influence of b^* on the θ field and consequently on the heat transfer rate from the cylinder. The higher the b^*, the more the viscosity decreases with temperature, and the consequence is the heat transfer enhancement, observed commonly for non-Newtonian fluids (Chhabra and Richardson 2008), and in agreement with previous studies (Soares et al., 2010).

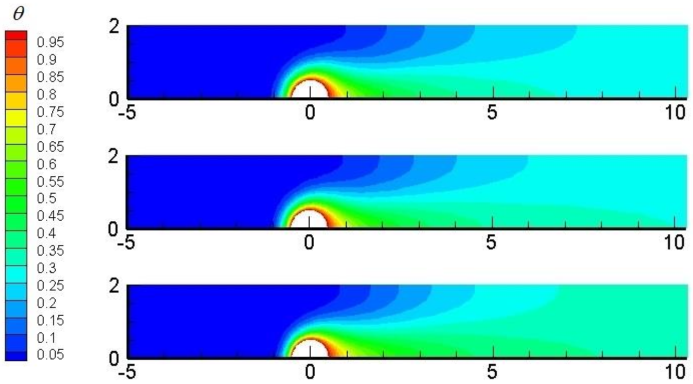


Figure 2. Dimensionless temperature field. (a) b^* = 0; (b) b^* = 1 and (c) b^* = 3.

Figure 3(a) shows the axial velocity profiles in $x^* = x/D$ = 3. The usual flat velocity profile of shear-thinning fluids is elongated by the decrease in viscosity by influence of temperature, which is more pronounced for higher b^*. Figure 3(b) shows the temperature profile in the same position, showing the heat transfer enhancement with the increase of b^*. To corroborate this assumption, an increase in the Nusselt number of 13% was observed for b^*= 3 as comparing to b^* = 1 and 77% as comparing to b^* = 0.

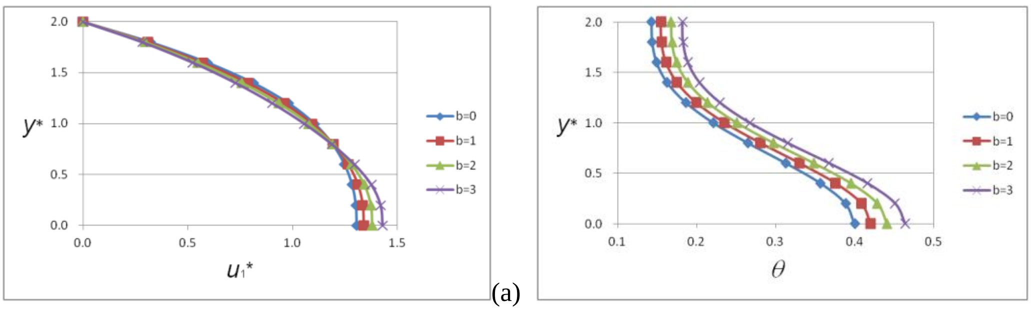


Figure 3. Transversal profiles in x^* = 3: (a) axial velocity and (b) temperature.

3.2 Thermodependent viscoplastic fluid

For a viscoplastic fluid, the number of parameters governing flow and heat transfer is greater than for a power-law fluid. When using the modified Herschel-Bulkley model of Eq. 5, there are 6 parameters, or factors, that influence the fluid dynamics (see Eq. 22). In order to evaluate the effect of each of these factors in the Nusselt number and in the pressure drop due to the cylinder, a factorial design 2^6 (Montgomery, 2001) was then employed. For each parameter was assigned a code letter, and ranges of values employed for each parameter were:

A - a^*: 0 and 3;
B - b^*: 0 and 3;
C - Re: 1 and 20;
D - Hb: 1 and 10;
E- Pr: 1 and 10.
F - n: 0.4 and 1.

A total of 64 simulations were necessary to obtain the results to generate the Pareto chart and results from the factorial analysis. Figure shows the Pareto chart for the effects of these 6 parameters on the resulting Nusselt number. It may be noticed that all parameters had effects higher 0.70 (see dashed line). Some interaction among parameters were detected, meaning that the effects of each of this parameters is different depending on the level of the other. The main interactive combinations were: Re and Hb (CD), Re and Pr (CE), Hb and n (DE). Surprisingly, a three factor interaction was detected among Re, Hb and Pr (CDE). The next main factor affecting the heat transfer result was the b^* parameter. It ca be noticed from the Pareto chart that b^* (B) has much influence than a^* (A) on the heat transfer. As a^* is the parameter governing temperature dependence of the yield stress and b^* is the parameter governing temperature dependence of consistency index, this result suggests that changes in viscosity inside the yielded zone affect more the heat transfer than changes in the yield stress. The explanation may lay in the fact that heat transfer occurs in the yielded region which is formed along the cylinder wall.

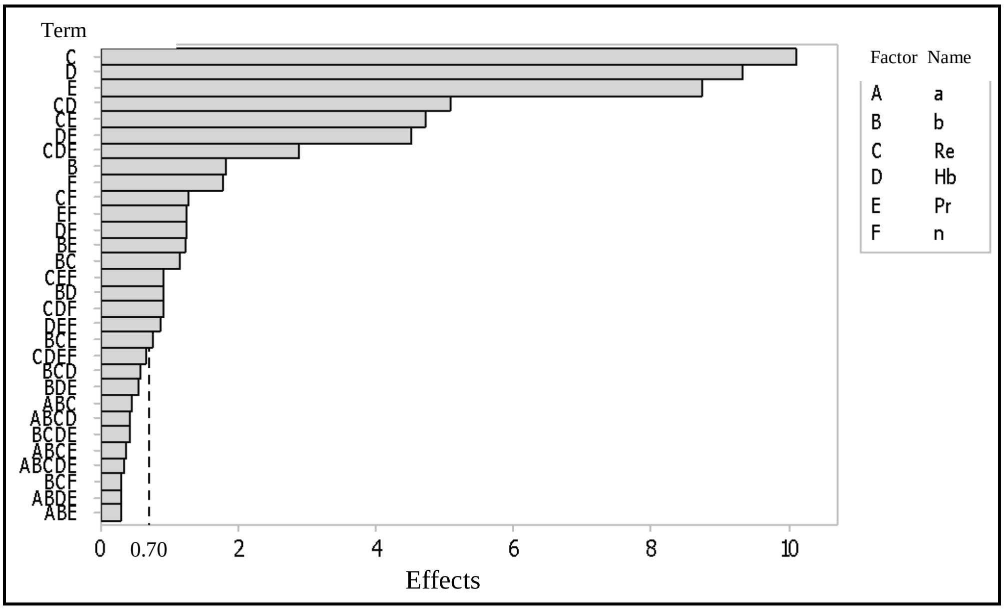


Figure 4. Pareto chart of effects on the Nusselt number.

Figure 5 is the Pareto chart for the effects of all parameters and their combinations on the dimensionless pressure drop over the cylinder. As it was expected, the Reynolds number (C) was the main parameter affecting pressure drop (C). Interactions of DF (Hb and n) and CDF (Re, Hb and n) were also important. The power law index was the following most important parameter, for it is responsible for shear-thinning, and the next parameter was the coefficient b^*, the rheological parameter controlling consistency temperature dependence. The lower in viscosity promoted by the increase in temperature decreases the pressure drop.

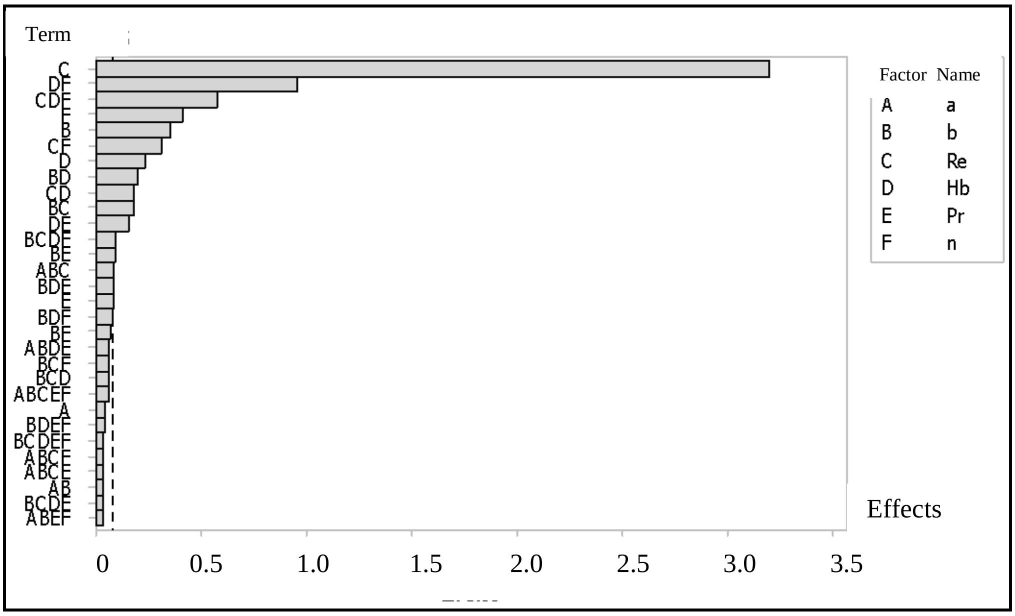
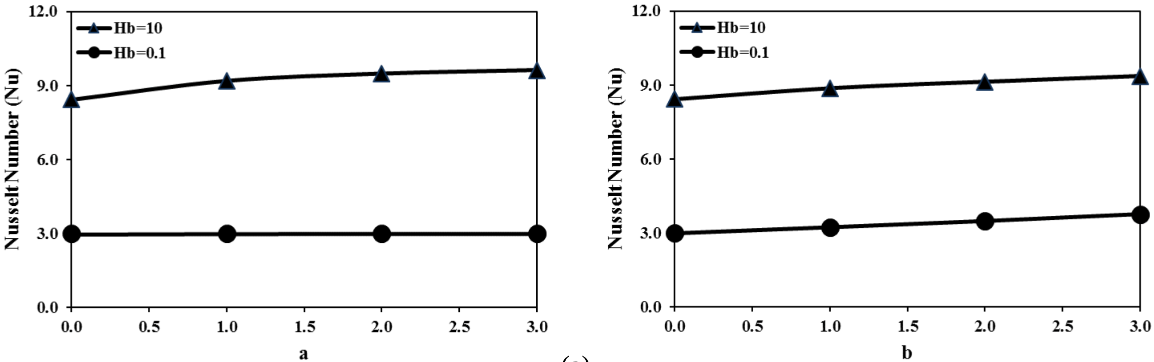


Figure 5. Pareto chart of effects on the pressure drop (Δp^*).

Figure 6 shows the effects of the coefficients a^* and b^* on the Nusselt number for a fixed Reynolds number (Re = 1), Prandtl number (Pr = 10), power-law index (n = 0.4), and two values of Herschel-Bulkley number, Hb = 1 and Hb = 10. In Fig. 6(a) b^* = 0, and in Fig. 6(b), a^* = 0. The influence of the temperature dependent parameters is more pronounced for the higher Hb in the case of a^* (Fig. 6(a)). For parameter b^*, the level of heat transfer enhancement is the same for either Hb. Viscosity reduction due to temperature dependent rheological parameters is a key to understand this behavior, for the higher the b^*, more viscosity decreases and heat convection in improved.



(a) (b)

Figure 6. Nusselt number as a function of temperature dependence parameters. (a) a^* and (b) b^*.

4. CONCLUSIONS

In this work a mathematical model for flow and heat transfer of non-Newtonian temperature dependent fluids. A stabilized finite element formulation, based on the GLS method was introduced in order to approximate problems concerning this type of flow. Because of temperature dependency of rheological properties, the flow and thermal problems are coupled. A decoupled algorithm was employed to solve the resulting equations.

This formulation was implemented in an in-house software, namely NNFEM, and some results were obtained concerning temperature dependent shear-thinning and viscoplastic fluids. From these results, it may be concluded that the parameter which governs the decrease in consistency index with increase in temperature affects the resulting pressure drop and Nusselt number more than other temperature dependency parameters.

From a 26 factorial analysis it may be concluded that temperature effects are considerable for non-Newtonian fluids flowing in the tested range of parameters.

5. ACKNOWLEDGEMENTS

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