DYNAMIC MODAL ANALYSIS BY CRAIG-BAMPTON METHOD

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Abstract. The Component-mode synthesis methods are applied to the finite element method in order to reduce the computational cost of dynamic analysis. This work aims at coupling reticulated structures by the Craig-Bampton method using an algorithm implemented in the matlab software for free vibration analysis. The internal degrees of freedom of each substructure were reduced and the substructures were coupled by degrees of freedom of contour. It was possible to observe that the method presents good precision in relation to the full Finite Element Method model and that a great reduction in the number of equations of the models is possible, which depends on the frequency range of interest.

1. INTRODUCTION

Equations that describe civil, mechanical, and aerospace engineering problems generally have no analytical solutions. It is necessary to use numerical methods to solve these problems. The Finite Element Method allows solving these equations. For large and complex problems in dynamic analysis, this method can present high computational cost. (Hollkamp and Gordon, 2008).

Reducing computational costs is always one of the goals of the analyzes, since it reduces the model execution time and reduces the space of use of the memories. Although nowadays presenting large resources, more is required each time. Rixen (2004) reports that it is possible to solve linear systems with millions of degrees of freedom, however, the complexities of systems are increasing along with the capabilities of computing.

The substructuring methods are coupling techniques applied to the Finite Element Method that allow to reduce computational costs. In the modal domain they are called component-mode synthesis. Each of the methods differ by using a set of truncated modes to obtain the approximate response of the real structure solution, which may be constraint modes, free-interface normal modes, fixed-interface normal modes, attachment modes, etc. In several applications each substructure is designed by different groups, being analyzed by its frequencies and modes for the subsequent coupling and obtaining the complete structure response (Bathe and Dong, 2014).

In the Craig Bampton method (Craig e Bampton, 1968) a truncated set of fixed-interface normal modes and constraint modes are used to obtain the reduced-order matrices of the system. This article aims evaluate accuracy of Craig-Bampton method in relation to the full Finite Element Method model in reticulated structures. In the next section, the component mode synthesis formulations specific to the Craig-Bampton method is presented.

2. THE CRAIG–BAMPTON METHOD

Considering a substructure (s) with $N_u^{(s)} = N_i^{(s)} + N_b^{(s)}$ degrees of freedom, the equilibrium equation must be partitioned according to equation (1) (Kuether and Allen, 2017):

$$\begin{pmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ib} \\ \mathbf{M}_{bi} & \mathbf{M}_{bb} \end{pmatrix}^{(s)} \begin{Bmatrix} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_b \end{Bmatrix}^{(s)} + \begin{pmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{pmatrix}^{(s)} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{Bmatrix}^{(s)} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_b \end{Bmatrix}^{(s)} \tag{1}$$

In that $\mathbf{u}_b$ refers $N_b^{(s)}$ to contour degrees of freedom, that is, they are shared with adjacent substructures, and $\mathbf{u}_i$ refers to $N_i^{(s)}$ internal degrees of freedom.

Klerk, Rixen and Voormeeren (2008) reports that a complete set of physical coordinates $\mathbf{u}$ is reduced to a smaller set of generalized coordinates $\mathbf{p}$. In the Craig-Bampton method the internal degrees of freedom of the substructures are reduced by the following transformation (Wenneker, 2013):

$$\mathbf{u}^{(s)} = \left\{ \begin{matrix} \mathbf{u}_i \\ \mathbf{u}_b \end{matrix} \right\}^{(s)} = \begin{pmatrix} \mathbf{\Phi}_{ik} & \mathbf{\Psi}_{ib} \\ \mathbf{0} & \mathbf{I}_{bb} \end{pmatrix}^{(s)} = \mathbf{T}_{CB}\mathbf{p} \tag{2}$$

In what $\mathbf{I}_{bb}$ is an identity matrix, $\mathbf{\Phi}_{ik}$ corresponds to the fixed-interface normal modes and $\mathbf{\Psi}_{ib}$ are the modes of restriction given by:

$$\mathbf{\Psi}_{ib} = \begin{pmatrix} -\left(\mathbf{K}_{ii}\mathbf{K}_{ib} \right) \\ \mathbf{I}_{bb} \end{pmatrix} \tag{3}$$

The fixed-interface normal modes are obtained by restricting the degrees of freedom of the boundary (Craig and Chang, 1976), achieving to the eigenvalue problem:

$$[\mathbf{K}_{ii} - w_j^2 \mathbf{M}_{ii}]\{\phi_i\}_j \qquad j = 1,2,...,N_i \tag{4}$$

Where N_i is the number of internal degrees of freedom

Considering the Rayleigh-Ritz solution (Bathe, 1996), one has the reduced-order matrices:

$$\hat{\mathbf{M}}_{CB}^{(s)} = \mathbf{T}_{CB}^T \mathbf{M}\mathbf{T}_{CB} = \begin{pmatrix} \mathbf{I}_{kk} & \hat{\mathbf{M}}_{kb} \\ \hat{\mathbf{M}}_{bk} & \hat{\mathbf{M}}_{bb} \end{pmatrix}^{(s)} \tag{5}$$

$$\hat{\mathbf{K}}_{CB}^{(s)} = \mathbf{T}_{CB}^T \mathbf{K}\mathbf{T}_{CB} = \begin{pmatrix} \mathbf{\Lambda}_{kk} & \mathbf{0}_{kb} \\ \mathbf{0}_{bk} & \hat{\mathbf{K}}_{bb} \end{pmatrix}^{(s)} \tag{6}$$

For the coupling of two substructures (1 and 2), the linear transformation necessary to solve the system of equations is written as (Craig and Kurdila, 2006):

$$\left\{ \begin{matrix} \mathbf{P}_{k_1}^{(1)} \\ \mathbf{P}_b^{(1)} \\ \mathbf{P}_{k_2}^{(2)} \\ \mathbf{P}_b^{(2)} \end{matrix} \right\} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \left\{ \begin{matrix} \mathbf{q}_{k_1}^{(1)} \\ \mathbf{q}_{k_2}^{(2)} \\ \mathbf{u}_b \end{matrix} \right\} \tag{7}$$

Where a transformation from dependent coordinates $\mathbf{p}$ to independent coordinates $\mathbf{q}$ is performed.

The matrices of reduced-order mass and stiffness coupled are given by:

$$\hat{\mathbf{M}}_{CB}^{(s)} = \begin{pmatrix} \mathbf{I}_{k_1 k_1} & \mathbf{0}_{k_1 k_2} & \hat{\mathbf{M}}_{k_1 b}^{(1)} \\ \mathbf{0}_{k_2 k_1} & \mathbf{0}_{k_2 k_2} & \hat{\mathbf{M}}_{k_2 b}^{(2)} \\ \hat{\mathbf{M}}_{b k_1}^{(1)} & \hat{\mathbf{M}}_{b k_2}^{(2)} & \hat{\mathbf{M}}_{bb}^{(1)} + \hat{\mathbf{M}}_{bb}^{(2)} \end{pmatrix} \tag{8}$$

$$
\hat{\mathbf{K}}_{CB}^{(s)} = \begin{pmatrix} \mathbf{\Lambda}_{k_1 k_1}^{(1)} & \mathbf{0}_{k_1 k_2} & \mathbf{0}_{k_2 b} \\ \mathbf{0}_{k_2 k_1} & \mathbf{\Lambda}_{k_2 k_2}^{(2)} & \mathbf{0}_{k_2 b} \\ \mathbf{0}_{b k_1} & \mathbf{0}_{b k_2} & \hat{\mathbf{K}}_{bb}^{(1)} + \hat{\mathbf{K}}_{bb}^{(2)} \end{pmatrix} \tag{9}
$$

3. NUMERICAL RESULTS

3.1 Coupling bi-supported beam

The CB method was applied by coupling two simply supported beam models using the formulation presented in the previous section. The schematic of this application is illustrated by Figure 1.
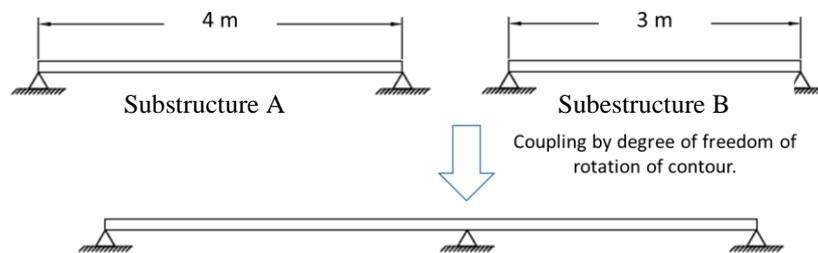

Figure 1: Representation of the coupling of simply supported beams by the Craig-Bampton method.

In this application, the coupling of the beam models occurred at the degree of freedom of rotation. The substructure A was modeled with the profile W 310x143 and the substructure B with the profile W 360x72, both were discretized into 30 finite elements of two nodes. The modulus of elasticity of 200 GPa was considered for the sections of the two substructures.

Table 1 presents the frequencies of the fixed-interface modes obtained for substructure A and B. For this application the frequency range of interest is from 0 to 300 Hz.

Table 1: Frequencies of fixed-interface normal modes relative to substructures A and B.

Mode Number	Simply Supported Beam A (4 m) Fixed-interface (Hz)	Simply Supported Beam B (3 m) Fixed-interface (Hz)
1	68.366	130.587
2	273.464	522.349
3	615.297	1175.291
4	1093.877	2089.437
5	1709.235	3264.844

Modes of 2 times the maximum frequency of interest for each substructure have been included in the transformation matrix, as Kuether and Allen (2014) recommend. Observing the frequency range of interest, the first two modes of each substructure were selected.

The full FEM model was also constructed in order to validate the results. From Table 2 it is possible to observe the convergence of the Craig-Bampton method in relation to the full Finite Element Method method considering the natural frequencies.

Table 2 shows that in the frequency range of interest the maximum difference between the frequencies was 0.139%.

Table 2: Percentage difference of frequencies of the C-B Method with respect to the Finite Element Method.

Mode Number	Craig-Bampton Frequency (Hz)	Full FEM model Frequency (Hz)	Difference %
1	77.397	77.394	0.003
2	177.334	177.297	0.021
3	303.114	302.692	0.139
4	596.565	573.942	3.942
5	1169.214	693.170	68.676

Figure 2 shows the first three modes of vibration from the Craig-Bampton method and from the full FEM model.
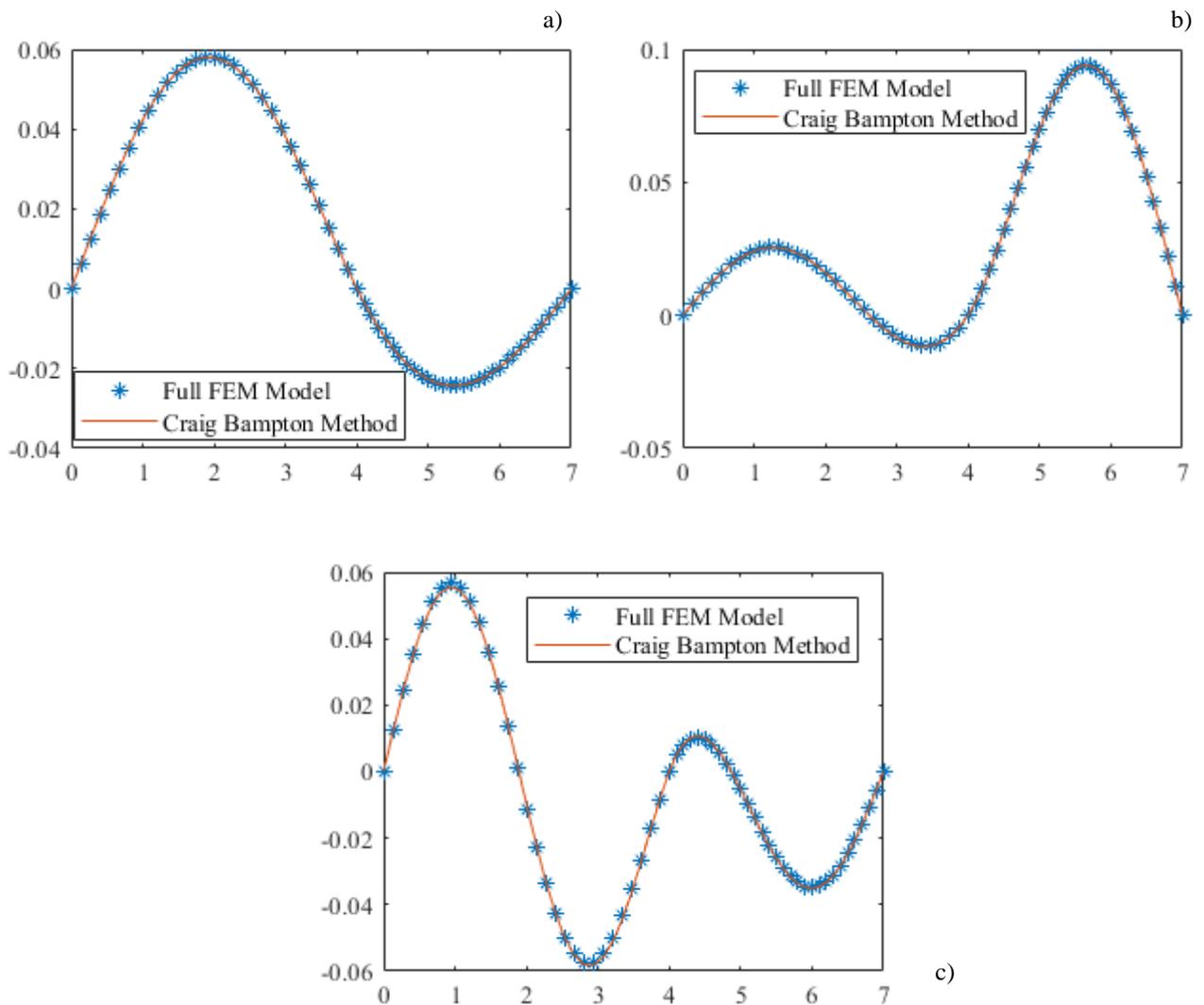

Figure 2: First – 77.394 Hz (a) second – 177.297 Hz (b) and third – 302.692Hz (c) modes of vibration for the coupled beams.

The first three modes obtained by the Craig-Bampton coupling were accurate compared to those obtained by the full Finite Element Method model. It is noted from Figure 3 that in the frequency range of interest the MAC values are close to 100% for these.
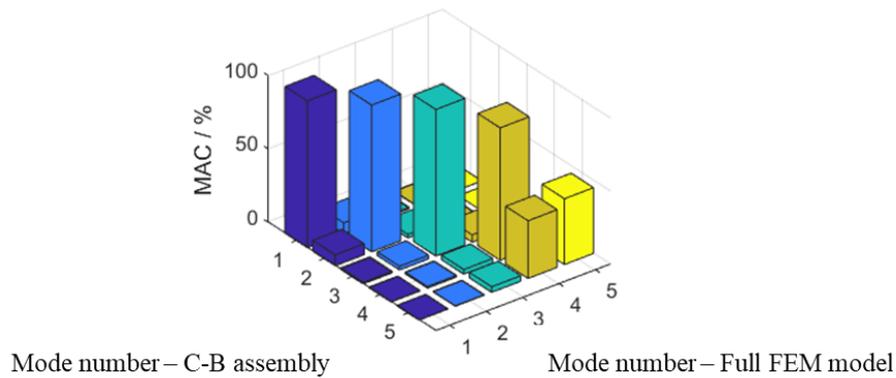
Figure 3: MAC values for the coupling of simply supported beams.

In this application the model obtained by the Craig-Bampton method was executed with 5 equations, while the complete finite element model was obtained with 119 degrees of freedom.

3.1 Coupling bi-supported beam

Next, the CB method was explored by coupling two bar models. The schematic of this application is illustrated by Figure 4.
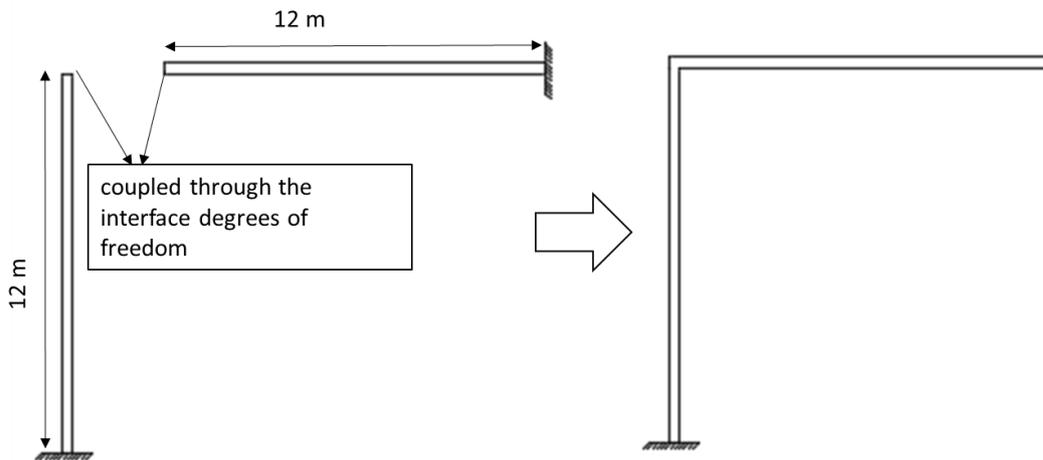

Figure 4: Representation of the coupling of planar bars.

In this example, the bars were discretized in 40 finite elements and modeled with the profile W 310x143. The substructures are coupled by degree of freedom of rotation and translate.

In Table 3, it is possible to observe the frequencies related to the first six fixed-interface modes obtained for the substructures.

Table 3: Frequencies of the fixed-interface modes for Substructures

Mode number	Substructures
	Fixed interface (Hz)
1	17.425
2	48.033
3	94.164
4	155.659
5	212.607
6	232.531

Considering the frequency range of interest from 0 to 110 Hz the first five mode are selected (Table 3).
The natural frequencies obtained through the full FEM model and via Craig-Bampton method are given in Table 4.

Table 4: Natural frequency percent difference obtained for coupled structure (0 to 110 Hz)

Mode number	C-B method assembly	Full FEM model	Difference (%)
	Frequency (Hz)	Frequency (Hz)	
1	11,984	11.984	0.000
2	17,322	17.322	0.000
3	38,624	38.620	0.011
4	47,052	47.051	0.002
5	78,888	78.761	0.161
6	85,348	85.182	0.194
7	103,575	103.077	0.481
8	107,749	107.211	0.499

Table 2 shows that within the frequency range of interest, the maximum difference between mode frequencies is 0.499%. In Figure 5, it can be seen that the MAC coefficient presented values close to 100% for the first eight modes
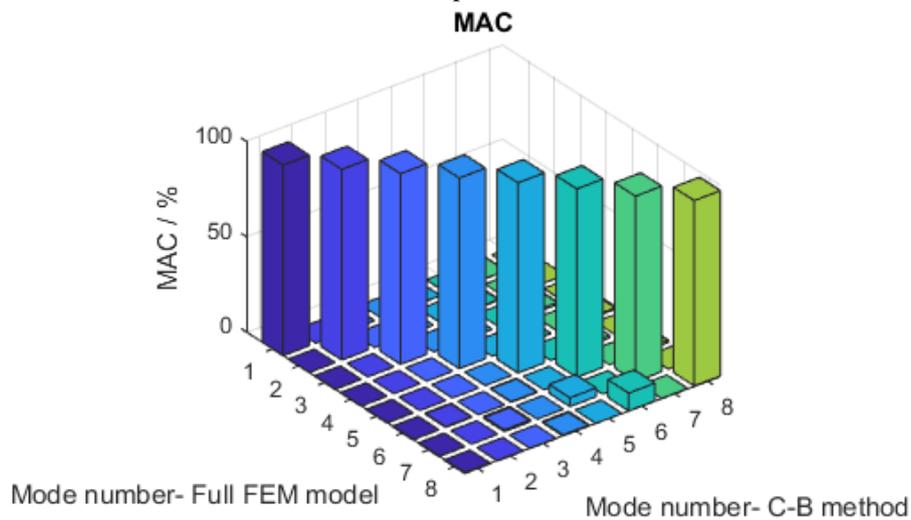

Figure 5: MAC values for the coupling of bars.

4. CONCLUSIONS

The component-mode synthesis was developed considering the formulations of the Craig Bampton method. The substructures were coupled by degrees of freedom of contour and the internal degrees of freedom were reduced, allowing to obtain complete response of the structures with lower computational cost.

In addition to allowing a large reduction of the number of equations of the models, for analysis from the method it is necessary to consider few modes in the transformation matrix, thus confirming the reduction of the computational cost.

It is observed that the method presents reliability in the frequency range of interest. The MAC values were close to 100 % showing that the modes correlate correctly. The maximum frequency difference in relation to the Finite Element method for complete structure did not exceed 1%.

Acknowledgements

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