NUMERICAL SIMULATION OF PIG MOTION IN GAS-FLOW PIPELINES USING THE FLUX-CORRECTED TRANSPORT METHOD

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Abstract. *In order to maintain efficiency and high productivity in pipeline operations, it is necessary to keep an updated maintenance program. The use of PIGs is a very common task, but not a simple one, since there are uncertainties and risks associated to its passage inside a long duct. For these reasons, it is important to follow the motion of a PIG inside a pipeline using numerical simulations. The pigging operation depends on the velocity at which the PIG travels throughout the line, which is coupled to the fluid flow itself. Therefore, it becomes crucial to accurately predict the PIG motion and the fluid flow within the pipeline. This paper presents a mechanical model, along with a numerical scheme, to obtain approximate solutions to the resulting initial-boundary-value problem that describes the PIG movement in a transient gas flow inside a pipeline. The model is discretized within the finite-volume framework and is solved using the Flux-Corrected Transport (FCT) method, which is second-order in space. The problem is formulated as an initial-boundary-value problem of hyperbolic nature. We present results for the PIG velocity history and for the distributions of flow pressure and velocity along the pipe.*

Keywords*: PIG motion in pipelines, gas-flow-pipelines, Flux-Corrected Transport Method, Numerical simulation, petroleum engineering*

1. INTRODUCTION

Pipelines are the most efficient and safe manner to transport fluids, such as oil, gas and other petroleum products, in the oil industry. To keep high efficiency and low costs, periodical pipeline maintenance must be performed in the line through pigging operations. Pigs are responsible for carrying instrumented devices to profile the pipeline in terms of wall thickness, weld imperfections and general structural damage, as well as for removing undesired obstructions to the flow, such as wax deposition and condensates. Since the efficiency of pigging operations relies on the velocity at which the pig travels along the line, which in turn is affected by the fluid flow conditions and vice-versa, it becomes crucial to properly predict the pig movement and its hydraulic consequences within the pipeline. The passage of the pig throughout the pipeline involves many variables, which makes the prediction of the pig's dynamics inside the pipeline extremely difficult.

Azevedo *et al.* (1996, 1997), Campo e Rachid (1998), Nguyen *et al.* (2001), Nieckele *el al.* (2001), Esmaeilzadeh *et al.* (2006, 2009), Nieckele and Tomalsquim (2008), Botros *et al.*(2009) are some of the authors that presented relevant contribution to the mathematical modeling and its numerical discretization for solving the initial-boundary-value problem that describes the pig motion in a transient gas flow inside a pipeline. Recently Bueno *et al.* (2012) carried out a long review of the pig models available in the literature and proposed a general mathematical model, coupled to a numerical strategy based on the Flux-Corrected Transport method – FCT (Boris and Book, 1973), to solve the pig motion in two-phase flows inside pipelines. The present work is a preliminary step towards the implementation of this strategy.

In this paper we propose a mathematical model that combines a system of one-dimensional partial-differential equations (PDE's) for the motion of a gas inside a pipeline with a general model for a pig the moves with the gas. For the sake of simplicity, we assume that the flow is incompressible and isothermal. The hyperbolic system of non-linear partial-differential equations for the gas motion is discretized using the finite-volume method and solved using the FCT method for the flux terms. This algorithm is linked to an IMSL routine (Fortran Numerical Math Library Version 7.0) that solves a system of non-linear algebraic equations for the pig. The simultaneous solution for the gas flow and the pig motion allows the pig velocity and position to be calculated at all times, together with the pressure and velocity distributions for the gas along the pipeline. Our numerical results are compared to OLGA's, a commercial software that is used as a reference in the petroleum industry for this type of simulation.

2. MATHEMATICAL EQUATIONS

The gas flow inside a pipeline is governed by a system of non-linear partial differential equations that express the conservation of mass, momentum and energy. These equations can be written in a simplified form for the flow in a pipe with a constant cross-section area if we assume that the flow is one-dimensional, time-dependent, compressible and isothermal. The latter assumption implies that the equation of state for the gas, considered to obey the ideal gas law, depends (linearly) on the pressure only and the energy equation can be disregarded in the analysis, as a first approximation. Based on these considerations, we may write this system of hyperbolic partial-differential equations according to

$$\frac{\partial(\rho)}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0,$$

(1)

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + p)}{\partial x} = -\rho g \sin(\beta) - \frac{\tau_w S}{A}.$$

(2)

In Eqs (1) and (2), x represents the axial coordinate along the pipe, t denotes time, and the dependent variables ρ, p and u refer to the gas density, pressure and velocity, respectively. In the first term of the right-hand side of Eq. (2), g represents the gravity acceleration and β the pipeline inclination, whereas in the last term of Eq. (2) S is the pipe's perimeter, τ_w is the shear stress on the pipe wall and A is the pipe's cross sectional area. The gas density ρ is related to the pressure p and temperature T by the ideal-gas law $p = \rho R T$, where R is the gas constant, but the temperature is assumed to be constant, as mentioned above. Additionally, we can write the closure equation for the wall shear stress as

$$\tau_w = \frac{1}{2} f \rho u |u|,$$

(3)

where

$$f = \max\left[\frac{16}{Re}, 0.001375\left[1 + \left(2 \times 10^4 \frac{\varepsilon}{D} + \frac{1 \times 10^6}{Re}\right)^{\frac{1}{3}}\right]\right]$$

(4)

and

$$Re = \frac{\rho D |u|}{\mu},$$

(5)

where f is the wall friction factor, proposed by Moody (1947), Re is the Reynold number, D is the pipe's diameter, ε is the superficial roughness of the pipe wall and μ is the gas dynamic viscosity.

2.1 Initial and boundary conditions

The set of Eqs. (1) - (5) form a closed system of equations with u and p as unknowns. This system is subject to the following boundary and initial conditions:
Boundary conditions:
Pipeline inlet: u
Pipeline outlet: p.

Initial condition:
Steady state solution obtained previously using the values of u and p set at the boundaries as initial condition.

3. NUMERICAL METHOD

The FCT method is a conservative method of second order in space based on the conservative discretization of the flux terms proposed by Boris and Book (1973, 1976), Book *et al.* (1975) and Lezeau and Thompson (1998). This method is used for solving systems of hyperbolic equations, such as those involving shock waves and discontinuities. More detailed information can be found in Sondermann *et al.* (2015).

3.1 Hyperbolic system of PDE´s

Equations (1) and (2) comprise a hyperbolic system of Partial Differential Equations (PDE´s) that is here discretized using the finite difference method. This type of system may be written in the conservative form

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{Q})}{\partial x} = \mathbf{S}(\mathbf{Q}),$$
(6)

with $(x,t) \in (0,L) \times (0,\infty)$.

Equation (6) is a general representation of the hyperbolic system of PDE´s. For our model, the non-linear system defined by Eqs. (1) and (2) may be rewritten in the form of Eq. (6), where the vectors $\mathbf{Q}$, $\mathbf{F}$ and $\mathbf{S}$ are the vector of conservative variables, the flux vector and the source-term vector, defined, respectively, as

$$\mathbf{Q} \equiv \begin{bmatrix} \rho & \rho u \end{bmatrix}^T,$$
(7)

$$\mathbf{F} \equiv \begin{bmatrix} \rho u & \rho u^2 + p \end{bmatrix}^T,$$
(8)

$$\mathbf{S} \equiv \begin{bmatrix} 0 & -f\dfrac{\rho u^2/2}{A} - \rho g \operatorname{sen}\beta \end{bmatrix}^T.$$
(9)

4. PIG MOTION MODEL

For mathematical modeling purposes, the presence of a pig inside the pipeline is considered to be a moving boundary. Figure 1 shows a schematic drawing of a generic situation where a pig occupies a certain position inside the pipeline at time t, where x represents the horizontal axis, p^-, p^+, u^-, u^+ are the pressure and gas velocity, upstream ($-$) and downstream ($+$) of the pig. The quantities L_c and L are the pig and pipe lengths, respectively, which are subject to the condition $L_c << L$.
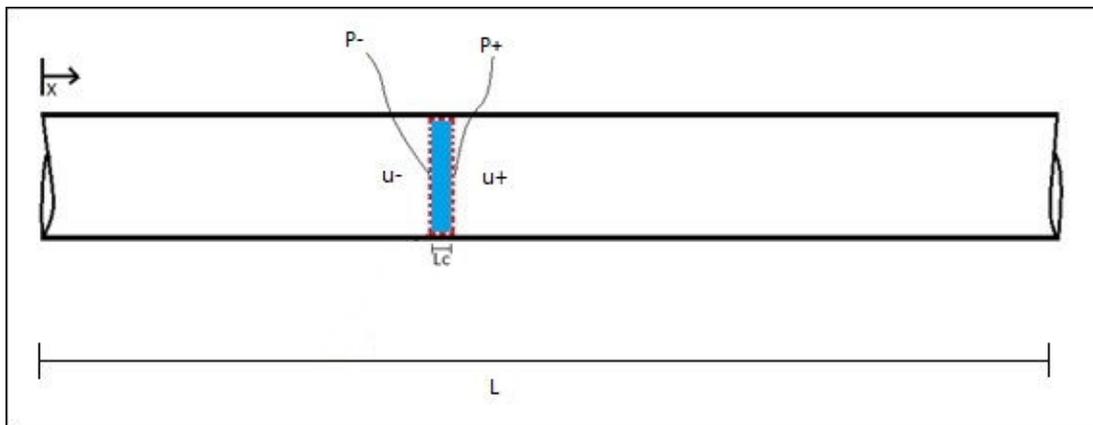

Figure 1. Pig positioning scheme inside the pipeline.

Pigs are known to have multiple kinds of shapes, material and constructive characteristics that must be taken into account. This is usually defined by the type of service that the pig will perform (cleaning, inspection, batch production, pipeline cleanup). We considered, in the present work, a pig with simple constructive characteristics. Figure 2 shows a foam pig without holes and it only allows bypass through the gap between the pig and the pipeline, represented by δ. The geometry of the pig is characterized by the pig radius r_P and the pipe by its internal diameter D.
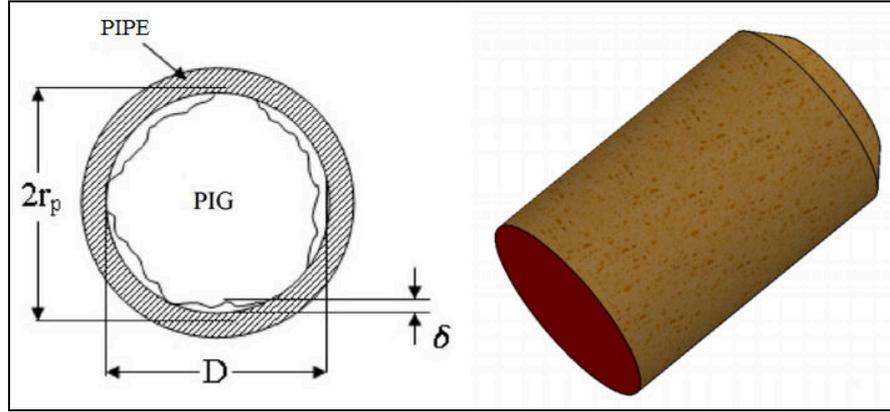
Figure 2. Foam Pig.

The moving boundary imposed by the pig causes discontinuities in pressure and fluid velocity upstream and downstream the PIG, which must be solved as a result of the coupling of the pig model and the flow equations discretized using the FCT method.

Based on the mass and momentum conservation laws, the equations below describe the pig motion model. The mass conservation equation for the pig can be expressed as

$$\rho^- A(u^- - v_P) = \rho^+ A(u^+ - v_P) = \dot{m}_{BP},$$

(10)

where ρ^-, ρ^+, u^- and u^+ represents density and fluid velocity upstream and downstream the pig, v_P is the pig velocity and $\dot{m}_{BP}$ the bypass mass flow rate through the gap.

We can rewrite Eq. (10) using the ideal- gas law $p = \rho RT$, which reads

$$\frac{p^-}{RT} A(u^- - v_P) = \frac{p^+}{RT} A(u^+ - v_P) = \dot{m}_{BP}.$$

(11)

The mass flow through the gap is defined as

$$\dot{m}_{BP} = \bar{\rho} Q_{gap},$$

(12)

where $\bar{\rho} = \hat{\rho}(\rho^-, \rho^+)$ is a function that relates the gas density behind and ahead of the pig, and the volumetric flow rate in the gap is given by

$$Q_{gap} = \pi D \left(\frac{\delta^3}{12\mu} \frac{|\Delta p|}{L_C} - \frac{\delta}{2} v_P \right).$$

(13)

The linear momentum conservation law applied to the pig gives

$$M_P \frac{dv_P}{dt} = (p^- - p^+)A - M_P g \operatorname{sen} \beta - F_H - F_M,$$

(14)

with the symbols M_P, F_H and F_M representing the pig mass, hydrodynamic force and friction force, respectively. The hydrodynamic force is defined as

$$F_H = (1 - \xi)\pi D L_C \left(\mu \frac{v_P}{\delta} - \frac{(p^- - p^+)}{2L_C} \delta \right),$$

(15)

where $\xi = \hat{\xi}(v_P)$ is a parameter that measures the effective contact between the pig and the pipeline wall. The friction force is defined according to

$$F_M = \begin{cases} \operatorname{sgn}(v_P)F_M^d, & \text{if } v_P \neq 0 \\ \operatorname{sgn}\left((p^- - p^+)A - F_H - M_P g \operatorname{sen} \beta\right)F_M^s, & \text{if } v_P = 0 \text{ and } \dfrac{dv_P}{dt} \neq 0, \\ (p^- - p^+)A - F_H - M_P g \operatorname{sen} \beta, & \text{if } v_P = 0 \text{ and } \dfrac{dv_P}{dt} = 0 \end{cases} \tag{16}$$

where F_M^s and F_M^d are, respectively, the static and dynamic friction forces, which are given by

$$F_M^s = \xi \pi D L_C \eta_s \frac{1}{1-v}\left(\frac{E^* \Delta r}{r_P} + v\frac{(p^- + p^+)}{2} \right), \tag{17}$$

$$F_M^d = \xi \pi D L_C \eta_d \frac{1}{1-v}\left(\frac{E^* \Delta r}{r_P} + v\frac{(p^- + p^+)}{2} \right). \tag{18}$$

In Eqs. (17) and (18), η_s and η_d denote the static and dynamic friction factors, respectively, v is the poisson ratio and E^* is the elastic modulus of the pig.

With Eqs. (11), (12) and (14), and the closure laws defined by Eq. (13) and Eqs. (15) – (18), we form a system of differential-algebraic equations that describes the pig's dynamics inside the pipeline during its motion with the gas flow.

5. SOLUTION METHOD FOR THE PIG MOTION INSIDE THE PIPELINE

As mentioned before, the pig is considered to be a moving boundary inside the pipeline. In order to couple the FCT solution downstream and upstream of the pig with the pig's motion solution, we developed a strategy that will be described below.
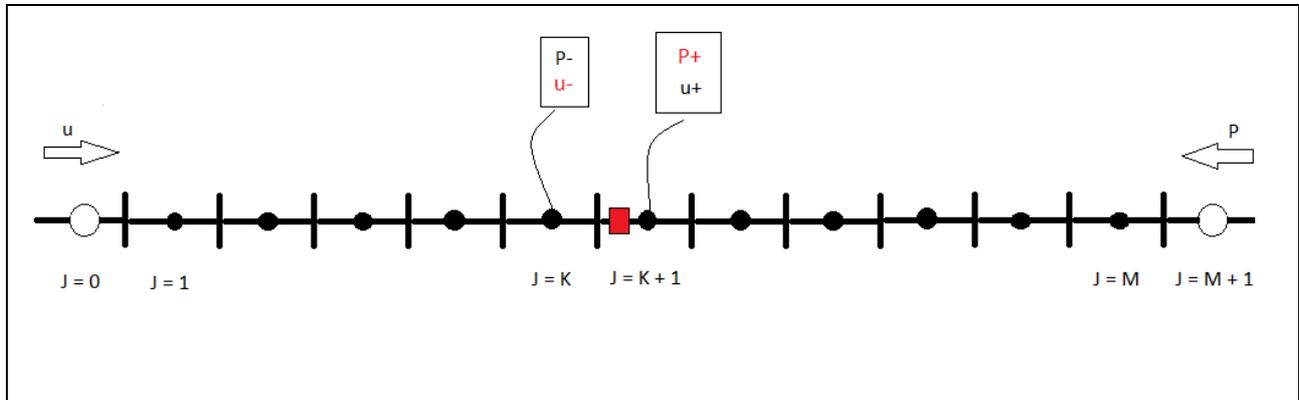

Figure 3. Pipeline discretization with a generic PIG location

Figure 3 shows a discretized pipeline with M cells and two ghost cells ($J = 0$ and $J = M + 1$). The pig was positioned between cells $J = K$ and $J = K + 1$. The velocity, u, and pressure, p, are the boundary conditions of the system imposed at the ghost cells.

The solution algorithm of the coupled problem is shown bellow.

1. Downstream gas flow ($J=1$, $K - 1$) is calculated (FCT). The boundary conditions are imposed on $J = K + 1^*$;
2. Upstream gas flow ($J=K + 2$, M) is calculated (FCT). The boundary conditions are imposed on $J = K^*$;
   Both cell was considered as ghost cells. After FCT solution, the previous values were restored.
3. With the values of u^- and p^+ from the previous time step, the algebraic differential system that describe the pig motion is solved and new values of u^+, p^- and v_p are calculated;
4. Using cells $J=K - 2$ and $J=K + 1$ as boundary conditions, only the value of u^- is updated;
5. The last step is to use cells $J=K$ and $J=K+3$ as boundary conditions to update the pressure value, p^+, from cell $J=K+1$.

6. RESULTS AND DISCUSSION

In Sondermann *et al.* (2015), the basic gas flow solved with the FCT method was validated against the analytical solutions to the Fanno line and the shock tube flows. This work brings confidence to the results that we obtain for the gas flow without the pig in the pipeline. Based on this model, it was possible to include the pig model and proceed with the pig motion analysis in gas-flow pipelines. In addition, we simulate a similar case using the software OLGA® to compare our results.

Table 1 shows the values of flow properties, boundary conditions and pipeline data used in our FORTRAN® program.

Table 1. Pipeline Data.

Dynamic Viscosity (Pa.s)	1.9×10^{-5}
Gas Constant (J/kg k)	287
Inlet Gas Velocity (m/s)	5
Outlet Pressure (Pa)	10×10^{5}
Diameter (m)	0.3032
Length (km)	20
Surface Roughness (m)	4.572×10^{-5}
Temperature (K)	273
Discretization Cells (-)	1000

In OLGA®, fluid's properties were inserted through a PVT file. This file must be generated with an additional package. For this simulation, we used the same pipeline data for both programs.

A typical operation procedure is to launch the PIG while the gas is pumped throughout the pipeline. As an approximation of this scenario, the initial condition of the simulations was the steady-state regime previously calculated through both softwares. Figures **Erro! Fonte de referência não encontrada.** and **Erro! Fonte de referência não encontrada.** show the distribution of pressure and gas velocity along the pipeline, respectively, with the pig in two different positions, compared to the results of OLGA's. It is possible to observe a discontinuity in both quantities caused by the presence of the PIG and the excellent agreement between the two models. **Erro! Fonte de referência não encontrada.** depicts the PIG velocity history in comparison with OLGA's, showing, again, an excellent agreement between both simulations.
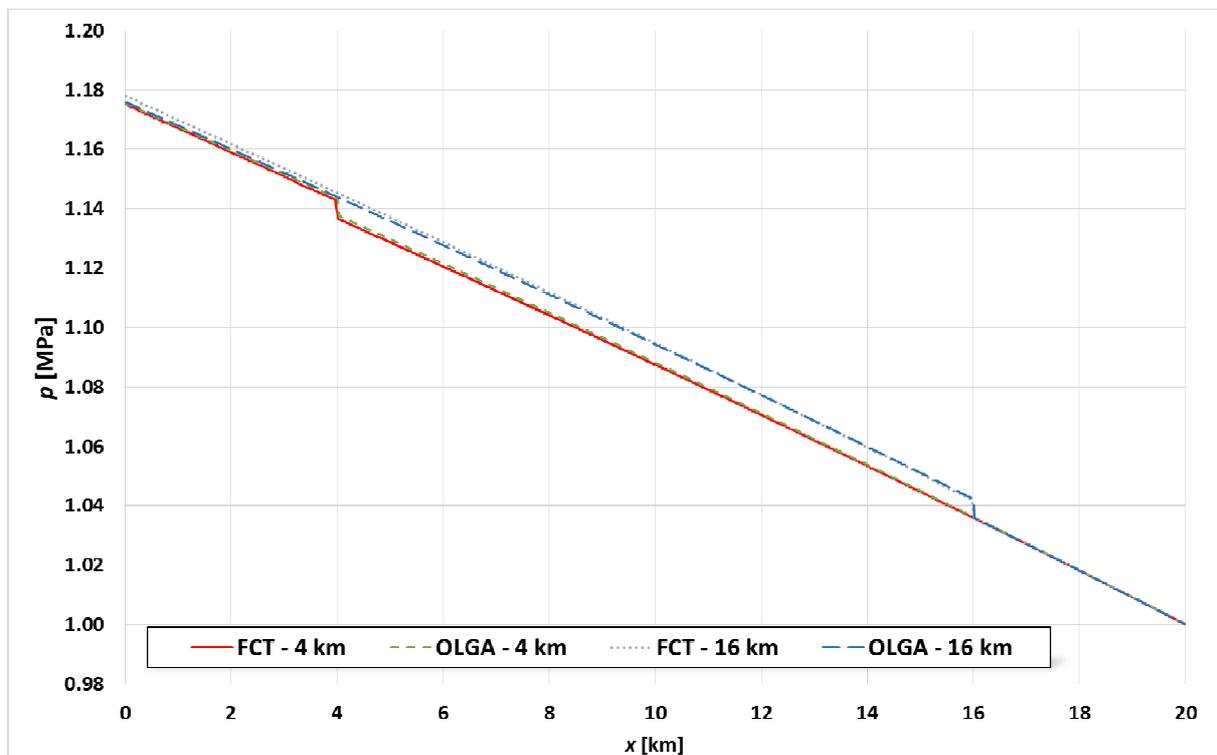
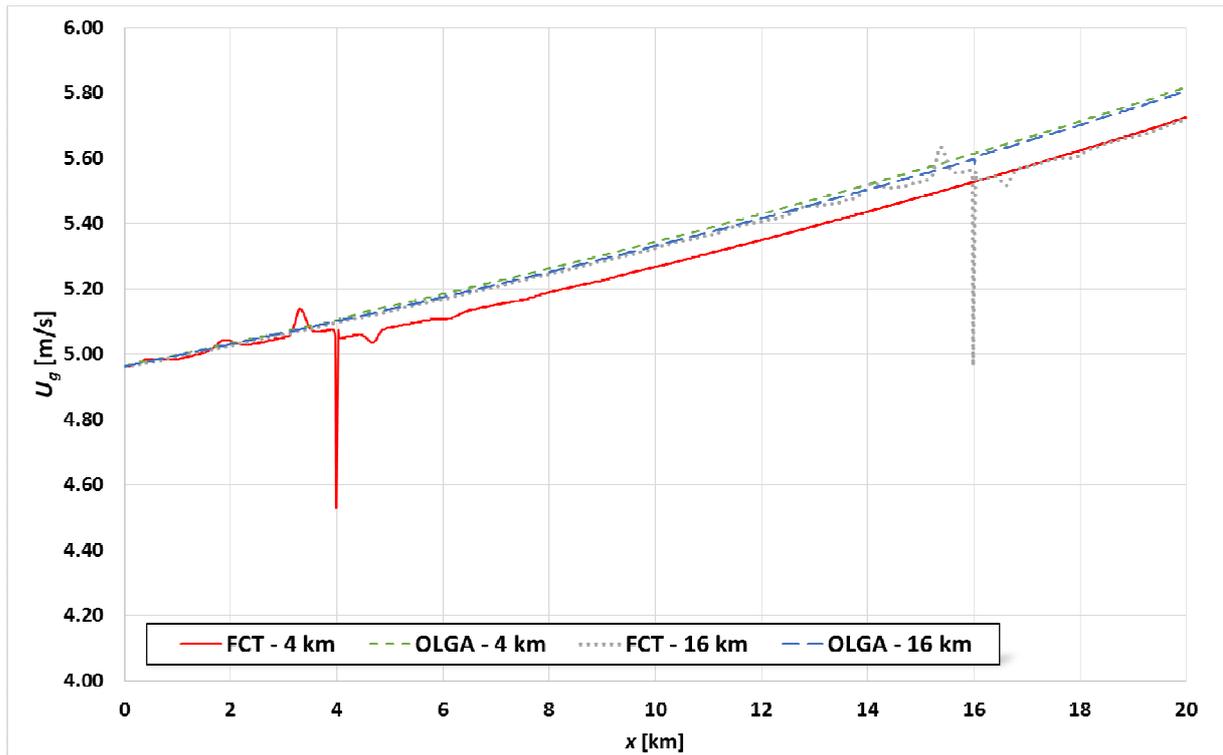Figure 4. Gas pressure distribution along the pipeline.

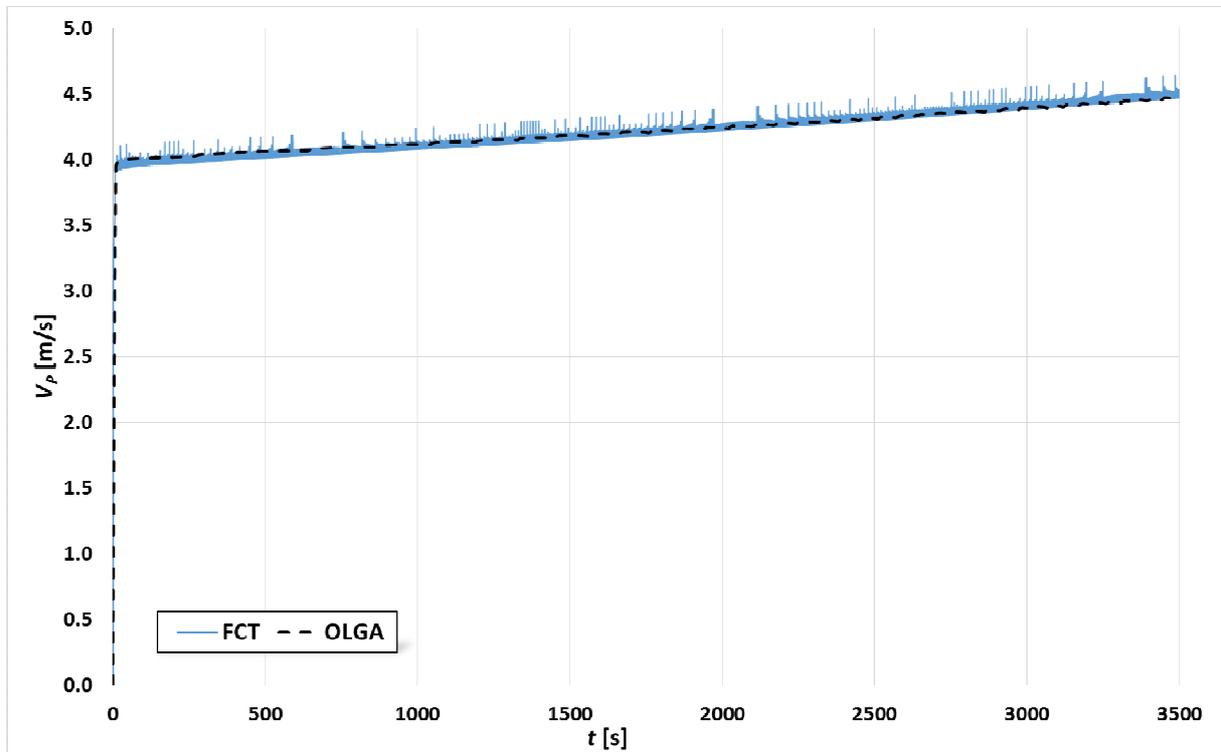Figure 5. Gas velocity distribution along pipeline.

Figure 6. Pig velocity.

7. CONCLUSIONS

The gas flow solution combined with the PIG's motion model were calculated using the Flux-Corrected Transport method and showed a very good agreement with the results obtained from OLGA®, despite the differences between the model proposed here and OLGA's, for both the gas flow and the pig model, in addition to differences in the numerical discretization. Overall, we may conclude that the agreement is quite impressive.

The results of this work gave us confidence to incorporate this pig model and numerical strategy into a more complex model to simulate the pig motion inside a pipeline with a two-phase flow. This is our main target.

8. ACKNOWLEDGEMENTS

8. REFERENCES

Azevedo, L. F. A., Braga, A.M.B, Nieckele, A.O., Naccache, M, F, 1996, "*Simple Hydrodinamic Models for the Prediction of Pig Motions in Pipelines*", Offshore Technology Conference, OTC 8232;

Book, D.L., Boris, J.P., Hain, K., 1975, "*Flux corrected transport II. -Generalizations of the method*", Journal of Computational Physics, Vol. 18, pp. 248-283;

Boris, J. P., Book, D. L., 1973, "*Flux correct transport I - SHASTA - A fluid transport algorithm that works*", Journal of Computational Physics, Vol. 11, pp. 38-69;

Boris, J.P., Book, D. L., 1976, "*Solution of continuity equation by the method of flux-corrected transport*", Journal of Computational Physics, Vol. 129, pp. 85-129;

Botros, K. K., Golshan, H., 2009, "*Dynamics of PIG Motion in Gas Pipeline*", AGA – Operations Conference & Biennial Exhibition;

Bueno, D. E. G., Figueiredo, A. B., Baptista, R. M, Rachid, F. B. F., Bodstein, G. C. R., 2012, "*Featuring PIG Movement in Two-Phase Gas Pipelines*", 9th International Pipeline Conference, IPC2012-90187;

Campo, E. V., 1998, *Modelagem do Movimento de PIGS em Regime Transiente*, Dissertação de M.Sc. PGMEC/UFF, Niterói, RJ, Brasil;

Esmaeilzadeh, F., Mowla, D., Asemani, M., 2009, *Mathematical Modeling and Simulation of PIGging Operation in Gas and Liquid Pipelines*, Journal od Petroleum Science and Engineering 69 (2009) 100-106;

Esmaeilzadeh, F., Mowla, D., Asemani, M., Shiraz U., 2006, *Modeling of PIG Operations in Natural Gas and Liquid Pipeline*, SPE INTERNATIONAL, SPE 102049;

Lezeau, P., Thompson, C.P., 1998. *Numerical Simulation of Multi-Phase Flow: Speed, Error Control & Robustness*. Cranfield University Technical Report;

Moody, L.F., 1947, "An approximate Formula for Pipe friction factors", Transactions of ASME, v. 69, pp.1005;

Tolmasquin, S. T., Nieckele, A. O., 2008, *Design and control of PIG operations through pipelines*, Journal of Petroleum Science and Engineering 62 (2008) 102-110;

Nieckele, A.O., Braga, A.M.B, Azevedo, L. F. A., 2001, *Transient PIG Motion Through Gas and Liquid Pipelines*, *Journal Of Energy Resources Technology*, DOI: 10.1115/1.1413466;

Petzold, L.R., 1982, *A Description of DASSL: A Differential-Algebraic System Solver,* In Procceding of the IMACS World Congress, Montreal, Canada;

Sondermann, C. N., Patricio, R. A. C., Figueiredo, A. B., Baptista, R., Rachid, F. B. F., Bodstein, G. C. R., "*Numerical Simulation of The Gas Flow in Pipelines Using The Flux-Corrected Transport Method*", 23rd ABCM International Congress of Mechanical Engineering, 2015;

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