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# EXPERIMENTAL STUDY OF EQUIVALENT 1DoF MAIN STRUCTURE COUPLED TO TLCD: VALIDATION OF OPTIMUM PARAMETER OBTAINED BY PARAMETRIC OPTIMIZATION

**Juliano Ferreira Martins**  
**Marcus V. G. de Moraes**  
**Suzana Moreira Avila**

Graduate Program in Engineering Materials Integrity, University of Brasília, Brasília, Distrito Federal, Brazil  
juliano.martins@hotmail.com, mvmoraes@unb.br, avilas@unb.br

**Abstract.** *Tuned Liquid Column Damper (TLCD) is a kind of passive absorber composed by a U-shaped tube filled with liquid, commonly, water. The device has been receiving attention from several researchers as an alternative to vibration reduction. TLCD is a non-linear mechanical system due to turbulent head-loss in oscillatory conditions. To avoid solving nonlinear simultaneous equations, solutions such as statistical linearization and parameter optimization have been proposed in previous works. Besides, on its investigations, it is derived a closed-form solution for optimized TLCD damping ratio and head loss coefficient for white noise. Optimal parameters of mass and aspect ratio to control the main structure subjected to several kinds of wind random excitation are presented in literature. This study performs an experimental investigation of equivalent 1 DoF mass-spring system (main structural subsystem) coupled to a TLCD performed on the Laboratory of Dynamics of the University of Brasilia. Each subsystem is experimentally identified for determining equivalent parameters to feed reduced 2DoF coupled system (main structure + TLCD). Analytical validation of TLCD dynamic behavior is compared to experimental results. Finally, with help of response map (kind of parametric optimization), optimized parameters of TLCD are determined to obtain an expressive vibration reduction of 2DoF coupled system compared to the uncontrolled system (main structure). The data from analytical and experimental analysis of a coupled structure + TLCD present great coherence.*

**Keywords:** *TLCD, Vibration Absorber, Passive Control, Structural Vibration*

## 1. INTRODUCTION

The vibration levels in slender structures, such as walkways, bridges, high towers, and wind turbines are getting more attention with span increase. Slender structures are characterized by having low frequencies and damping, not avoiding unexpected responses. To preserve the structure lifespan, it is necessary to study additional mechanical devices capable to reduce the vibration level of the main structure. These equipment, known as vibration absorbers, are important for the health of the structure. They help to prevent disasters on structures under natural hazards as wind and earthquake.

The devices can be classified as active, passive, hybrid or semi-active. Tuned Liquid Column Damper (TLCD) is a kind of passive absorber composed by a U-shaped tube filled with liquid, commonly, water. TLCD is a non-linear mechanical system due to the turbulent head-loss in oscillatory conditions (Sakai *et al.* (1989)). Restoring and damping forces occurs due to the liquid mass when it oscillates with opposite phase to the structure, settled by tuned parameters (Okay *et al.* (2017)). The TLCD holds researchers attention since it presents a low cost, easy handling, and lightweight. The headloss depends on the opening of the orifice in its center (Yalla and Kareem (2000); Mensah and Dueñas-Osorio (2014)).

TLCD equipment has been receiving attention among researchers. Balendra *et al.* (1995) present an application of TLCD to buildings, which a comparison between TLCD and TMD showed that the reduction of response of the TLCD could behave in a similar mode compared to the TMD. Gao *et al.* (1997) present an analytical solution to harmonic excitation using an equivalent damping approach. Balendra *et al.* (1995) show that for a better accomplishment the TLCD frequency has to be tuned as well as the tower frequency. However, although the TLCD system is not tuned, it obtains good performance. The authors also expose this lead to advantage since the structural frequency keeps changing throughout its lifespan.

Yalla and Kareem (2000) investigate a set of optimum parameters. Mensah and Dueñas-Osorio (2014) examine a wind turbine structural response coupled to TLCD, and they observe reliability enhances. Shum (2009) presents the optimal parameters of a TLCD-structure system using a non-linear closed form solution for white noise excitation. Alkmim (2017) extend optimal parameters of TLCD-structure for other kinds of random excitation. Da Silva and De Moraes (2018) compare numerical FE fluid-structure coupled solution by reporting to experimental results with a good agreement. There are several application of TLCD in seismic behaviours (Adam *et al.* (2017); Mendes (2018); Altunişik *et al.* (2018);

Espinoza *et al.* (2018)), wind excitation (Samali *et al.* (1992); Di Matteo *et al.* (2014); Park *et al.* (2018); Di Matteo *et al.* (2017)), and base interactions (Coudurier *et al.* (2018); Furtmüller *et al.* (2019)).

This study performs an experimental investigation of equivalent 1 DoF mass-spring system (main structural subsystem) coupled to a TLCD. Each subsystem is experimentally identified for determining equivalent parameters to feed reduced 2DoF coupled system (main structure + TLCD). Analytical validation of TLCD dynamic behavior is compared to experimental results. Finally, with the help of response map, optimized parameters of TLCD are determined to obtain an expressive vibration reduction of 2DoF coupled system compared to the uncontrolled system (main structure).

## 2. Mathematical Formulations

### 2.1 Tuned Liquid Column Damper Coupled to Main Structure

Figure 1 presents the schematically representation of the coupled system (main structure + TLCD) defining with dimensional and dynamic parameters. Deduced from the Bernoulli equation or the energy principles (Di Matteo *et al.* (2016)), governing equation of liquid column relative motion is done below:

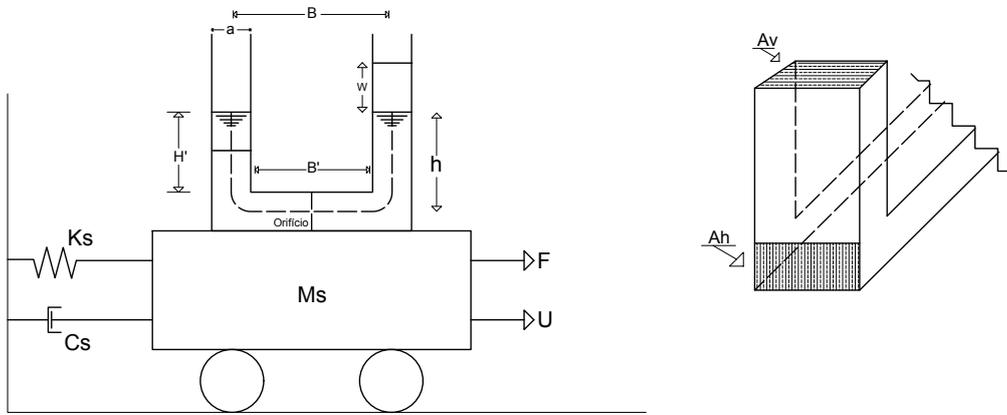


Figure 1: System Configuration

$$\rho AL\ddot{w} + \frac{1}{2}\rho A\epsilon |\dot{w}| \dot{w} + 2\rho Agw(t) = -\rho Ab\ddot{u}, \quad (1)$$

where  $w$  is denoted to liquid relative displacement,  $u$  is structure horizontal displacement of main structure,  $\epsilon$  is head loss coefficient,  $\rho$  is fluid density,  $A$  is cross sectional area, and  $g$  local gravity constant. The geometric parameters of TLCD reservoir are defined by the variables fluid column height  $h$ , the horizontal length  $B$ , and the total length  $L = B + 2h$ . The expression  $-\rho AB\ddot{u}(t)$  is the fluid coupling term. The fluid parameters are:  $m_w = \rho AL$  fluid mass,  $0.5\rho A\epsilon |\dot{w}(t)| = c_w$  fluid damping coefficient,  $2\rho Ag = k_w$  fluid stiffness.

To perform the structural vibration control, the main system is reduced to a SDOF model (single degree of freedom). The linearized equation of motion of the reduced main structure equipped with TLCD for lateral vibration is done as:

$$(m_s + m_w)\ddot{u} + \rho Ab\ddot{w} + c_s\dot{u} + k_s u(t) = F(t), \quad (2)$$

where  $m_s$  is structure mass,  $k_s$  structural stiffness,  $c_s$  structural damping coefficient, and  $F(t)$  the excitation load applied on main structure. In literature, the principal dimensionless parameters is defined as mass ratio  $\mu = m_w/m_s$  and aspect ratio  $\alpha = B/L$

The fluid damping is non-linear, but it can be replaced by an linear equivalent damping ratio (Gao *et al.* (1997); Shum (2009); Di Matteo *et al.* (2016)). The fluid damping coefficient is defined as  $c_w = 0.5\rho A\epsilon |\dot{w}(t)| = 2m_w\xi_w\omega_w$  where fluid damping coefficient  $\xi_w = 2/(3\pi)\epsilon w_0 L$  may be similar to  $2m_w\xi_w\omega_w$ , where  $w_0$  is the response amplitude of the liquid and  $\epsilon$  the headloss coefficient. Besides,  $F$  is the force acting on the primary system where the coefficient is  $\Delta = 1$  for base excitation and  $\Delta = 0$  for primary system excitation Yalla and Kareem (2000). Grouping Eq.(1) and (2), the dimensionless motion equation is obtained by:

$$\begin{bmatrix} 1 + \mu & \alpha\mu \\ \alpha\mu & \mu \end{bmatrix} \begin{pmatrix} \ddot{u} \\ \ddot{w} \end{pmatrix} + \begin{bmatrix} 2\omega_s\xi_s & 0 \\ 0 & 2\omega_w\xi_w \end{bmatrix} \begin{pmatrix} \dot{u} \\ \dot{w} \end{pmatrix} + \begin{bmatrix} \omega_s^2 & 0 \\ 0 & \omega_w^2\mu \end{bmatrix} \begin{pmatrix} u \\ w \end{pmatrix} = \begin{pmatrix} \delta_{st} \\ \Delta\mu\delta_{st} \end{pmatrix}, \quad (3)$$

with  $\delta_{st} = F_0/k_s$ .

## 2.2 Effective Length Description - ( $L_e$ )

As a way of reducing the error between the experimental and analytical investigation, Chaiviriyawong *et al.* (2008) demonstrate, by numerical analysis, that the part of the fluid in the lower corners (Fig. 1) does not contribute with inertia to the oscillatory fluid system. The lower corners exhibit decreased velocities compared to the center of the main flow line. In another work, Chaiviriyawong *et al.* (2008) present a deduction of this velocity field. Integration of the velocity field of this flow line results in an effective length ( $L_e$ ) against the use of the elliptical flow field estimation method.

Therefore, the authors conclude that the value of the effective length ( $L_e$ ) is given by:

$$L_{ef} = vB' + 2H' + \frac{\pi a}{2}, \quad (4)$$

where  $v = A_v/A_h$ ,  $a$  is the tube width and  $B'$  and  $H'$  the distances. It is noticed that all the terms are geometric factors of the sizing of the absorber, in this way, the indications arranged in Fig. 1 are presented as well. In addition,  $A_v$  corresponds to the area of the column section and  $A_h$  the cross-sectional area of the horizontal portion of the tube.

## 3. Experimental Setup

The experimental studies were performed on the Laboratory of Dynamics at the University of Brasilia. Each subsystem was investigated separately to feed the 2DoF model. The structure is built upon 10 pieces of steel modules demonstrated in Figure 2. The steel is a SAE1020 with a Young modulus ( $E$ ) equals to 205GPa and a specific weight ( $\rho_{aco}$ ) equals to  $7870kg/m^3$ .

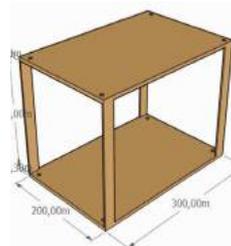


Figure 2: Real dimensions of the modulus used on the main structure

The main structure is presented in Figure 3a with 6 extra masses of 20 kg each, and on Fig. 3b the coupled system is identified.

The data from the structure tests were obtained on free and forced vibration. On both cases, it was used the accelerometer 626B03 (Low Frequency PCB) to measure the structural displacement, and on forced vibration tests the load cell 208C01 (Low Frequency PCB) were responsible to measure the excitation force. It was used the LabView software to obtain those results.

Besides, the TLCD was constructed from a glass reservoir. The main characteristic of the TLCD is its "U" shape, so this was created by adding a cell of Styrofoam. Figure 4 shows the geometric dimensions and the schematic view of the projected TLCD. The data from the TLCD was obtained by capturing the trajectory of a fish bait fixed on the tank, allowing it to move only up and down.

The study on the absorber was carried on free vibration analysis. The experiment was performed by shifting the water column inside the tube ( $H'$ ) and recorded on video with 60 frames per second. The data collected from the experiment area first submitted to the free software CVMob where the points are obtained.

Then, the identification of the modal parameters of the experiment is done using the Curve Fit Tool from MATLAB (*Mathworks*®) paid software. In this step, the Equation (5) that behaves a damped system is entered as input data.

$$x(t) = ae^{-bt} \sin(ct + d) + e. \quad (5)$$

Therefore, it is expected after the parameter configuration that the tool prints a behavior similar to the arrangement of the points obtained in the experiment, overlapping them. From the factors of the input equation, we can estimate the natural frequency ( $\omega_n$ ) and the damping ratio of the system ( $\xi_a$ ).

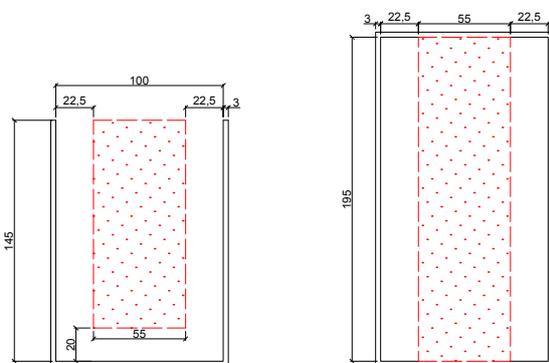


(a) Main structure with coupled masses



(b) Main structure coupled to TLCD

Figure 3: Experimental Setup



(a) Geometric dimensions of experimental TLCD



(b) Schematic view of the experimental TLCD

Figure 4: TLCD characterization

## 4. Experimental Identification

### 4.1 Parametric Optimization

First, the results from the main structure are present in Fig. 5. It is possible to identify the first three natural frequencies on the data. This result was obtained by a free vibration method where it is imposed an initial displacement on the structure. The time of capture the data is 20 seconds. On this result, it is possible to understand that the frequency that refers to the first modal shape appears to be more dominant.

Furthermore, on the TLCD studies, it was investigated the equipment under a series of fluid heights ( $H'$ ). The results are presented in Fig. 6 concerning the two methods of considering the fluid length on the tube, theoretical and effective. The experimental results are presented contrasted with a shaded area. This shaded area represents the wrapping that may be considered true by the curve fit function. Although the experimental results seem not to be on the analytical curves, it can be observed that the experimental results have a curve trend among its representation.

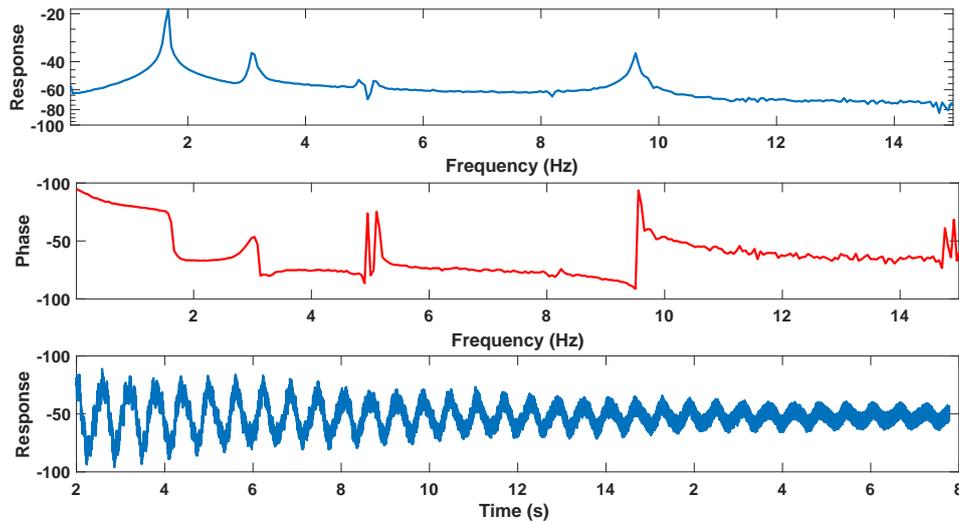


Figure 5: Main structure experimental response by free vibration method

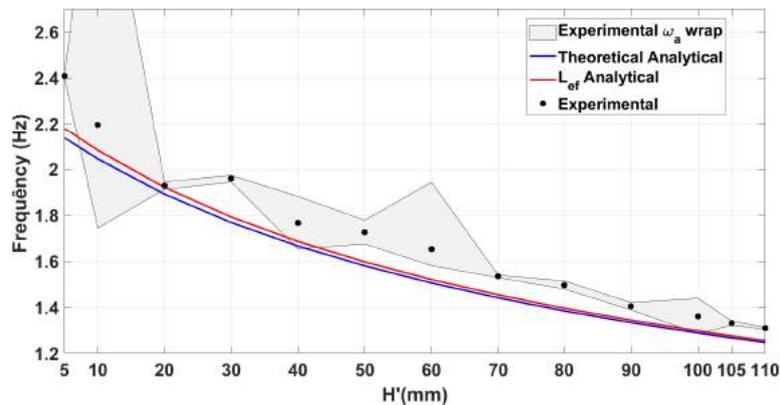


Figure 6: TLCD frequencies for different fluid height ( $H'$ ): analytic (L), effective analytic ( $L_{ef}$ ), experimental.

Following the studies, it is carried out experiments on the coupled structure. The coupled system works as it is represented on the Fig. 1, with the structure reduced to 1DoF by modal reduction technique Soong and Dargush (1997). From the preliminary investigation, it is defined that the structural values are:  $m_s = 176,935Kg$ ,  $k_s = \omega_s^2 m_s = 1,833 \times 10^4 N/m$  e  $c_s = 2m_s \xi_s \omega_s = 36,73 Ns/m$

The first studies are to define the optimum fluid height to find the optimal tuning. With this, numerical studies are performed, and it is exposed on Figure 7. On this Figure, it is shown the results for the 1DoF compared to the coupled system with some heights. As it can be seen, the height that presents the better response attenuation is the one with  $H' = 50mm$ . Hence, this height is known as the optimum height of  $H'_{opt}$  since it is the one that is best tuned to the main structure frequency.

Knowing that optimum fluid height it is possible to define the  $\alpha$ , and the response map could be generated. The response map technique exhibits a range of parameters from a settled  $\alpha$ , showing the amplitude values by the combination of  $\mu$ ,  $\gamma$ , and  $\xi_w$ . Hence, after the map being formulated, it is proposed to find the point of minimum amplitude response for a given  $\mu$ . On this case, the  $H' = 50mm$  corresponds to a  $\mu = 0.0046$ . The response map is represented in Fig. 8, with the respect that the  $\mu$  value is too small.

The data collected from the map is processed on the Eq. 3 with the main structure parameters, and it is possible to see the behavior of the response from different  $\gamma$  values. It is represented in Fig. 9. It shows the value for the optimum area, which is the point of minimum amplitude for that  $\mu$ . It also exposes a point before and after the optimum region for the same  $\mu$ , in order words, it varies only the  $\gamma$ .

## 4.2 Experimental Verification

Expecting to obtain the behavior observed in the numerical analysis, the experimental results are presented. First, the investigation that results from free vibration analysis. The Figure focus on the peak area of the FRFs (Frequency Response Function). Although these results seem to represent the same values, it is observed that a reduction is already presented

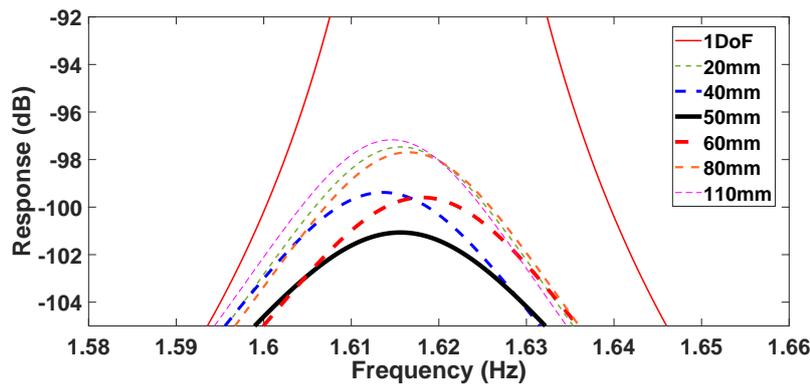


Figure 7: Numerical frequency response from the coupled system with various fluid height  $H'$  compared to a 1DoF response

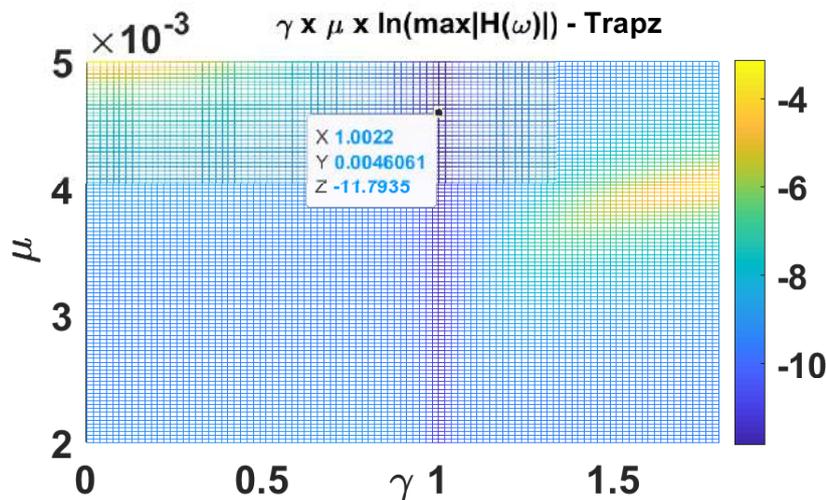


Figure 8: Response map of the coupled structure to TLCD for the  $H'_{opt}$  with  $\alpha = 0.39$  with responses on function of  $\gamma$  e  $\mu$ .  $\mu$  ranging between 0.002 and 0.005.

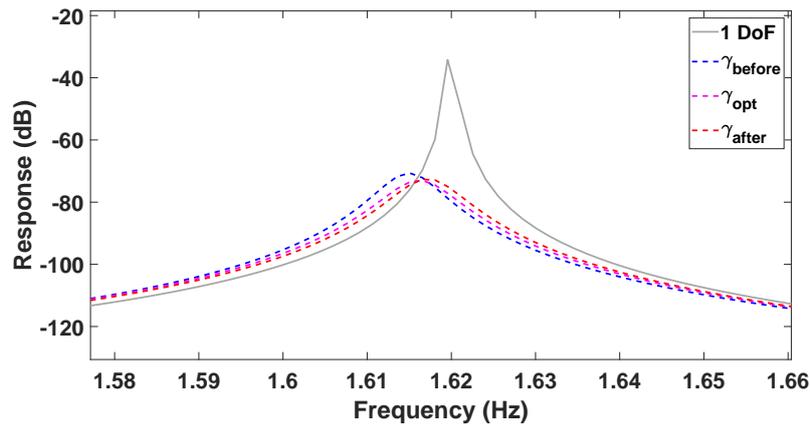


Figure 9: FRF comparison from response map solution of optimum  $\gamma$  and  $\gamma$  before and after the optimum region

from the coupled system compared with the 1DoF system.

On the forced vibration analysis it was used a shaker that gives harmonic forces input. The inputs were controlled and for each excitation frequency input, an amplitude was acquired. With this, it was possible to construct the FRF. On this experiment, the Lissajous graphics helped on the identification of the resonance frequencies. To explore the effect of the absorber under these excitation and analyze its effectiveness, the experiment is performed with the  $H'_{opt}$ ,  $H' = 40mm$  and  $H' = 60mm$ . Figure 11 represents the acquired data from the experiment.

To focus on the peak region of the experimental FRF, Fig. 12 shows this detail. With this, it is possible to identify that the  $H'_{opt}$  has the minor response value among the other tests.

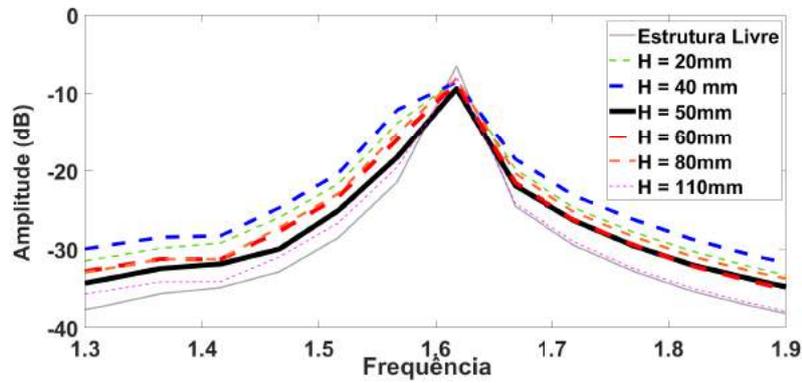


Figure 10: Experimental frequency response of 1DoF and coupled structure to TLCD with various fluid heights  $H'$  via free vibration analysis

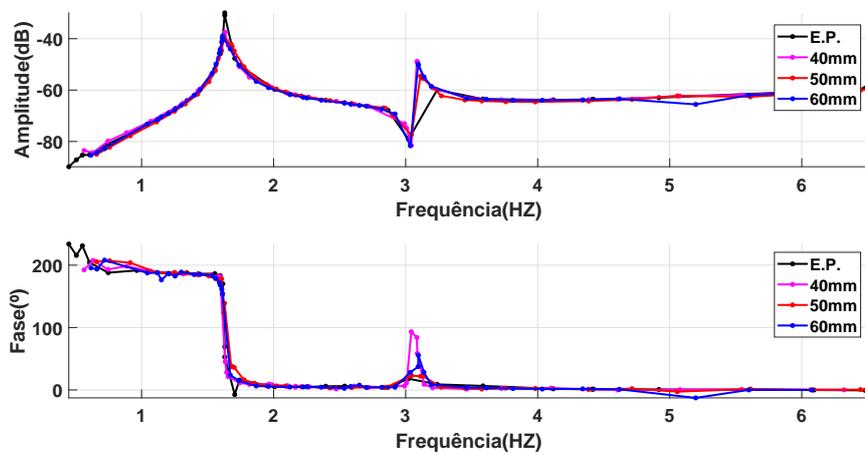


Figure 11: Experimental frequency response of 1DoF compared to coupled structure to TLCD to  $H' = 40mm$ ,  $H'_{opt}$  and  $H' = 60mm$ .

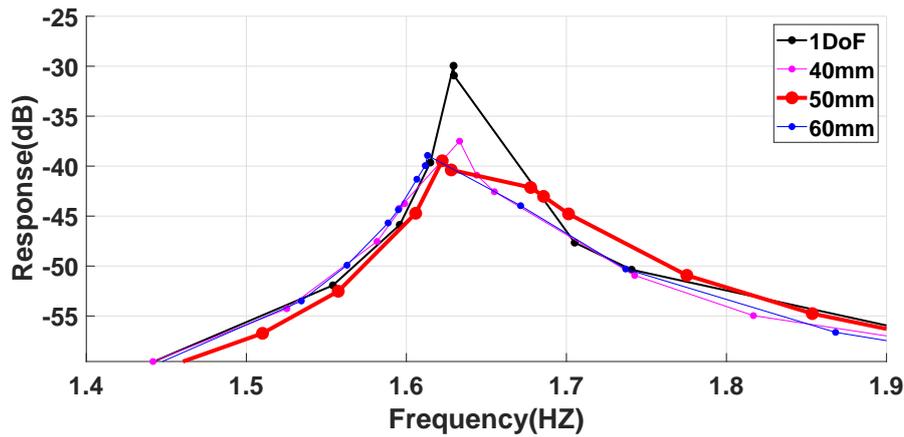


Figure 12: Detail of the peak region of the FRF exposed on Fig. 11.

Table 1 exposes the response values with the excitation frequency for a point before and after the resonance region. It also presents the values of the peaks acquired from the experimental sets.

Table 1: Responses displacements of the mains structure 1DoF and the coupled structure to TLCD via forced vibration

	DoF	Coupled $H' = 40mm$	Coupled $H'_{opt}$	Coupled $H' = 60mm$
Frequency (Hz)	1,6295	1,6332	1,6224	1,6132
Displacement (dB)	-29,9542	-37,5016	-39,4653	-38,9272

From Table 1 it is possible to analyze that the structure coupled to the TLCD presents a decrease in its response. Moreover, the optimum height shows the best reduction as expected from numerical results. The difference between the 1DoF and the optima response is 9,51 dB, which represents an attenuation of 31,75%. Although the other results are not considered as optimum such as  $H' = 40mm$  and  $H' = 60mm$ , the decrease is possible to be identified in these cases.

Thus, the experiment shows reliable results, and it can be inferred that the response decrease is given by the tuning of the main structure and TLCD frequency since the  $\mu$  is too small.

Figure 13 shows the time response from the 1DoF system and the coupled system. It is also seen the response reduced. The reduction in RMS terms is observed on 31,6%.

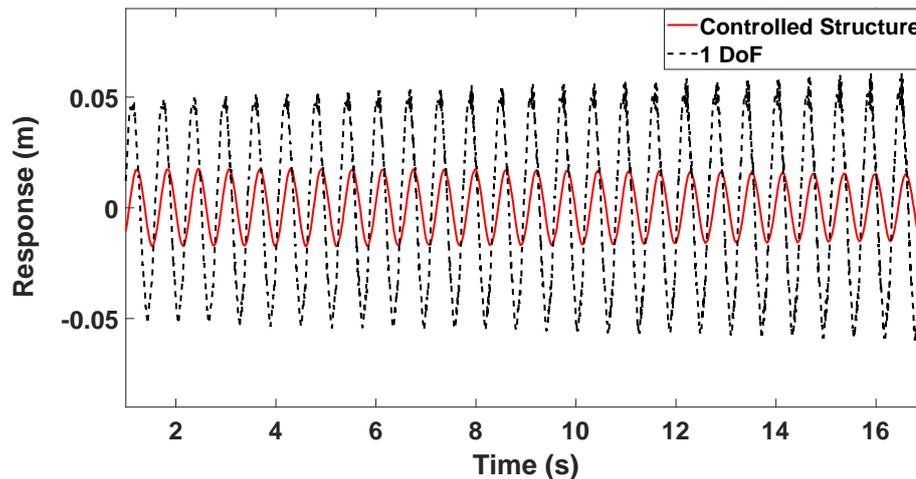


Figure 13: Time response for the 1DoF and the controlled structure

## 5. Concluding Remarks and Perspectives

This study performs an experimental investigation of equivalent 1 DoF mass-spring system (main structural subsystem) coupled to a TLCD (Fig. 6). Each subsystem is experimentally identified for determining equivalent parameters to feed reduced 2DoF coupled system (main structure + TLCD).

An experimental setup is provided with the presentation of the used equipment and software. It also exposes how the data is acquired from the experiments. The structure and the absorber are identified.

The results are exposed. First, a parametric optimization is presented to find numerical results capable of previewing the experimental investigation. With this, the optimum fluid column height is defined as  $H' = 50mm$ . A response map is presented listing a range of parameters.

Last, experimental verification is done. With the results, it is possible to identify that the  $H' = 50mm$  is the most effective as expected. Its is important to note that on the free vibration analysis the repeatability of the results is not guaranteed due to the technique for pulling the structure to obtain an initial displacement.

The forced vibration express a reduction of 9,51 dB, and in terms of RMS, the difference is 31,6% from the 1DoF and the controlled signal. It is observed that despite the low value of  $\mu$  the control on the structure is effective. The relation of masses from the absorber and the structure express the  $\mu$  value. It is observed that on low  $\mu$  rates, the control is done by only adjusting the frequencies.

Further works are needed to investigate the absorber subject to random loads such as earthquakes and winds. Besides, another work perspective is the study of viscous damping by numerical and experimental means to verify the limitations of stochastic linearization.

## 6. ACKNOWLEDGEMENTS

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