



25<sup>th</sup> ABCM International Congress of Mechanical Engineering  
October 20-25, 2019, Uberlândia, MG, Brazil

## Multi-Copter Forces and Moments Coefficient Estimation Using Adaptive Network-based Fuzzy Inference Systems

Vitor Taha Sant'Ana  
Felipe Machini M. Marques  
Bruno Luiz Pereira  
Roberto Mendes Finzi Neto  
Federal University of Uberlândia  
vitortaha@ufu.br  
machini@ufu.br  
brunolp@ufu.com  
finzi@ufu.br

**Abstract.** This article presents the modeling of the forces and moments coefficients from a radio controlled multi-copter vehicle obtained from the training data of an experimental flight. A method based on Fuzzy Inference Systems is proposed aiming to obtain an heuristic model capable of describing the acting forces and moments based on the input command signals. The main difference of the methodology proposed in this work is the optimization used to obtain the fuzzy parameters known as Adaptive Network-based fuzzy inference system (ANFIS), which was later known as Neuro-Fuzzy. Results have shown that the proposed method is capable of estimate the parameters from experimental data after data handling.

**Keywords:** Neuro-Fuzzy, Thrust Coefficient, Multi-Copter, UAV.

### 1. INTRODUCTION

This work propose a dynamic model without knowing the multi-copter characteristics, as aerodynamic airfoil from the propellers, inertia matrix and mass, the physical properties of the aircraft and/or the motor-propulsive group (Brandon and Morelli, 2012). There are many works as Conroy *et al.* (2018) that have proposed experimental procedures to estimate the motor propulsive parameters on controlled environment. However, one can state that these physical parameters may change according to the operational conditions of the aircraft. Hence, the main concept concerning this work is to obtain a dynamical model for a multi-copter UAV susceptible to different flight missions conditions using artificial intelligence tools.

The artificial intelligence and the inherent techniques have spread on aerospace technologies in the past few decades. Initially applied on airfoil coefficients prediction as presented by Rajkumar and Bardina (2002) and, after that, the idea to use other methods has matured and systems based on fuzzy rules have emerged for aircraft dynamical modelling applications. The ANFIS, distinctively, is a computational intelligence technique capable to be trained as the Neural Networks, and to understand the multi-copter dynamics according to the rule base build by an specialist. The ANFIS could be fed with any multi-copter experimental data, once the input and output correlations are done (rule base) by an UAV dynamic expert.

The present work is outlined as follows: firstly the ANFIS structure and mathematical approach is presented illustrating how the forces and moments model can be trained by the proposed method; later, a multi-copter dynamical model along with the external forces and moments acting on the body is proposed highlighting which variables shall be used to train the model and wich parameters expect to obtain; then, the apparatus used for data acquisition is described followed by the results obtained after the data handling and model training.

### 2. METHODOLOGY

Basically, the proposed methodology consists in the use of the Neuro-Fuzzy training process to estimate the forces and moments coefficients of the vehicle dynamical model. In order to accomplish this, first the ANFIS methodology is presented and, later, the dynamical model of the multi-copter which represents the movement of the system is derived describing the main acting forces and moments.

## 2.1 Adaptive-Network-based Fuzzy Inference System

The Adaptive Network-based fuzzy inference system (ANFIS) Jang (1992) was designed through the junction of the Neural Network architecture with the Fuzzy Inference System (FIS) logic. The figure below shows the ANFIS architecture according with Pereira (2017):

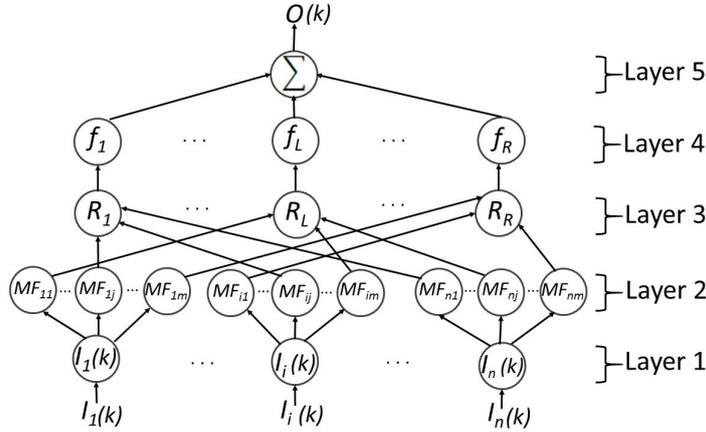


Figure 1. ANFIS architecture

The purpose of each layer, using the Takagi-Sugeno inference is given by the the following definitions:

**First Layer:** Each node receives one input variables  $I_i(k)$  that will be used in the training process. The output from  $i$  node from the first layer at time  $k$ ,  $u_i^{(1)}(k)$  is given by:

$$u_i^{(1)}(k) = I_i(k) \quad (1)$$

**Second Layer:** In this layer the fuzzification of input variables is performed, that is, the real numbers are transformed into Fuzzy subsets with a certain degree of pertinence. In this step, the membership functions (MF) are built for the description of the inputs> Consider that the membership functions are approximated by gaussians, the output of node  $ij$  from layer 2 at time  $k$ ,  $u_{ij}^{(2)}(k)$ , is given by:

$$u_{ij}^{(2)}(k) = e^{-\frac{(u_i^{(1)}(k) - m_{ij}(k))^2}{\sigma_{ij}^2(k)}} \quad (2)$$

where  $m_{ij}(k)$  e  $\sigma_{ij}(k)$  are the mean and standard deviation, respectively, of the Gaussian membership function  $MF_{ij}$ .

**Third Layer:** Here the propositions of type If...Then are realized, forming the rule base for ANFIS. For each rule, operator AND and OR are treated as minimum or maximum, respectively, so the output of the  $L$  node from this layer,  $u_L^{(3)}(k)$ , is a function of the layer 2 selected output from rule  $R_L$ .

**Fourth Layer:** The nodes from this layer are treated as constants, and are defined as a function  $f_L : R^n \rightarrow R$  such that  $f_L = f(I_1, \dots, I_i, \dots, I_n, w_{1L}, \dots, w_{jL}, \dots, w_{oL}, k)$ , where  $w_{1L}, \dots, w_{jL}, \dots, w_{oL}$  are weights that will be determined in the ANFIS training phase. Thus the output from node  $L$  of layer 4,  $u_L^{(4)}(k)$ , is calculated by:

$$u_L^{(4)} = u_L^{(3)}(k) f_L(I_1, \dots, I_i, \dots, I_n, w_{1L}, \dots, w_{jL}, \dots, w_{oL}, k) \quad (3)$$

**Fifth Layer:** The last layer releases the ANFIS answer, given by equation:

$$O(k) = \frac{\sum_{L=1}^R u_L^{(4)}(k)}{\sum_{L=1}^R u_L^{(3)}(k)} \quad (4)$$

## 2.2 Multi-Copter Equations

According to Valavanis (2007), for any aircraft configuration, either rotary or fixed wing, the equations of motion can be derived using Newton-Euler formulation applied to a six degree of freedom rigid body. The external forces and moments acting on the body center of gravity (CG) are associated with the aircraft geometry (aerodynamic properties), propulsion system and control surfaces or actuators. Multi-copter UAVs, specifically, are equipped with a group of motor-propulsive systems, which are the source of the required thrust to overcome or balance the aircraft weight and drag. Further, the unbalance of the generated thrust by each propeller induces pitching, rolling and yawning moments around the body CG allowing the multi-copter to shift its position on space.

### 2.2.1 Forces

In general, micro autonomous flying vehicles uses a motor-propulsive set, consisted by electric spinning rotors equipped with propellers. The aerodynamic interaction of the propellers with the surrounding air is responsible to deliver the resultant thrust force to overcome the vehicle weight and put the aircraft into flight. In particular, brushless motors have become the standard for multi-rotor vehicles. So, the delivered thrust force from the motor is affected by the propulsive set parameters as: the motor electrical resistance, no-load current, motor efficiency and the propeller geometry (Conroy *et al.*, 2018). According to Marques (2018), the rotor propeller, since it has an aerodynamic shape, generates lift and, consequently, produces a thrust vector normal to the propeller radius which can be represented as:

$$T = \left( \frac{K_\tau K_v \sqrt{2\rho\pi R_m^2} \Omega}{K_t} \right)^2 \quad (5)$$

where  $K_\tau$  is a proportional constant relating the thrust force and the torque produced by the motor,  $K_v$  and  $K_t$  an electrical constant associated with the applied voltage and the motor's shaft angular speed and the torque produced by the motor and the input current, respectively,  $\rho$  is the surrounding air density,  $R_m$  the propeller radius and, finally,  $\Omega$  the propeller angular speed.

In conclusion, the thrust force produced by each rotor is proportional to the square of the propeller angular speed by a constant  $k$ . Thus, the resulting vector by the  $n$  motor-propulsive groups, on the body coordinate frame is:

$$\vec{T} = \sum_{i=1}^n T_i = \begin{bmatrix} 0 \\ 0 \\ \sum k\Omega_i^2 \end{bmatrix} \quad (6)$$

Furthermore, concerning the acting external forces, the drag due to the incoming viscous flow generates a force in the opposite direction of the body movement. For small multi-copters, the viscous drag can be considered as proportional, by a constant  $k_d$ , to the linear velocity on the three directions of the body coordinate frame ( $u$ ,  $v$  and  $w$ ). Hence, the drag force vector, acting on  $CG$ , is expressed as:

$$\vec{F}_D = \begin{bmatrix} -k_{dx}u & -k_{dy}v & -k_{dz}w \end{bmatrix}^T \quad (7)$$

### 2.2.2 Moments

As a consequence of the acting forces and the aircraft movement, the body experiments the effect of acting moments on its  $CG$  which can be summarized as: the torque due the thrust force variation, the gyroscopic effect and the torque due the propeller drag.

Firstly, the described thrust force on Eq. 6 generates a torque associated with the rotor distribution on the multi-copter frame w.r.t. the its  $CG$  expressed by the  $\gamma_i$  angle and the arm length  $l$ , which physically represents the moment arm. Hence, the generated moment is:

$$\vec{\tau}_T = \begin{bmatrix} \sum_{i=1}^n l \sin(\gamma_i) k\Omega_i^2 \\ \sum_{i=1}^n -l \cos(\gamma_i) k\Omega_i^2 \\ 0 \end{bmatrix} \quad (8)$$

Since the aircraft angular speed varies while the multi-copter propellers are spinning, the gyroscopic effect will be present on the model as a consequence of the propeller's angular momentum variation during flight. As consequence, a gyroscopic torque is produced acting on the aircraft center of gravity which can be expressed as (Roskam, 2001):

$$\vec{\tau}_G = \begin{bmatrix} Q \sum_{i=1}^n J_{m_i} \Omega_i \\ P \sum_{i=1}^n J_{m_i} \Omega_i \\ 0 \end{bmatrix} \quad (9)$$

being  $J_{m_i}$  the propeller moment of inertia,  $P$  and  $Q$  the multi-copter angular speed on  $x$  and  $y$  direction of the body coordinate frame, respectively.

As consequence of the aerodynamic drag force produced by the propeller when it is rotating around its spinning axis, a fan torque is produced on the  $z$  direction of the body coordinate frame. Once again, the resultant torque is proportional to the square of the angular speed by a constant  $b$ , and can be represented by (Marques, 2018):

$$\vec{\tau}_G = \begin{bmatrix} 0 & 0 & \sum_{i=1}^n -\text{sign}(\Omega_i) b\Omega_i^2 \end{bmatrix}^T \quad (10)$$

### 3. Apparatus and Procedures

This section describes the apparatus and procedures used to conduct the experimental data acquisition. The selected object of study was a commercial DJI™ F450 quadrotor multi-copter as shown in Fig. 2. The multi-copter is radio guided and embedded with a Pixhawk-PX4 micro-controller which is responsible for the system stabilization and used for data acquisition. The PX4 is endowed with two IMU (Inertial Measurement Unit), a magnetometer and a GPS capable of measuring the Euler angles, angular velocity, linear accelerations in body axis, the yaw angle and the multi-copter inertial position.



Figure 2. Quadrotor multi-copter with the embedded measuring systems.

The flights were conducted in an external environment where the vehicle was able to flight as long as the battery endured (approximately 8 minutes). All the flight envelope was recorded and the data was later uploaded to a microcomputer using a SD card with the aid of the Mission Planner © software to acquire the data for the Neuro-Fuzzy training process.

Table 1. Quad-rotor properties.

Property	Parameter	Value	Unit [SI]
Number of rotors	$n$	4	-
Mass	$m$	1.14	$kg$
Inertia on $x_b$ axis	$J_{xx}$	0.007	$kg.m^2$
Inertia on $y_b$ axis	$J_{yy}$	0.007	$kg.m^2$
Inertia on $z_b$ axis	$J_{zz}$	0.013	$kg.m^2$
Arm Length	$l$	0.220	$m$

### 4. Results

The Neuro-Fuzzy training process requires two steps. The first one is to verify the correlation between the input variables with the output variable, the correlation applied in this work is known as Pearson Correlation Coefficient, and this correlation needs to be higher than 0.85 to proceed a good training.

To verify the correlation between the input and output, the Fig. 3 is presented, showing the directly influence of the PWM signal, once the signal increases, the thrust in the Z body axis also increases. At last, the PWM signal is the sum of all 4 motors from the quadrotor multi-copter, each one varying between 1200 till 1800.

The second step to train the Neuro-Fuzzy, after the evaluation of the correlation between input and output variables, is the optimization of the fuzzy parameters, which in this case are the means and standard deviations of the Gaussian membership functions. The training graph is presented in Fig. 4.

The means and standard deviations optimized are shown in Tab. 2 and Tab. 3.

After the training, another flight test data were selected to validate. In Fig. 5 can be observed a certain difference between the Neuro-Fuzzy estimated values and the flight data, it is a small error when observed the scale of the Y axis ( $F_Z$ ).

The correlation between thrust ( $F_Z$ ) and PWM signal provides the value of the constant presented in the Eq. 6. The Fig 6 shows the thrust force in function of the PWM signal.

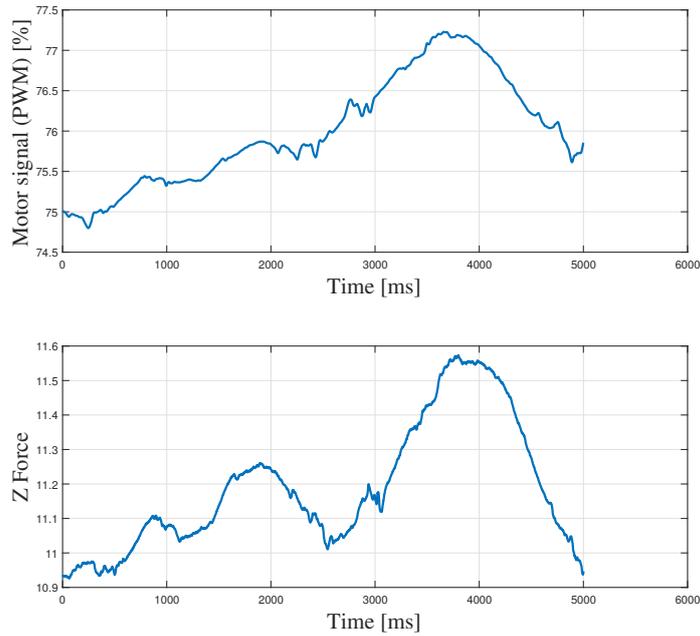


Figure 3. Input and output correlation graph

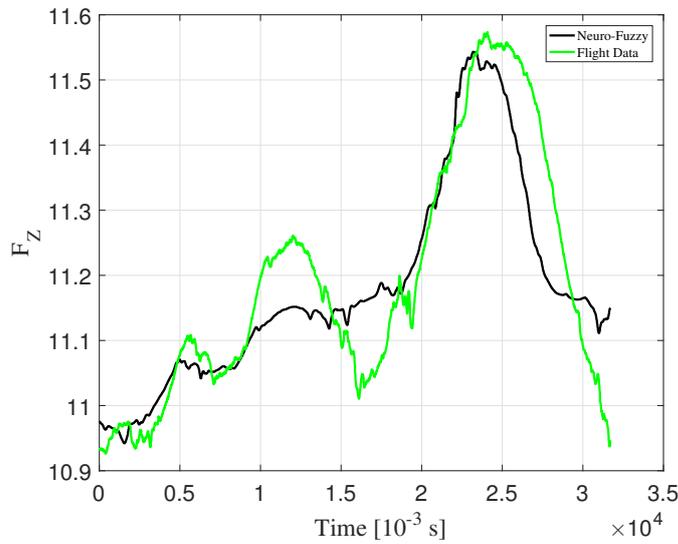


Figure 4. Neuro-Fuzzy training graph

Table 2. Input means and standard deviations values for  $F_Z$  training.

	PWM
Mean	$10^3 \times [5.9244, 6.0103, 5.8000, 5.9619, 6.0452, 6.2173]$
Std	$10^3 \times [0.0402, 0.1000, 0.1000, 0.0428, 0.0385, 0.0785]$

Table 3. Output zero order polynomial values.

	$F_Z$
Coefficient	10.0, 10.5, 10.6, 11.4, 11.6, 11.9

The derived of the curve presented in Fig. 6 gives the constant  $k$  from Eq. 6, which can be applied in the moment calculation.

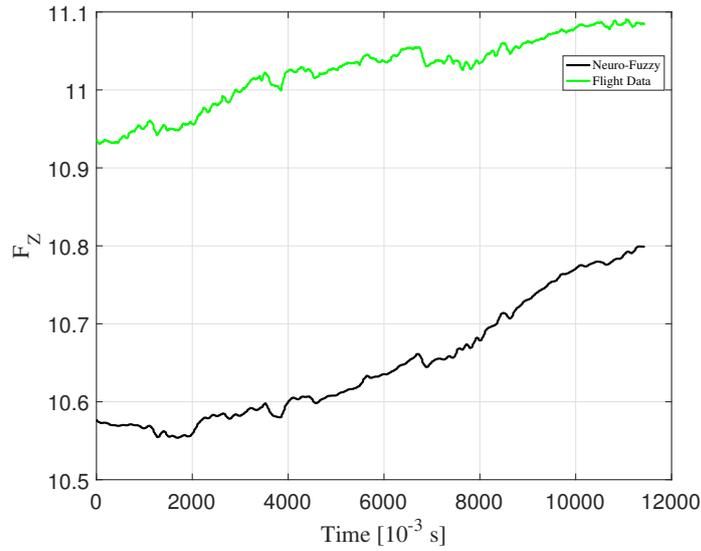


Figure 5. Neuro-Fuzzy validation graph

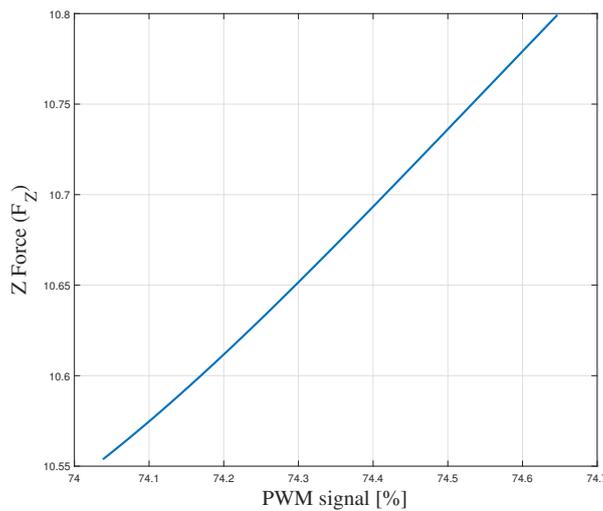


Figure 6. Correlation between Z force and PWM signal.

## 5. CONCLUSIONS

The authors built a model to describe the multi-copter dynamics. For that, the thrust force, presented in Sections 2.2.1 and 2.2.2, was modeled as a set of membership functions. After the modeling, the value of the constant  $k$  was estimated, and the moments could be calculated to represent the system behavior for several operational conditions, all provided by the Neuro-Fuzzy systems as described in Section 2.1. The training and validating data for the model estimation was acquired experimentally using a real quad-copter UAV. As a result, the model was fed with the control input variables, and had as output the thrust force. Once the dynamical model is based on experimental data, the presented work provided a model with higher fidelity which will be useful for flight simulators and control applications in future works.

## 6. ACKNOWLEDGEMENTS

The authors thanks the Graduate Program of the Faculty of Mechanical Engineering (FEMEC) and the financial support provided by the government agencies (CAPES, CNPq and FAPEMIG).

## 7. REFERENCES

- Brandon, J.M. and Morelli, E.A., 2012. “Nonlinear Aerodynamic Modeling From Flight Data Using Advanced Piloted Maneuvers-and Fuzzy Logic”. *NASA Center for AeroSpace Information*.
- Conroy, J., Kehlenbeck, A., Humbert, J.S., Conroy, J., Kehlenbeck, A., Humbert, J.S. and Nothwang, W., 2018. “Characterization and enhancement of micro brushless DC motor response”. , No. June 2014. doi:10.1117/12.2051263.
- Jang, J.S.R., 1992. “Fuzzy Controller Design without Domain Experts”. *IEEE International Conference on Fuzzy Systems*, pp. 289–296.
- Marques, F.M.M., 2018. *Modeling, Simulation and Control of a Generic Tilting Rotor Multi-Copter*. Master’s thesis, Federal University of Uberlândia.
- Pereira, B., 2017. *Aplicação de Sistemas Neuro-Fuzzy e Evolução Diferencial na Modelagem e Controle de Veículo de Duas Rodas*. Master’s thesis, Federal University of Uberlândia, Uberlandia, MG.
- Rajkumar, T. and Bardina, J., 2002. “Prediction of Aerodynamic Coefficients using Neural Networks for Sparse Data”. *NASA Center for AeroSpace Information*.
- Roskam, J., 2001. *Airplane Flight Dynamics and Automatic Flight Controls*. DARcorporation, Lawrence, KS, 3rd edition. ISBN 1884885179 9781884885174.
- Valavanis, K.P., 2007. *Advances in Unmanned Aerial Vehicles*, Vol. 33. Springer. ISBN 978-1-4020-6113-4.

## 8. RESPONSIBILITY NOTICE

The following text, properly adapted to the number of authors, must be included in the last section of the paper:  
The author(s) is (are) the only responsible for the printed material included in this paper.