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NONLINEAR DYNAMIC STRUCTURES BEHAVIOR WITH CRACK BREATHING USING THE HARMONIC BALANCING METHOD

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Abstract. *Fatigue cracks are one of the most important problems in mechanical components, since they can cause severe losses, both personal and economic. Therefore, it is essential to know the behavior element to have tools that allow predicting the damage level before it completely failure. The beams are elements affected by the described problem, as they are subject to alternative compression and tension stresses. In this article, the nonlinear dynamic behavior of a cantilever beam modelled as a mass-spring system with two and three degrees of freedom, respectively, are considered. The effects of the crack on the beam stiffness are evaluated by a bilinear function and a harmonic excitation force is considered. The Harmonic Balancing Method (HBM) was used to determine the Nonlinear Normal Modes (NNMs) in the domain of time, phase portrait, configuration space and in the three-dimensional projection of phase space. The perspective of this work is to access how the crack effects the nonlinear behavior of a beam, specially the Nonlinear Normal Modes.*

Keywords: *Nonlinear Dynamics, Crack Breathing, Harmonic Balancing Method, Nonlinear Normal Modes*

1. INTRODUCTION

The crack formation process is closely related to the fatigue failure phenomenon. In ductile materials as medium carbon steels, cracks start as microscopic discontinuities and grow rapidly when subjected to cyclic loading. Thus, a study of the types and causes of crack propagation is fundamental (Gasch, 1993). Damage to structures like breathing cracks influences their natural characteristics, such as frequency, damping and mode shapes. One of the earliest known theories for cracked beams is that of Christides and Barr (Dimarogonas, 1996). In many cases, a structure in its operating conditions, which initially behaves in a predominantly linear way, begin to present nonlinear characteristics with the appearance of crack. Due to the nonlinear dynamic behavior of the structure to crack breathing apply Harmonic Balancing Method (HBM) to detect, locate and estimate cracking depth and numerical-computational simulation we will use software: Matlab and Maple.

2. CRACK BREATHING PROBLEM

Nonlinear phenomena are widely found in practical engineering applications. A typical example is the fatigue crack which opens and closes under dynamic loads, exhibiting breathing characteristics (Pugno *et al.*, 2000). Breathing cracks are usually initiated due to persistent cyclic loading (fatigue) in the structures and these cracks increase over time and may lead to structure collapse. In recent years, there has been a significant interest in detecting these cracks in time for corrective actions to be taken, avoiding injury and risks to operational safety. The crack breathing creates a bilinear rigidity effect, Fig. 1. When the cracked beam is under compression with the crack closed, i.e., the contact of the two surfaces of the crack, the rigidity is assumed to be k; the stiffness is multiplied by the parameter α $(0 \leq \alpha \leq 1)$ when the cracked beam is under tension and with open crack. If $\alpha = 1$ the system is linear, when this linear system is excited by a harmonic force, the response will be a harmonic function of the same frequency. If $\alpha < 1$, then the response is expected

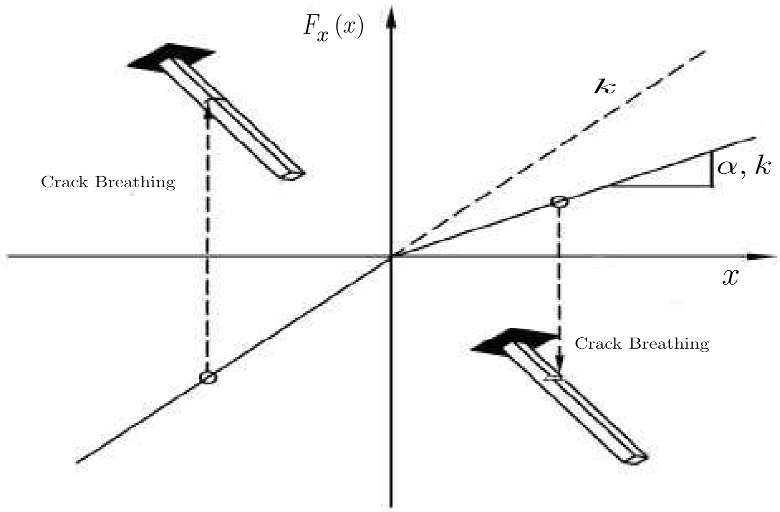
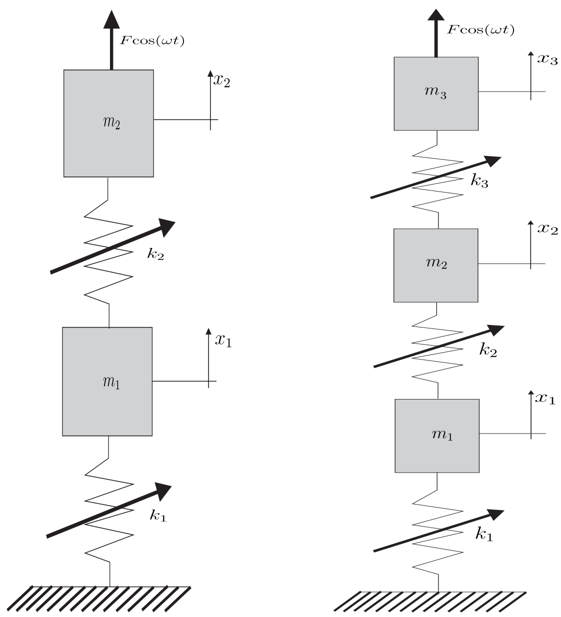


Figure 1. Cantilever beam with crack breathing (Prawin and Rama Mohan, 2015).

3. TWO AND THREE DEGREE OF FREEDOM MASS SPRING SYSTEMS

To better understand the nonlinear dynamic behavior of a cracked beam, we study the dynamic behavior of spring mass systems of two and three degrees of freedom with bilinear springs excited by a harmonic force $f(t) = F\cos(\omega t)$, Fig. 2 Although these are simple systems with few degrees of freedom (in relation to beam), these are useful for the understanding of nonlinear physical phenomena, as well as the evaluation of the location of cracks. Applying Newton's second law to the systems shown in, Fig. 2(a) and Fig. 2(b) the equations of motion are given by Eq. (1) and Eq. (2), respectively:



(a) (b)

Figure 2. Spring mass systems with two (a) and three (b) degrees of freedom with bilinear springs.

$$\begin{aligned} m_1\ddot{x}_1 + k_1x_1 - k_2(x_2 - x_1) &= 0 \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) &= F\cos(\omega t) \end{aligned} \tag{1}$$

$$\begin{aligned} m_1\ddot{x}_1 - k_2(x_2 - x_1) + k_1 x_1 &= 0 \\ m_2\ddot{x}_2 - k_3(x_3 - x_2) + k_2(x_2 - x_1) &= 0 \\ m_3\ddot{x}_3 + k_3(x_3 - x_2) &= F\cos(\omega t) \end{aligned} \tag{2}$$

where x_1, x_2 and x_3 are the mass displacements of m_1, m_2 and m_3, respectively. The dots indicate the derivative with respect to time, k_1, k_2 and k_3 are values the stiffness, $f(t) = F\cos(\omega t)$ is the harmonic external force that drives the system, F is the amplitude of the excitation force and ω is the excitation frequency. Accordingly to (Liu and Dowell, 2005), the treatment of these equations through HBM needs to be previously admentionalized, making $\tau = \omega t$ and assuming that $0 \leq t \leq T$, where $T = 2\pi/\omega$ is the period, then $0 \leq \tau \leq 2\pi$. The effect of bilinear rigidity as a function of displacement produced by the springs between the masses m_1, m_2 and m_3 Fig. 2 is modeled by:

$$F_{bil_1}(x_1) = \begin{cases} \alpha_1\, k_1\, x_1(\tau), & 0 \leq \tau \leq t_1, \\ k_1\, x_1(\tau), & t_1 \leq \tau \leq 2\pi - t_1, \\ \alpha_1\, k_1\, x_1(\tau), & 2\pi - t_1 \leq \tau \leq 2\pi. \end{cases} \tag{3}$$

$$F_{bil_2}(x_2 - x_1) = \begin{cases} \alpha_2\, k_3\, (x_2(\tau) - x_1(\tau)), & 0 \leq \tau \leq t_1, \\ k_2\, (x_2(\tau) - x_1(\tau)), & t_1 \leq \tau \leq 2\pi - t_1, \\ \alpha_2\, k_2\, (x_2(\tau) - x_1(\tau)), & 2\pi - t_1 \leq \tau \leq 2\pi. \end{cases} \tag{4}$$

$$F_{bil_3}(x_3 - x_2) = \begin{cases} \alpha_3\, k_3\, (x_3(\tau) - x_2(\tau)), & 0 \leq \tau \leq t_1, \\ k_3\, (x_3(\tau) - x_2(\tau)), & t_1 \leq \tau \leq 2\pi - t_1, \\ \alpha_3\, k_3\, (x_3(\tau) - x_2(\tau)), & 2\pi - t_1 \leq \tau \leq 2\pi. \end{cases} \tag{5}$$

Writing Eq. (1) and Eq. (2) in the bilinear stiffness model, respectively, where $''$ indicates the derivative with respect to the dimensionless time, we have:

$$\begin{aligned} m_1 x_1^{''} + F_{bil_1}(x_1) - F_{bil_2}(x_2 - x_1) &= 0 \\ m_2 x_2^{''} + F_{bil_2}(x_2 - x_1) &= F\cos(\tau) \end{aligned} \tag{6}$$

$$\begin{aligned} m_1 x_1^{''} - F_{bil_2}(x_2 - x_1) + F_{bil_1}(x_1) &= 0 \\ m_2 x_2^{''} - F_{bil_3}(x_3 - x_2) + F_{bil_2}(x_2 - x_1) &= 0 \\ m_3 x_3^{''} + F_{bil_3}(x_3 - x_2) &= F\cos(\tau) \end{aligned} \tag{7}$$

Assuming that the response of the nonlinear dynamic systems Eq. (6) and Eq. (7) is harmonic, the Harmonic Balancing Method (HBM) can be applied. This method expresses the periodic motion of a system by means of a finite Fourier series (Nayfeh and Mook, 1995). Thus, the response of the system Eq. (6) is given by Eq. (8) and Eq. (9) and the system response Eq. (7), is given by Eq. (8), Eq. (9) and Eq. (10), respectively:

$$x_1(\tau) = b_0 + \sum_{n=1}^{H} a_n \cos(n\,\tau) + \sum_{n=1}^{H} b_n \sin(n\,\tau) \tag{8}$$

$$x_2(\tau) = d_0 + \sum_{n=1}^{H} c_n \cos(n\,\tau) + \sum_{n=1}^{H} d_n \sin(n\,\tau) \tag{9}$$

$$x_3(\tau) = e_0 + \sum_{n=1}^{H} f_n \cos(n\,\tau) + \sum_{n=1}^{H} e_n \sin(n\,\tau) \tag{10}$$

where H is the number of harmonics, $a_i, b_i, c_i, d_i, e_i,\ f_i$, for $i = 1, \cdots, n$ and b_0, d_0, e_0, determined coefficients by solving a system of equations.

As the response dynamical systems Eq. (6) and Eq. (7) are periodic, then F_{bil_1}, F_{bil_2} and F_{bil_3} are also periodic and can be approximated by a Fourier series. Approaching, F_{bil_3} by a Fourier series Eq. (11) and form calculates analogous to Fourier series F_{bil_1} and F_{bil_2}.

$$F_{bil_3}(x_3 - x_2) = F_0 + \sum_{n=1}^{H} F_{cn} \cos(n\tau) + \sum_{n=1}^{H} F_{sn} \sin(n\tau) \tag{11}$$

the Fourier coefficients F_0, F_{cn} and F_{sn} are obtained as follows (Liu and Dowell, 2005):

$$F_0 = \frac{1}{2\pi}\left\{\int_0^{t_1} \alpha_3 k_3 x(\tau)d\tau + \int_{t_1}^{2\pi-t_1} k_3 x(\tau)d\tau + \int_{2\pi-t_1}^{2\pi} \alpha_3 k_3 x(\tau)d\tau\right\} \tag{12}$$

$$F_{sn} = \frac{1}{\pi}\left\{\int_0^{t_1} \alpha_3 k_3 x(\tau)\sin(n\tau)d\tau + \int_{t_1}^{2\pi-t_1} k_3 x(\tau)\sin(n\tau)d\tau + \int_{2\pi-t_1}^{2\pi} \alpha_3 k_3 x(\tau)\sin(n\tau)d\tau\right\} \tag{13}$$

$$F_{cn} = \frac{1}{\pi}\left\{\int_0^{t_1} \alpha_3 k_3 x(\tau)\cos(n\tau)d\tau + \int_{t_1}^{2\pi-t_1} k_3 x(\tau)\cos(n\tau)d\tau + \int_{2\pi-t_1}^{2\pi} \alpha_3 k_3 x(\tau)\cos(n\tau)d\tau\right\} \tag{14}$$

where $x(\tau) = x_3(\tau) - x_2(\tau)$. Considering that, $k_3 = 0.5, \alpha_3 = 0.3$ e $t_1 = \pi/2$, we obtain F_0, F_{cn} and F_{sn}.

Substituting, F_{bil_3}, F_{bil_2}, F_{bil_1}, Eq. (8), Eq. (9) and Eq. (10) in nonlinear dynamical systems Eq. (6) and Eq. (7), respectively and balancing the harmonics, the temporal response of the nonlinear dynamical systems shown in Fig. 2.

4. NONLINEAR NORMAL MODES

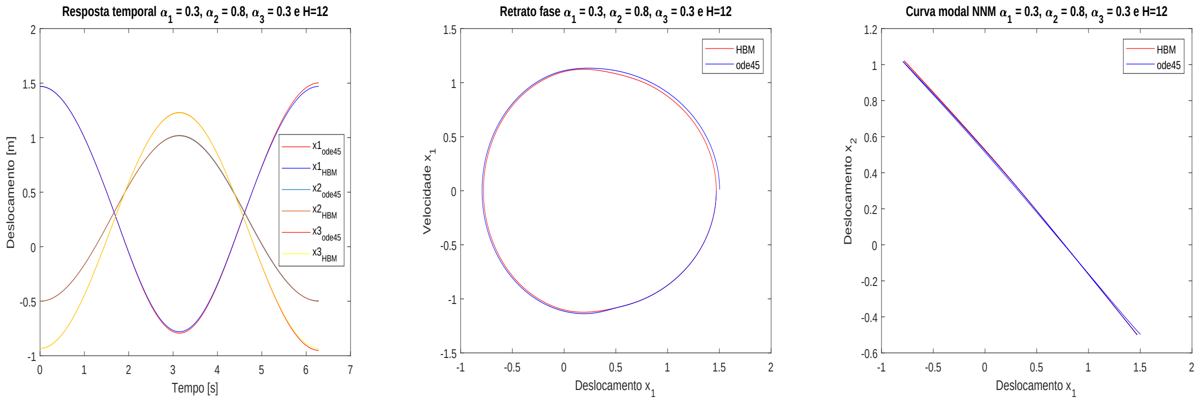
Normal nonlinear modes (NNMs) provide a solid theoretical and mathematical tool for interpreting a of nonlinear dynamic phenomena, but they have a clear and simple conceptual relationship with the linear normal modes (LNMs). There are two main definitions of NNMs, due to (Rosenberg, 1996), (Shaw and Pierre, 1994). Rosenberg defined NNMs as vibrations in unison of the system; i.e., synchronous periodic motions in which all generalized coordinates vibrate equiperiodically, achieving instantaneous zero velocity at the same instant of time. The modeling and understanding of the effects of nonlinearity on real vibratory structures is one of the challenges in structural engineering. In this context, normal nonlinear modes (NNMs) are widely used as a tool for understanding the forced responses of nonlinear systems, damage detection in structural engineering.

5. NUMERICAL SIMULATIONS

To evaluate the NNMs, it is necessary to excite the structure close to one of its natural frequencies. Considering the following parameters: $k_1 = k_2 = k_3 = 0.5\,N/m$, $\alpha_1 = 0.3$, $\alpha_2 = 0.8$, $\alpha_3 = 0.3$, $F = 1\,N, m_1 = m_2 = m_3 = 1\,kg$, $\omega = 1\,Hz$ e $H = 12$, the temporal response of the nonlinear dynamic system is obtained Eq. (7). The temporal response $x_1(\tau), x_2(\tau)$ and $x_3(\tau)$ is a combination of cosine and deriving one obtains $\dot{x}_1(\tau)$, $\dot{x}_2(\tau)$ and $\dot{x}_3(\tau)$ and a combination of sines, then $\dot{x}_1(0) = \dot{x}_2(0) = \dot{x}_3(0) = 0$. Substituting, $\tau = 0$, we obtain $x_1(0) = 1.470240424, x_2(0) = -0.4996747456$ e $x_3(0) = -0.9326683506$. Thus, the initial condition of the dynamic spring mass system of three degrees of freedom with bilinear springs is given by $(x_1(0), \dot{x}_1(0), x_2(0), \dot{x}_2(0), x_3(0), \dot{x}_3(0))$. The Fig. 3 shows the temporal response, phase portrait and modal curve of NNMs com with the above mentioned parameters obtained using HBM and compared with ode45 which is a function based on the Runge-Kutta method for the solution of ordinary differential equations. It can be seen from Fig. 3(a) that the amplitude is smaller compared to the temporal response, Fig. 4(a). For the case where $\alpha_1 = 0.3, \alpha_2 = 0.8$ and $\alpha_3 = 0.3$, the orbit in the phase portrait is a slightly deformed ellipse, Fig. 3(b), NMNs tend to approach straight lines, Fig. 3(c). For the case where $\alpha_1 = \alpha_2 = \alpha_3 = 0.8$, in the phase portrait the orbit tends to approach a circumference, Fig. 4(b), and the NMNs are straight, Fig. 4(c).

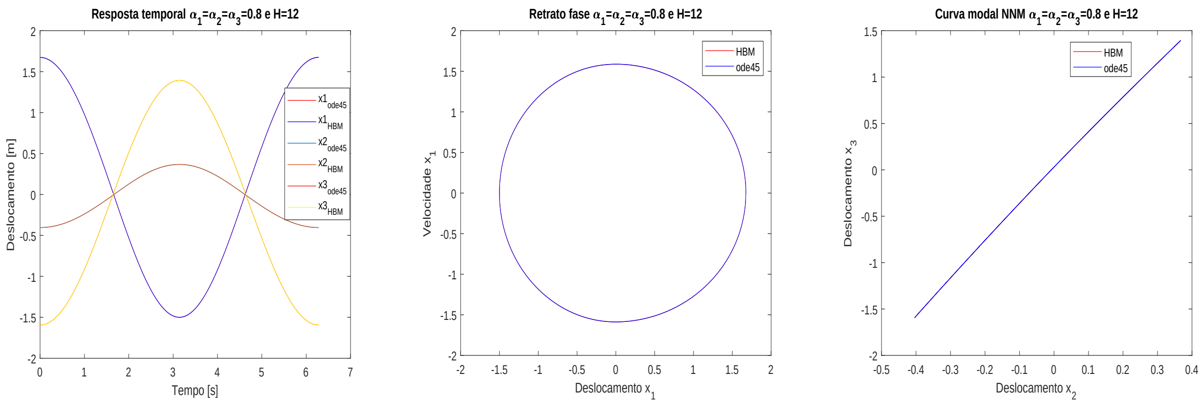
Analyzing the two-degree spring mass system with bilinear springs, it is observed that for the case where $\alpha_1 = 0.6, \alpha_2 = 0.8$, the amplitude is larger Fig. 5(a) compared to the case $\alpha_1 = \alpha_2 = 0.3$ as shown in Fig. 6(a), the phase portrait is a deformed ellipse, Fig. 5(b), the NMNs are straight lines, Fig. 5(c), and considering $\alpha_1 = \alpha_2 = 0.3$, the phase portrait shows a strong deformation of the ellipse, Fig. 6(b) and NNMs are curves, Fig. 6(c).

In Fig. 7(a) and Fig. 7(b), Fig. 7(c) show the periodic motions of NNMs represented in a three-dimensional projection of the phase space of nonlinear dynamic systems Eq. (6) and Eq. (7), respectively.



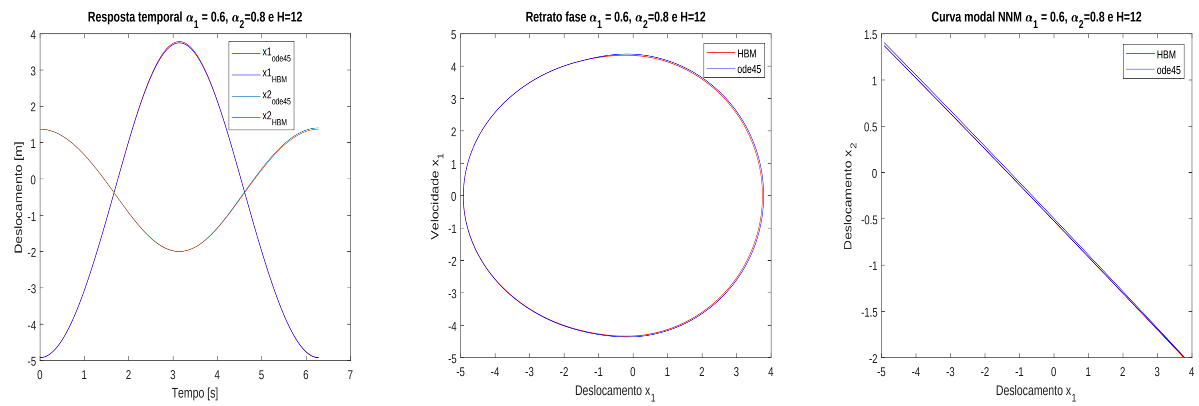
(a) (b) (c)

Figure 3. Temporal response (a), phase portrait (b) and modal curve (c) of NNMs: $\alpha_1 = 0.3, \alpha_2 = 0.8, \alpha_3 = 0.3$ and H=12.



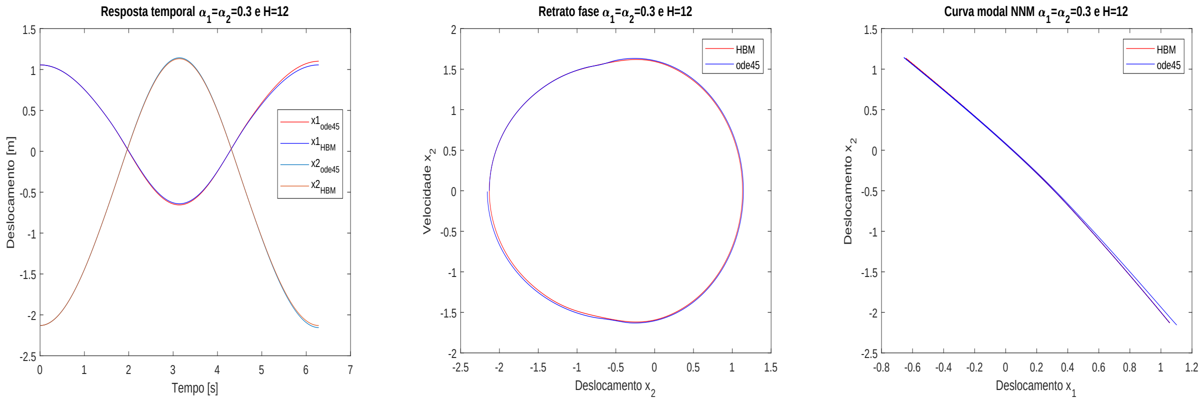
(a) (b) (c)

Figure 4. Temporal response (a), phase portrait (b) and modal curve (c) of NNMs: $\alpha_1 = 0.8, \alpha_2 = 0.8, \alpha_3 = 0.8$ and H=12.



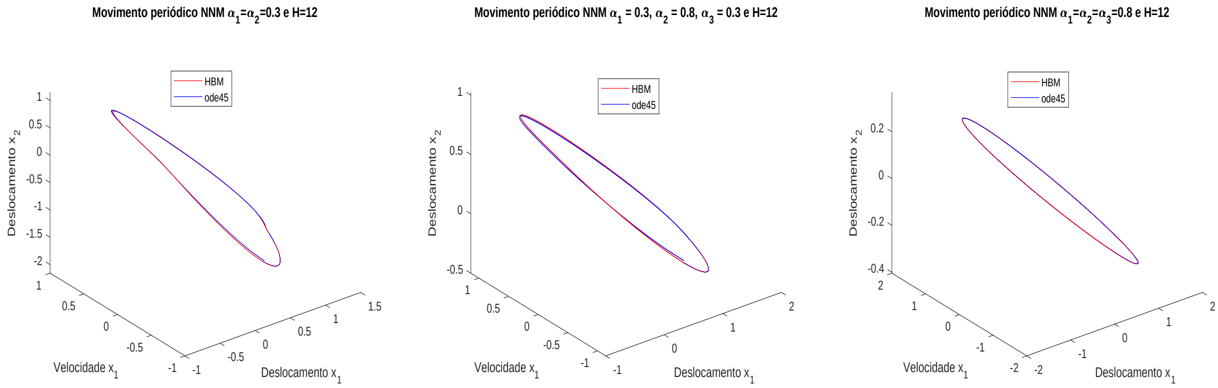
(a) (b) (c)

Figure 5. Temporal response (a), phase portrait (b) and modal curve (c) of NNMs: $\alpha_1 = 0.6, \alpha_2 = 0.8$ and H=12.



(a) (b) (c)

Figure 6. Temporal response (a), phase portrait (b) and modal curve (c) of NNMs: $\alpha_1 = 0.3, \alpha_2 = 0.3$ and H=12.



(a) (b) (c)

Figure 7. Periodic motion of NNMs spring mass systems with two (a) and three (b), (c) degrees of freedom with bilinear springs.

6. CONCLUSIONS

This work presented a methodology for the study of nonlinear dynamic behavior of structures with crack breathing considering mass-spring systems of two and three degrees of freedom with bilinear rigidity springs excited by a harmonic force. The nonlinear dynamic behavior of the breathing crack in the structure is studied analyzing the temporal response, phase portrait, modal curve in the configuration space and periodic movements of NNMs. The motions of NNMs can be open or closed curves for values of α_1, α_2 e α_3 close to zero and straight to values close to one, showing a relation with the Linear Normal Modes.

7. ACKNOWLEDGEMENTS

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