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BALANCING OF ROTATING MACHINES USING A KRIGING META-MODEL

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Abstract. *Unbalance is one of the most common malfunctions affecting rotating machines in the context of industrial plants. It is a phenomenon that can lead to high vibration amplitudes resulting in the fatigue of rotor elements. Different approaches have been proposed to solve unbalance problems, such as the so-called influence coefficient method, modal balancing, and four-run without phase. This paper presents an alternative balancing methodology for rotating machines, aiming at overcoming the time limitation faced by the most frequently used methods. This alternative technique uses a surrogate model to balance the rotating machine. In this case, several scenarios are created to simulated possible unbalance conditions of the rotor. The corresponding vibration responses are collected to develop a Kriging surrogate model. In the present contribution, the unbalance conditions were generated using the Monte Carlo simulation based on a uniform field. The numerical investigation was applied in a rotor composed by a flexible horizontal shaft, two rigid discs, and two self-alignment ball bearings. The results indicate the effectiveness of the proposed technique.*

Keywords: *rotating machines, balancing techniques, Kriging meta-model, Monte Carlo simulations.*

1. INTRODUCTION

According to Eisenmann and Eisenmann (1998), balancing is a systematic procedure used to approximates barycenter of a given rotor system to its geometric centerline. Consequently, the forces and resulting vibrations amplitudes applied to the bearings are attenuated. Different balancing techniques were proposed over the years, such as the so-called signal-based methods, e.g., modal balancing, four-run without phase, and influence coefficients method (Steffen Jr and Lacerda, 1996; Wowk, 1998; Bently and Hatch, 2002). Although widely used in industry, the signal-based techniques present some adverse aspects that encourage researchers to propose new balancing approaches. These methods demonstrated to be time-consuming since trial weights are positioned at specific locations along the rotor to determine the sensitivity of the vibration responses to unbalance variations. Another adverse aspect is that signal based balancing techniques consider a linear relationship between unbalance excitation and the resulting vibration. However, nonlinear effects are commonly present in the vibration responses of rotors.

During the last years, the research interest in balancing techniques decrease. However, there are many different ways and new perspectives to deal with it. For instance, balancing can be performed without using trial weights (Sadarriaga (2010)) or based on robust approaches (Carvalho et al., 2018), in which the effects of uncertain parameters or uncertain conditions can be incorporated in the balancing results.

In the present contribution, a balancing approach based on surrogate models is proposed. This technique is performed by generating different unbalance scenarios, which are samples created by using the Monte Carlo (MC) and Latin Hypercube simulations approaches. Correction masses and corresponding angular positions are obtained for different unbalance scenarios. For this aim, a conventional signal-based balancing approach is used. The unbalance scenarios and the associated correction masses (with their corresponding angular positions) are used as inputs and outputs, respectively, to formulate a Kriging surrogate model (Simpson et al., 2001; Wang et al., 2008; Xiaobo, 2017; Sinou, Nechak, and Besset, 2018). Thus, the obtained Kriging surrogate model is used for future balancing procedures. In the present work, a horizontal rotating machine considering two balancing planes and two measurement planes was used to verify the effectiveness of the proposed balancing approach. Additionally, the influence coefficient (IC) method was used to obtain the correction masses and their corresponding angular positions.

2. ROTOR MODEL

The finite element model of rotating machines encompasses different sub-systems, such as the shaft, discs, couplings, and bearings. The differential equation that describes the dynamic behavior of flexible rotors supported by ball bearings is presented by Eq. (1) (Lalanne and Ferraris, 1998).

$$\mathbf{M}\ddot{\mathbf{q}} + \left[\mathbf{D} + \Omega\,\mathbf{D}_g\right]\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{W} + \mathbf{F}_u + \mathbf{F}_s \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ is the damping matrix, $\mathbf{D}_g$ is the gyroscopic matrix, $\mathbf{K}$ is the stiffness matrix, and $\boldsymbol{\Omega}$ is the shaft rotation speed. $\mathbf{W}$ stands for the weight of the rotating parts, $\mathbf{F}_u$ represents the rotating unbalance forces, and $\mathbf{F}_s$ represents the supporting forces applied to the rotor by the bearings, and $\mathbf{q}$ is the generalized displacement vector.

The shaft finite element model is formulated from the Timoshenko beam theory with two nodes and four degrees of freedom per node (i.e., two displacements and two rotations). Due to the size of the matrices involved in the equation of motion, the pseudo-modal method (Lalanne and Ferraris, 1998) is used to reduce the dimension of the finite element model. Through this procedure, a reduced equation of motion is obtained, as illustrated by Eq. (2).

$$\mathbf{M}_m\,\ddot{\boldsymbol{\eta}} + \left[\mathbf{D}_m + \Omega\,\mathbf{D}_{g\,m}\right]\dot{\boldsymbol{\eta}} + \mathbf{K}_m\boldsymbol{\eta} = \mathbf{W}_m + \mathbf{F}_{u\,m} + \mathbf{F}_{s\,m} \tag{2}$$

in which $\boldsymbol{\eta}$ is the generalized displacement vector in modal coordinates ($\mathbf{q} = \boldsymbol{\Phi}\,\boldsymbol{\eta}$) and $\boldsymbol{\Phi}$ is the modal matrix containing the n first vibration modes of the non-gyroscopic and non-damped system. Additionally,

$$\begin{gathered}\mathbf{M}_m = \boldsymbol{\Phi}^T\mathbf{M}\,\boldsymbol{\Phi} \qquad \mathbf{D}_m = \boldsymbol{\Phi}^T\mathbf{D}\,\boldsymbol{\Phi} \qquad \mathbf{D}_{g\,m} = \boldsymbol{\Phi}^T\mathbf{D}_g\,\boldsymbol{\Phi} \\ \mathbf{K}_m = \boldsymbol{\Phi}^T\mathbf{K}\,\boldsymbol{\Phi} \qquad \mathbf{W}_m = \boldsymbol{\Phi}^T\mathbf{W} \qquad \mathbf{F}_{u\,m} = \boldsymbol{\Phi}^T\mathbf{F}_u \qquad \mathbf{F}_{s\,m} = \boldsymbol{\Phi}^T\mathbf{F}_s\end{gathered} \tag{3}$$

where $\mathbf{M}_m$ is the modal mass matrix, $\mathbf{D}_m$ is the modal damping matrix, $\mathbf{D}_{g\,m}$ is the modal gyroscopic matrix, $\mathbf{K}_m$ is the modal stiffness matrix, $\mathbf{W}_m$ is the modal weight vector, $\mathbf{F}_u$ is the modal unbalance forces, and $\mathbf{F}_s$ is the modal supporting forces.

3. IC METHOD

The IC method is widely used in industry to balance flexible rotors, leading to satisfactory and reliable results. Once the balancing planes, measuring planes, and trial weights are defined, the additional information required by the IC method is the vibration amplitudes and corresponding phase angles associated with the unbalanced system. The vibration responses are considered as inputs in the model.

Few definitions are essential to understand the mathematical formulation of the IC method. Trial weights are masses fixed to proper planes defined to apply a known unbalance force in the rotating machine. Measuring planes are locations suitable for the installation of vibration sensors. Correction weights are masses fixed at certain angular positions so that the vibration amplitude is reduced to a satisfactory level. The correction masses and corresponding angular positions are determined by the IC method. Balancing planes are planes in which the correction weights are installed (Wowk, 1998).

Equation (4) presents the relationship between the original rotor unbalance distribution $\mathbf{U}^p$, the associated vibration amplitudes $\mathbf{V}^j$, and the so-called influence coefficients $\boldsymbol{\alpha}^{jp}$. The influence coefficients are complex values (i.e., amplitude and angular position information) that relate the resulting vibration amplitudes measured at the position j due the unbalance force generated by the mass fixed at the position p.

$$\mathbf{V}^j_{v\,x\,1} = \boldsymbol{\alpha}^{jp}_{v\,x\,n}\mathbf{U}^p_{n\,x\,1} \tag{4}$$

where v is the number of measurement points (j = 1, …, $\mathbf{v}$) and n is the number of balancing planes (p = 1, …, n). In this case, $\boldsymbol{\alpha}^{jp}$ must be identified to perform the balancing.

For a given rotation speed Ω, Eq. (4) can be rewritten as follows:

$$\mathbf{V}_0 = \begin{Bmatrix} V_0^1 \\ V_0^2 \\ \vdots \\ V_0^v \end{Bmatrix} = \begin{bmatrix} \alpha^{11} & \alpha^{12} & \cdots & \alpha^{1n} \\ \alpha^{21} & \alpha^{22} & \cdots & \alpha^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^{v1} & \alpha^{v2} & \cdots & \alpha^{vn} \end{bmatrix} \begin{Bmatrix} U_0^1 \\ U_0^2 \\ \vdots \\ U_0^n \end{Bmatrix} = \boldsymbol{\alpha}\mathbf{U}_0 \tag{5}$$

in which $\mathbf{V}_0$ is the vector of vibration responses associated with the original unbalance distribution $\mathbf{U}_0$ of the rotor.

The influence coefficients matrix $\boldsymbol{\alpha}$ is determined by using a trial weight m_t attached first in the balancing plane $p = 1$. The angular position of m_t is taken as a reference value for the correction weights. This trial weight introduces an additional unbalance force in the rotor given by $W^1 = m_t h$, where h is the eccentricity of the trial weight. For the same rotation speed Ω, the new unbalance condition of the rotor is shown in Eq. (6).

$$\mathbf{V}_1 = \begin{Bmatrix} V_1^1 \\ V_1^2 \\ \vdots \\ V_1^v \end{Bmatrix} = \begin{bmatrix} \alpha^{11} & \alpha^{12} & \cdots & \alpha^{1n} \\ \alpha^{21} & \alpha^{22} & \cdots & \alpha^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^{v1} & \alpha^{v2} & \cdots & \alpha^{vn} \end{bmatrix} \begin{Bmatrix} U_0^1 + W^1 \\ U_0^2 \\ \vdots \\ U_0^n \end{Bmatrix} = \boldsymbol{\alpha}\mathbf{U}_1 \quad (6)$$

where $\mathbf{V}_1$ is the vector of vibration responses associated with the new unbalance distribution $\mathbf{U}_1$.

Disregarding the mass of the trial weights as compared with the total mass of the rotating machine, the matrix $\boldsymbol{\alpha}$ can be considered constant in Eq. (5) and Eq. (6). Therefore, Eq. (7) is obtained by subtracting Eq. (6) from Eq. (5), as follows:

$$\mathbf{V}_1 - \mathbf{V}_0 = \begin{Bmatrix} V_1^1 - V_0^1 \\ V_1^2 - V_0^1 \\ \vdots \\ V_1^v - V_0^v \end{Bmatrix} = \boldsymbol{\alpha} \begin{Bmatrix} W^1 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \quad (7)$$

The trial weight is then removed from the balancing plane $p = 1$, and the process is repeated for the remaining balancing planes. Therefore, $\boldsymbol{\alpha}$ can be fully determined. The generalization of the IC method is presented by Eq. (8), in which α, V, and W are complex quantities.

$$\alpha^{jp} = \frac{V_1^j - V_0^j}{W^p} \quad (8)$$

The correction masses m_c are obtained by inverting $\boldsymbol{\alpha}$ and multiplying the resulting equation by the initial vibration response $\mathbf{V}_0$ (see Eq. (5)). If the number of balancing planes is equal to the number of vibration sensors (i.e., $n = v$ in Eq. (4)), $\boldsymbol{\alpha}$ is inverted directly. Otherwise, the pseudo-inverse technique should be used. Different rotation speeds Ω can be used simultaneously in the IC method, providing a broadband balancing efficiency. Also, additional information on the IC method can be found in Eisenmann and Eisenmann Jr (1998), Wowk (1998), Ehrich (1992), Muszynska (2015), and Bently and Hatch (2002).

In the proposed approach, the vibration responses are used to define an unbalance condition. Thus, the formulation of the IC method is not modified.

4. KRIGING MODEL

Surrogate models are an interesting alternative to replace complex mathematical models of mechanical systems for simple ones, reducing the associated computational cost. The Kriging formulation represents one of the promising surrogate modeling approaches (Simpson et al., 2001; Wang et al., 2008; Xiaobo, 2017; Sinou, Nechak, and Besset, 2018). In this technique, input and output values are correlating using mathematical functions, such that an output value can be determined for any input within a defined space. According to Simpson et al. (2001), the metamodeling process is composed of three steps:

i. Sampling: definition of the numerical or experimental procedure to determine the samples associated with the input and output values of the real system or its model;
ii. Formulation: definition of the functions used for correlating the considered input and output values;
iii. Adjustment: correlation of the input and output values through the functions defined in the formulation step to determine the surrogate model.

The Kriging formalism is used in the present work as the surrogate technique. The main difference between the Kriging formalism and other approaches relies on the exploitation of spatial correlations between the function values to adjust the average behavior of the regression model. The general expression for the Kriging method is given by Eq. (2).

$$y(\mathbf{x}) = f(\mathbf{x}) + Z(\mathbf{x}) \quad (2)$$

where $y(\mathbf{x})$ is the output values associated with the input vector $\mathbf{x}$, $f(\mathbf{x})$ is a known polynomial function of $\mathbf{x}$, and $Z(\mathbf{x})$ is the realization of a customarily distributed Gaussian random process with mean zero and variance σ^2.

Predicted estimates $\hat{y}(\mathbf{x})$ at new values of $\mathbf{x}$ (new input data points) can be estimated as shown in Eq. (3). For this aim, a predetermined set of samples of the original model is required. The input and output vectors of the samples are represented by Eq. (4).

$$\hat{y}(\mathbf{x}) = g^T(\mathbf{x})\hat{\beta} + \mathbf{r}^T(\mathbf{x})\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{G}\hat{\beta}) \tag{3}$$

where $g(\mathbf{x})$ and $\hat{\beta}$ are associated with the chosen polynomial function, $\mathbf{r}^T(\mathbf{x})$ is the correlation vector of length N_s between an untried value of $\mathbf{x}$ and the sampled data points $\mathbf{S} = [\mathbf{s}^1, \ldots, \mathbf{s}^{Ns}]^T$, and $\mathbf{G}$ is an $N_s \; x \; p$ matrix containing the function g applied in the input sampled data points. The main correlation functions are shown in Tab. 1.

Table 1 – Correlation models.

Correlation models	$R_j\left(\theta_j, x_j, p_j\right)$	
Linear	$\max\left\{0, 1-\theta_j\left\lvert x_j - p_j\right\rvert\right\}$	
Gaussian	$\exp\left(-\theta_j\left\lvert x_j - p_j\right\rvert^2\right)$	
Exponential	$\exp\left(-\theta_j\left\lvert x_j - p_j\right\rvert\right)$	
Cubic	$1-3\xi_j^2+2\xi_j^3$	$\xi_j = \min\left\{1, \theta_j\left\lvert x_j - p_j\right\rvert\right\}$

The input and output vectors of the samples are represented in Eq. (4).

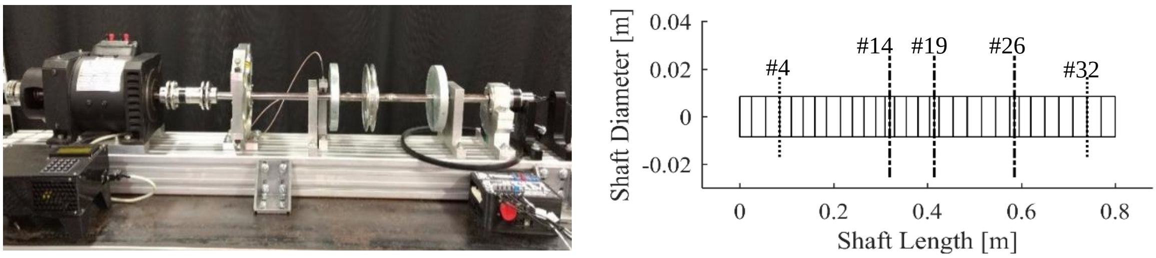
$$\begin{aligned} &\mathbf{S} = [\mathbf{s}^1, \ldots, \mathbf{s}^{N_s}]^T \\ &\mathbf{s}^i = [s_1^{\,i}, \ldots, s_k^{\,i}] \\ &\mathbf{Y} = [y^1, \ldots, y^{N_s}]^T \end{aligned} \tag{4}$$

where $R\left(\boldsymbol{\theta}, \mathbf{x}, \mathbf{p}\right)$ is the spatial correlation function defined by Eq. (5).

$$R\left(\boldsymbol{\theta}, \mathbf{x}, \mathbf{p}\right) = \prod_{j=1}^{k} R_j\left(\theta_j, x_j, p_j\right) \tag{5}$$

5. NUMERICAL RESULTS

The rotor test rig taken as reference for this numerical analysis is presented in Fig. 1a. The flexible shaft of the test rig was mathematically represented by using 33 finite elements (see Fig. 1b; steel shaft with 800 mm length and 17 mm of diameter; E = 205 GPa, ρ = 7850 kg/m^3, υ = 0.29). Three rigid discs are coupled to the shaft, namely D_1 (node #14; 2.637 kg; according to the FE model), D_2 (node #26; 2.649 kg; both of steel and with 150 mm diameter and 20 mm thickness; ρ = 7850 kg/m^3), and D_3 (node #19; 0.478 kg; aluminum disc). The system is supported by two self-alignment ball bearings B_1 and B_2 located at nodes #4 and #32, respectively. Displacement sensors are orthogonally mounted on the node #8 (S_{8X} and S_{8Z}) and node #12 (S_{12X} and S_{12Z}) to collect the shaft vibration.



a) b)

Figure 1 – Rotating machine used in the numerical simulations: a) Test rig; b) FE model.

The unknown parameters of the rotor FE model were obtained by using a model updating procedure, in which the heuristic optimization technique Differential Evolution (Storn and Price, 1995) was used. In this case, the stiffness and damping coefficients of the bearings, the proportional damping added to **D** (coefficients γ and β; $\mathbf{D}_p = \gamma\,\mathbf{M} + \beta\,\mathbf{K}$; see Eq. (1)), and the angular stiffness k_{ROT} due to the coupling between the electric motor and the shaft (added around the orthogonal directions X and Z of the node #1) were considered as unknown parameters.

The proposed identification process (i.e., the comparison between simulated and experimental frequency response functions, FRFs) was performed 10 times, considering 100 individuals in the initial population of the optimizer. The objective function adopted by the optimization process takes into account only the regions close to the FRFs peaks associated with the four first rotor natural frequencies. The experimental FRFs were measured on the test rig for the rotor at rest by applying impact forces along the X and Z directions of both discs, separately. The response signals were obtained by the two proximity probes positioned in the same directions of the impact forces, resulting in 8 FRFs (range of 0 to 200 Hz and steps of 0.25 Hz). Table 2 presents the parameters determined using the model updating procedure.

Table 2 – Parameters determined using the model updating procedure.

Parameters	*Values*	*Parameters*	*Values*	*Parameters*	*Values*
$^{*}k_X$ / B_1	9.982×10^5	$^{*}k_X$ / B_2	2.431×10^6	γ	2.598
$^{*}k_Z$ / B_1	2.004×10^6	$^{*}k_Z$ / B_2	9.997×10^7	β	2.12×10^{-10}
$^{**}d_X$ / B_1	81.196	$^{**}d_X$ / B_2	164.941	$^{***}k_{ROT}$	986.778
$^{**}d_Z$ / B_1	199.042	$^{**}d_Z$ / B_2	105.531		

*k: stiffness [N/m]; **d: damping [Ns/m]; ***k_{ROT}: stiffness [N/rad].

Figure 2 presents the Campbell diagram of the rotating machine considering the parameters updated through the optimization process, in which the first two forward critical speeds were determined at, approximately, 1810 rev/min and 5855 rev/min, respectively. More details about the model updating procedure adopted in this work can be found in (Cavalini Jr. et al., 2015; Cavalini Jr. et al., 2016). Figure 3 compares one simulated and experimental FRF considering the parameters shown in Tab. 2, validating the performed updating procedure.

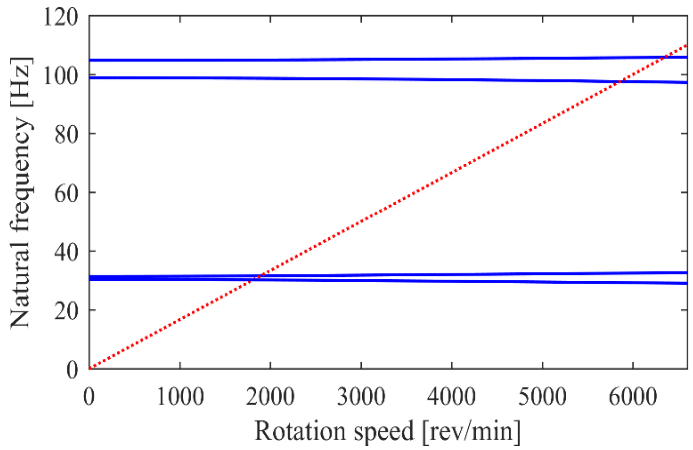


Figure 2 – Campbell diagram of the rotating machine.

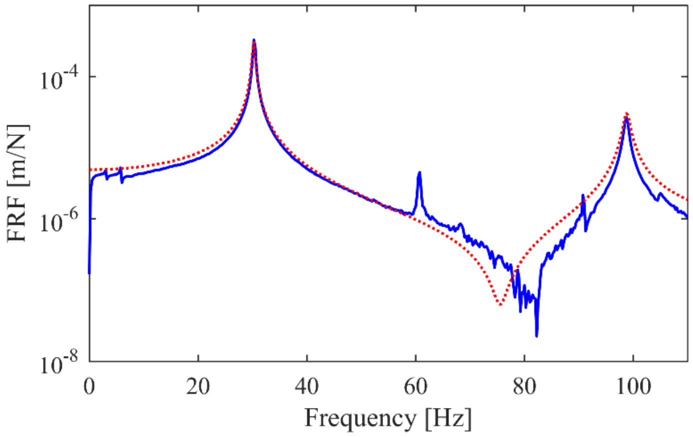


Figure 3 – Updated model (......) and experimental (——) FRFs (impact along the X direction of D_1 and sensor S_{12X}).

In the present work, the linear correlation function was used associated with a second-order polynomial function to obtain the associated Kriging surrogate model. For this aim, 350 unbalance scenarios were used to obtain the surrogate model. The vibration amplitudes and corresponding phase angles measured by using the sensors S_{8X}, S_{8Z}, S_{12X}, and S_{12Z} were considered as the inputs of the surrogate model. The correction masses and corresponding angular positions determined for both discs by applying the IC method were considered as outputs.

Figure 4 shows the vibration amplitudes of the rotating machine obtained before and after balancing performed using the constructed Kriging surrogate model, in which the efficiency of the proposed approach is demonstrated. In this case, the same 350 unbalance scenarios used to create the surrogate model were considered for testing purposes.

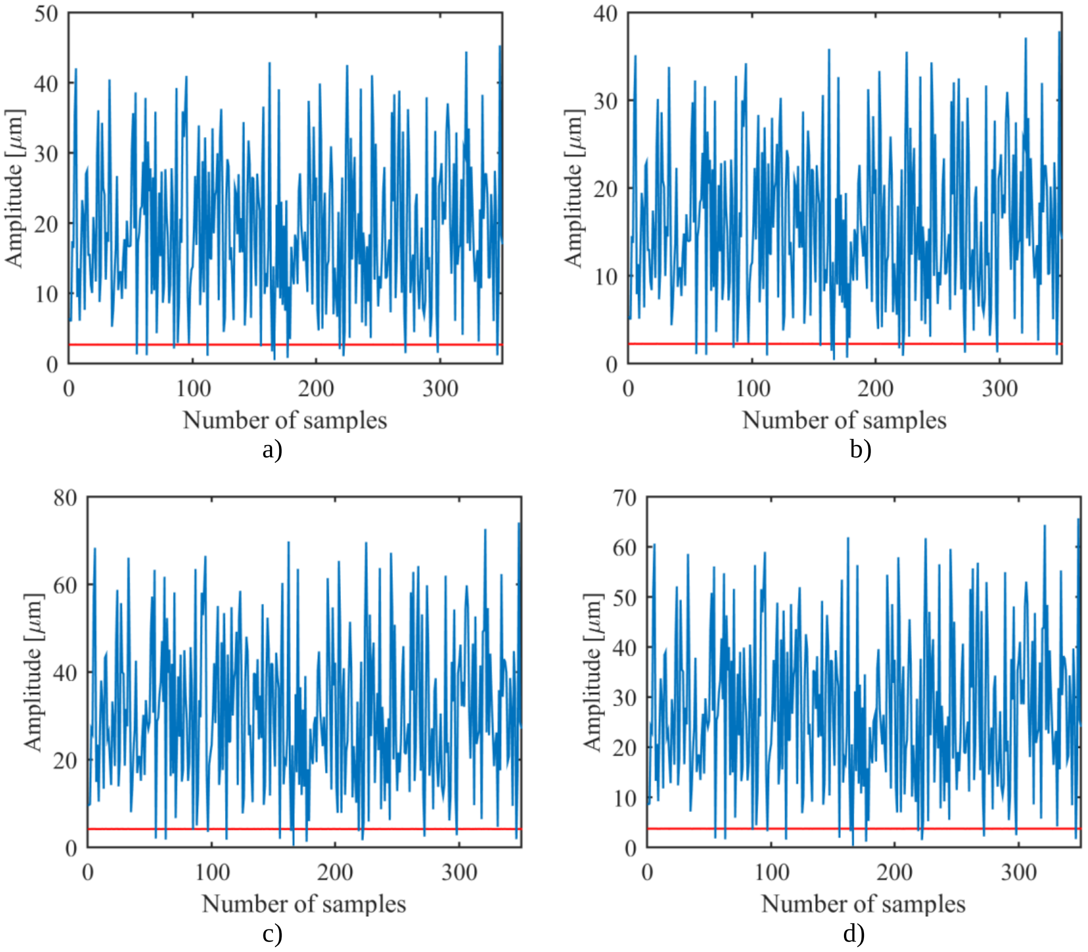


Figure 4 – Results obtained using the proposed balancing approach for the scenarios adopted to create the Kriging surrogate model: a) sensor S_{8X}; b) sensor S_{8Z}; c) sensor S_{12X}; d) sensor S_{12Z}; unbalanced (——) and balanced rotor (——).

Table 3 presents an unbalance condition different from the ones used to create the Kriging surrogate model. The correction masses and corresponding angular positions obtained using the surrogate model is shown in Tab. 4. Figure 5 presents the vibration amplitudes obtained by applying these correction masses in the rotating machine.

Table 3 – Unbalance condition inserted in the machine.

Parameters	*Correction*
Unbalance / D_1 [g.mm]	600
Phase angle / D_1 [degrees]	90
Unbalance / D_2 [g.mm]	200
Phase angle / D_2 [degrees]	0

Table 4 – Correction masses and angular positions determined by using the surrogate model.

Parameters	*Correction*
Unbalance / D_1 [g.mm]	68.2
Angular positions / D_1 [degrees]	-82.7
Unbalance / D_2 [g.mm]	25.6
Angular positions / D_2 [degrees]	121.9

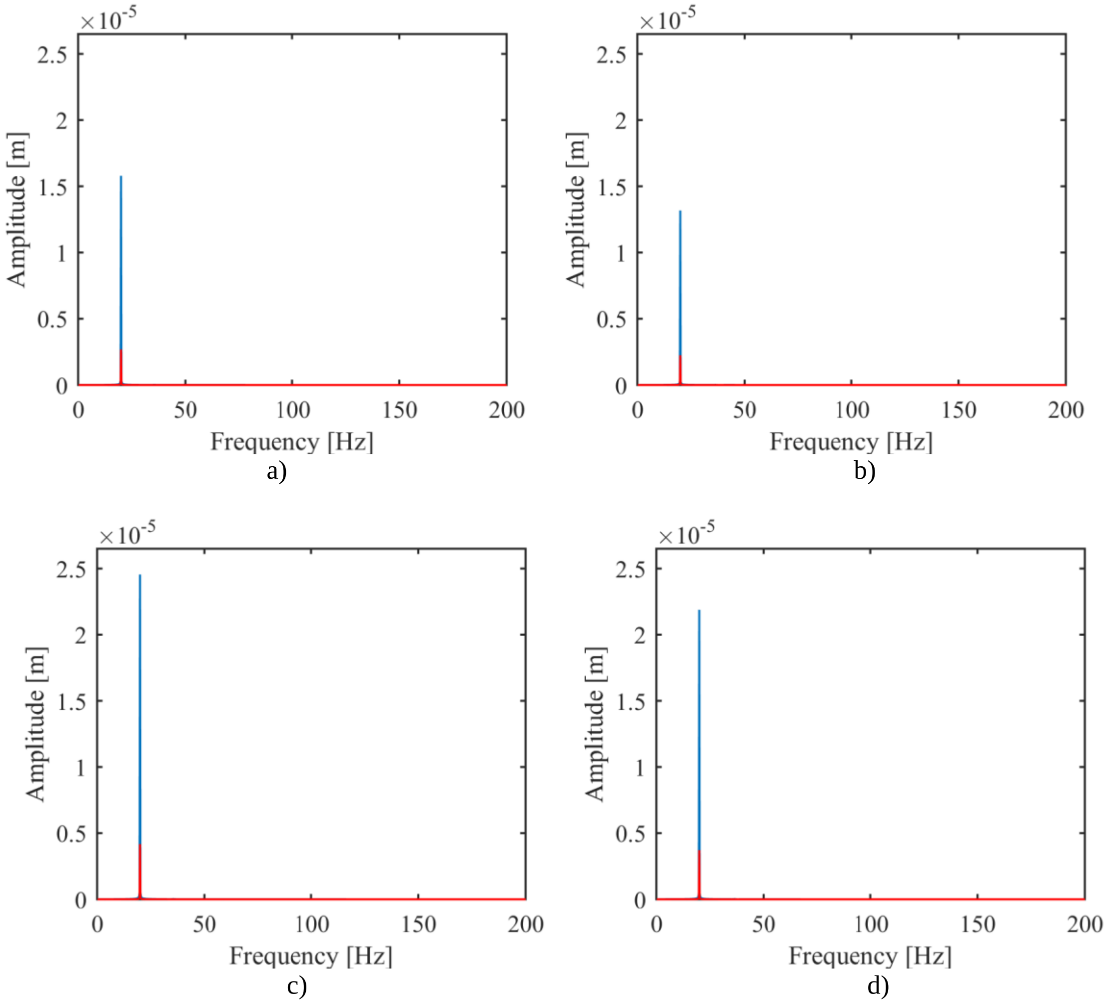


Figure 5 – Results obtained using the proposed balancing approach (frequency domain): a) sensor S_{8X}; b) sensor S_{8Z}; c) sensor S_{12X}; d) sensor S_{12Z}; unbalanced (——) and balanced rotor (——).

Table 5 summarizes the results presented in Fig. 5. Note that the proposed approach was able to balance the rotating machine efficiently. In this case, vibration reductions of approximately 83% were achieved for the four sensors. Figure 6 presents the comparison between time-domain vibration responses of the rotating machine before and after balancing.

Table 5 – Comparison between rotor vibration responses before and after balancing.

	S_{8X}	S_{8Z}	S_{12X}	S_{12Z}
Unbalance rotor [μm]	15.79	13.18	24.55	21.88
Balance rotor [μm]	2.68	2.24	4.17	3.72
Reduction [%]	83.03	83.00	83.01	82.99

6. CONCLUSIONS

In this work, a Kriging surrogate model was proposed to improve the efficiency of balancing procedures devoted to rotating machines. The methodology demonstrated to be efficient to balance the considered rotating machine. More studies will be performed to test the Kriging surrogate model over different conditions. The minimum number of samples required for creating the surrogated model will be evaluated. Experimental verification of the conveyed approach is also scheduled.

7. ACKNOWLEDGMENTS

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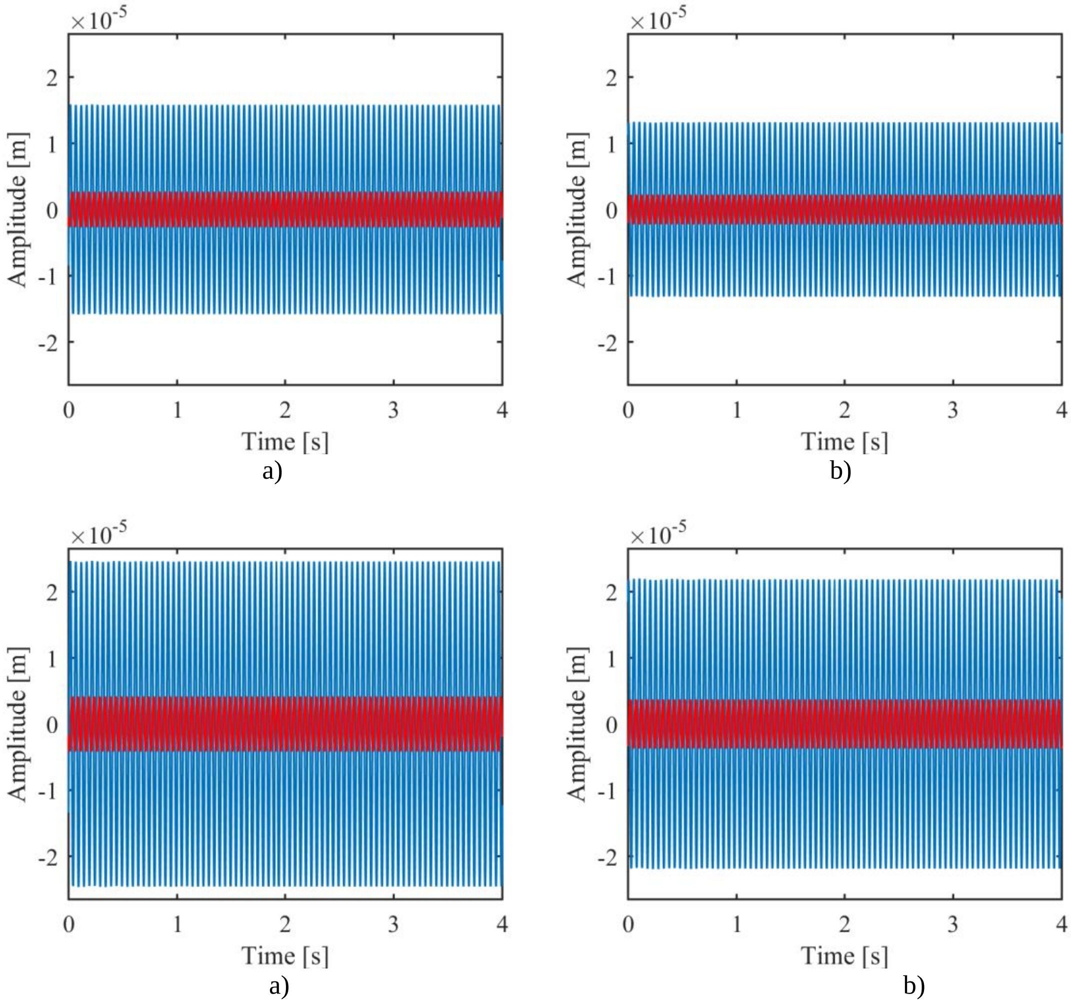

Figure 6 – Results obtained using the proposed balancing approach (time domain): a) sensor S_{8X}; b) sensor S_{8Z}; c) sensor S_{12X}; d) sensor S_{12Z}; unbalanced (——) and balanced rotor (——).

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