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DIMENSIONAL SYNTHESIS OF THREE-AXLE SEMI-TRAILER STEERING MECHANISMS

Taynah Barbosa Brandão Lima
Gabriel Fontanelle Pereira
Fernando Vinicius Morlin
Marina Baldissera de Souza
Daniel Martins
Universidade Federal de Santa Catarina, Departamento de Engenharia Mecânica, Centro Tecnológico, Campus Universitário - Trindade, CEP: 88.040-900, Florianópolis/SC - Brasil
taynah.lima@posgrad.ufsc.br - gabriel.f.p@posgrad.ufsc.br - fernando.v.morlin@posgrad.ufsc.br - marina.bs@posgrad.ufsc.br - daniel.martins@ufsc.br

Abstract. *This paper presents the procedures for the dimensional synthesis of the steering systems of a three-axle semi-trailer for trucks, considering that only the central axle is non-steerable. The steering systems are four-bar mechanisms, whose steer angles are calculated using the Freudenstein Equation. However, they do not guarantee that an ideal condition, known as Ackermann condition, is met when the vehicle is turning. Therefore, the dimensional synthesis is done through optimization, aiming to define the dimensions both steering systems should have to the vehicle respects as much as possible the Ackermann condition when the truck is turning. The interior-point algorithm is used to minimize the RMS error between the steer angles obtained by the Ackermann condition and the Freudenstein Equation. Sizing via optimization reduces the RMS error by 90,6% and the resulting dimensions indicate that there is room for improvement.*

Keywords: *Steering systems, Trucks with multiple axles, Ackermann condition, dimensional synthesis, optimization*

1. INTRODUCTION

According to the National Transport Confederation, in Brazil, road transportation accounts for approximately 61.1% of the freight transport matrix. In this scenario, the cargo vehicles known as trucks, in their different settings, are of great importance, since most of the loads carried by this modal system use this type of vehicle.

All the 74 Combinations of Cargo Vehicles (CVC) homologated in Brazil by DENATRAN have different semi-trailers configurations with axles or non-steering wheels, with the exception of CVC's that use semi-trailers with three distanced axes, where at least one must be, necessarily, directional or self-directional, that is, with the ability to steer. According to Manenti (2018), the use of directional axles results in greater maneuverability, improved drivability and reduced wear on tires caused by the "drag" phenomenon due to high lateral forces present in the vehicle dynamics of this type vehicle.

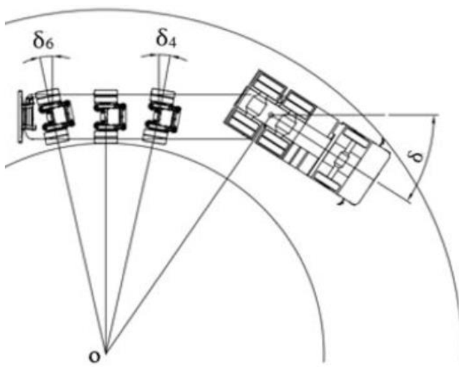

Figure 1: Three axles semi-trailer with central non-steering axle. Adapted from Manenti (2018).

There are several works in the literature of dimensional synthesis, among them Sleesongsom and Bureerat (2016) sized a rack-and-pinion steering linkage solving a multi-objective optimization problem to simultaneously minimize a steering error and a turning radius. Ettefagh and Javash (2014) determined the best function between input and output links of a four-bar Ackermann steering mechanism using two heuristic optimization methods and de Juan *et al.* (2009) synthesized three different steering mechanisms via an optimization method based on exact gradient determination. The length of the links were among the design variables. However, these studies evaluate vehicles with two axles, where the only steerable axle is the front-wheel.

The contribution of this paper is the dimensional synthesis of the steering systems of a vehicle with more than one steerable axle. The vehicle used as a case study is a three-axle semi-trailer for trucks, with central non-steering axle. An example of this kind of semi-trailer is shown in Fig. 1. The dimensional synthesis consists of an optimization problem where the objective function to be minimized is the RMS error between the steer angles defined by the Ackermann condition and the Freudenstein Equation. The design variables are the dimensions of the steering mechanisms.

The remainder of this paper is structured as follows: Section 2. briefly reviews the Ackermann condition and presents how the condition is defined for the three axles semi-trailer of Fig. 1. The steps to obtain the Freudenstein Equation are detailed in Section 3. The process of dimensional synthesis and the construction of the optimization are explained in Section 4. The solution of the dimensional synthesis is shown in Section 5. where the result of the optimization is also analyzed. Finally, Section 6contains some considerations about the results achieved in this paper.

2. ACKERMANN CONDITION

Consider a front-wheel-steering vehicle that is turning to the left, as shown in Fig. 2. When the vehicle is moving very slowly, there is a kinematic condition that relates the inner and outer wheels, allowing them to turn slip-free. The normal line to the center of each tire-plane must intersect at a common point and all the axles, except one, must be steerable in order to provide slip-free turning at zero velocity (Jazar, 2017). This geometric relation is called the Ackermann condition and is expressed by:

$$\cot \delta_o - \cot \delta_i = \frac{w}{l} \quad (1)$$

where the distance w between the steer axes of the steerable wheels is the *track*, the distance l between the front and real axles is the *wheelbase*, δ_i is the steer angle of the inner wheel and δ_o is the steer angle of the outer wheel.

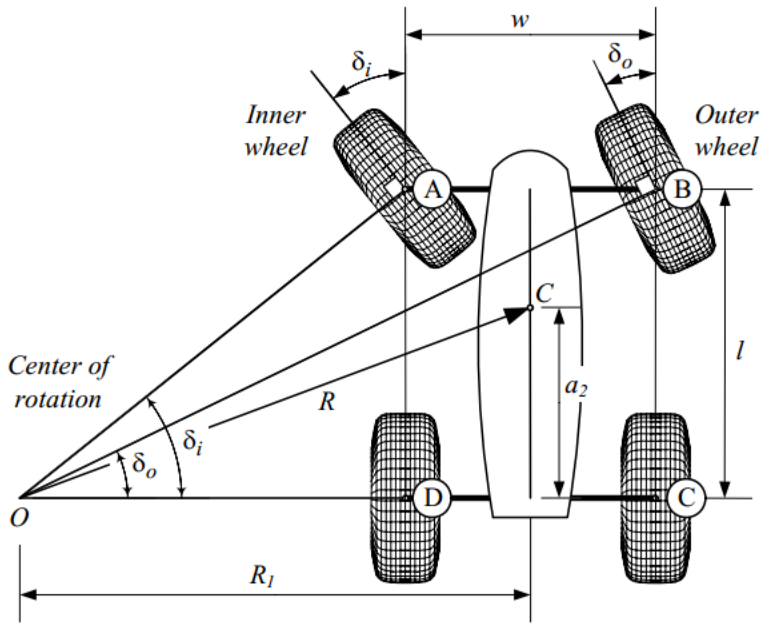

Figure 2: A front-wheel-steering vehicle and steer angles of the inner and outer wheels. Available at Jazar (2017).

For a n-axle vehicle with one non-steerable axle, there are $n-1$ geometric steering conditions. The present analysis was made considering a three-axle system, in which the first and the last one are steerable. The mass center C is assumed to be located over the non-steerable axle 2. For the Ackermann angles calculation, each steerable axle was separately analyzed in relation to the central axle, once it passes through the non-steerable axle, as shown in the Fig. 3.

The inner Ackermann angles can be defined as a function of the turning radius R, as follows:

$$\delta_{A_{i_1}} = \frac{a_1}{R - \frac{w}{2}} \quad (2)$$

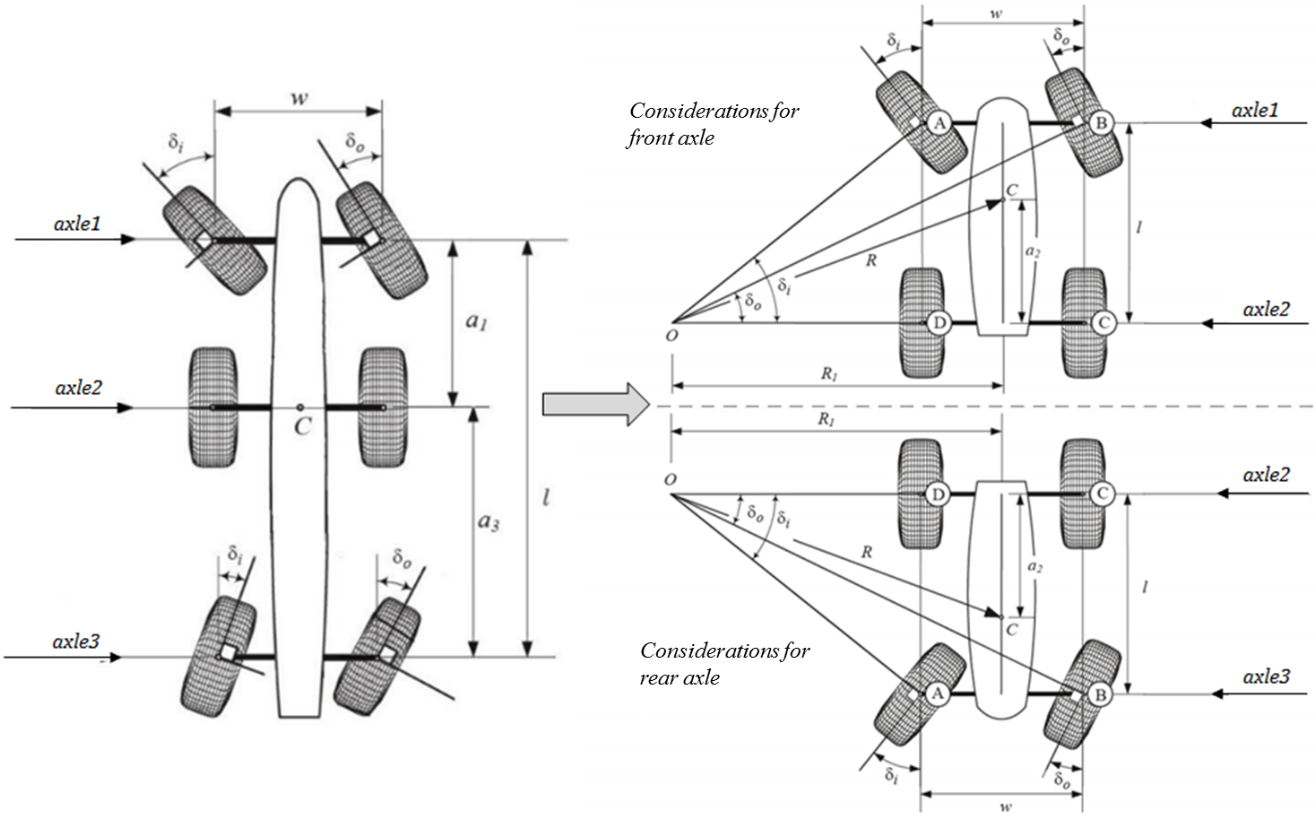

Figure 3: Driving dimensions of the 3 axles semi trailer. Adapted from Jazar (2017)

$$\delta_{A_{i_3}} = \frac{a_3}{R - \frac{w}{2}} \tag{3}$$

where a_i is the distance between the axle i and the mass center C.
Finally, we have the two Ackermann conditions, from which the outer angles can be calculated:

$$\cot\delta_{A_{O_1}} - \cot\delta_{A_{i_1}} = \frac{w}{a_1} \tag{4}$$

$$\cot\delta_{A_{O_3}} - \cot\delta_{A_{i_3}} = \frac{w}{a_3} \tag{5}$$

3. FREUDENSTEIN EQUATION

According to Jazar (2017), most of the mechanisms used in the mechanical sub-assemblies of vehicles are composed by the four-bar mechanism. This mechanism is illustrated in Fig. 4.

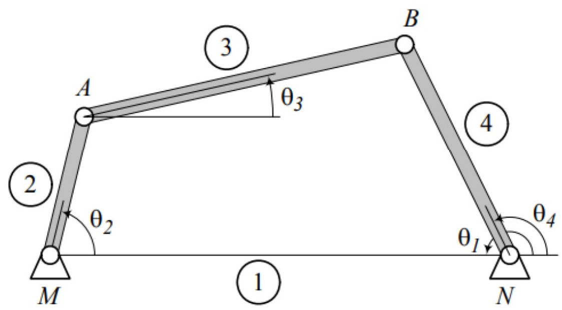

Figure 4: Four bar linkage. Available at Jazar (2017)

In this mechanism the link 1 (MN) is used as reference link, from which all angles and variables will be measured. The link 2 (MA) is called the input link, and it is controlled by the input angle θ_2. The link 4 is called the output link and its position is defined as a function of the output angle θ_4. The link 3 is called the coupler link, with angular position θ_3, connecting the links 2 and 4.

The angular position of the output link, θ_4, is a function of the length of the links and the value of the input variable θ_2. The angle θ_4 can be calculated using Eq. (6). to (9):

$$\theta_4 = 2\arctan(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}) \tag{6}$$

where:

$$A = J_3 - J_1 + (1 - J_2)cos\theta_2 \tag{7}$$

$$B = -2\sin\theta_2 \tag{8}$$

$$C = J_1 + J_3 - (1 + J_2)\cos\theta_2 \tag{9}$$

From the vector form of the four bars linkage it is possible to define the Eq. (10) to (12), where a is the length of the link 2, b is the length of the link 3, c is the length of the link 4 and d is the length of the link 1, i.e., the dimensions of the four-bar linkage.

$$J_1 = \frac{d}{a} \tag{10}$$

$$J_2 = \frac{d}{c} \tag{11}$$

$$J_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac} \tag{12}$$

At the same time, from Eq. (6) to (12), it is possible to derive the Freudenstein Equation, which is presented in Eq. (13). With this equation it is possible to calculate the coupling angles and output angles, depending on the input angle, as long as the link lengths of the four-bar mechanism are known.

$$J_1\cos\theta_4 - J_2\cos\theta_2 + J_1 = \cos(\theta_4 - \theta_2) \tag{13}$$

In order to use the Freudenstein Equation, Fig. 4 has been adapted, as shown below in Fig. 5.

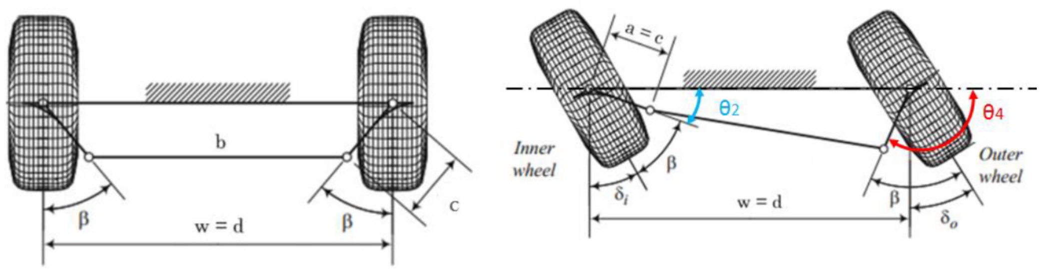

Figure 5: Adaptation of the nomenclature of dimensions of the four bar linkage. Adapted from Jazar (2017).

After these considerations, we can calculate the steer angles of the wheels as a function of β, θ_2 and θ_4. We have:

$$\delta_{F_i} = 90° - \theta_2 - \beta \tag{14}$$

$$\delta_{F_O} = 90^\circ + \beta - \theta_4 \tag{15}$$

Where δ_{F_i} is the steer angle of the inner wheel and the δ_{F_O} is the steer angle of the of the outer wheel. In addition, the dimension b of Fig. 5 is determined as a function of w, β and a, we have:

$$b = w - 2a\sin\beta \tag{16}$$

4. DIMENSIONAL SYNTHESIS

The dimensional synthesis aims to define the dimensions of the steering mechanisms that make the difference $\delta_{F_O} - \delta_{A_O}$ as close as possible to zero, i.e., make the steer angles defined by the Freudenstein Equation as close as possible to the ones defined by the Ackermann condition. The dimensional synthesis is done via an optimization process, whose objective function is the weighted mean of the RMS error of each steerable axle:

$$f_{obj} = P_1 e_{axle_1} + P_3 e_{axle_3} \tag{17}$$

The weights of Eq. (17) are the ratio between the distance of the steerable axles from the non-steerable axle and the vehicle's length, i.e. $P_1 = a_1/l$ and $P_3 = a_3/l$.

The RMS error for each steerable axle is defined by Eq. (18), where n is the number of points considered.

$$e_{axle_k} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(\delta_{F_{O_k}j} - \delta_{A_{O_k}j})^2}, \text{ for } k = 1, 3 \tag{18}$$

The value adopted is $n = 95$, representing 95 different values of inner angles obtained from Eq. (2) and (3), having as input a range of turning radius 3 m $\leq R \leq$ 50 m. The turning radius increases with a step of 0.5 m. It is assumed that the inner angles for the Ackermann condition and the angle θ_2 in Freudenstein Equation are the same, leading to the definition of δ_{F_i} by Eq. (14) for the two steerable axles. Knowing the inner angles, the outer angles for the Ackerman condition δ_{A_O} are defined by Eq. (4) and (5). The outer angles δ_{F_O} for both steerable axles considering the Freudenstein Equation are defined using Eq. (15).

The design variables of the optimization problem are a_1, a_3, β_1 and β_3, considering Fig. 5. The variables a_1 and a_3 correspond to the offset arm length of the steering mechanism of the front and rear axles, respectively. The angle of the front arms is represented by β_1 and the angle of the rear arms is represented by β_3. The initial value for the angles β_1 and β_3 indicates that the vehicle has a straight line trajectory. The dimensions w and l are fixed and defined as $w = 2.45$ m and $l = 2.5$ m.

The objective function is subject to the lower and upper limits of these design variables. The lengths a_1 and a_3 must be within the range 0.1 m $\leq a_k \leq$ 1 m and the angles β_1 and β_3 within $0^\circ \leq \beta_k \leq 60^\circ$, for $k = 1, 3$. The initial values adopted for the design variables are $a_{1_0} = a_{3_0} = 0,2$ m and $\beta_{1_0} = \beta_{3_0} = 10^\circ$.

Therefore, the optimization problem can be written as:

Min $f_{obj}(\vec{x})$ $\quad \vec{x} = \{a_1, a_3, \beta_1, \beta_3\}^T$
Subject to

$$\begin{cases} 0.1 \text{ m} - a_1 \leq 0 \\ 0.1 \text{ m} - a_3 \leq 0 \\ 0^\circ - \beta_1 \leq 0 \\ 0^\circ - \beta_3 \leq 0 \\ a_1 - 1 \text{ m} \leq 0 \\ a_3 - 1 \text{ m} \leq 0 \\ \beta_1 - 60^\circ \leq 0 \\ \beta_3 - 60^\circ \leq 0 \end{cases}$$

The interior-point algorithm (Wright, 2005) is used to minimize the objective function of Eq. (17).

5. RESULTS AND DISCUSSION

Figures 6 and 7 show the steer angle of the outer wheel calculated by the Ackermann condition and the Freudenstein Equation using as input data the steer angle of the inner wheel. Figure 6 concerns the front axle (axle 1) and Fig. 7, the rear axle (axle 3). Since axle 3 is behind the axle where the displacement of the center of the curve occurs, as shown in Fig. 3, the rear wheels steer towards the opposite side of the front wheels. In this way, the steer angles of the rear wheels are negative when considering the same reference adopted for the front wheels. That is the reason why the ordinate axis of Fig. 7 contains negative values.

The slashed lines in Fig. 6 and 7 represent the Ackermann condition. The values obtained by the Freudenstein Equation are plotted before and after optimization. The use of the initial values of the design variables in Freudenstein Equation results in the slash-dotted lines of Fig. 6 and 7, corresponding to the steer angle of the outer wheels before optimization. The divergence between these two curves correspond to an initial weighted RMS error of 2.8307.

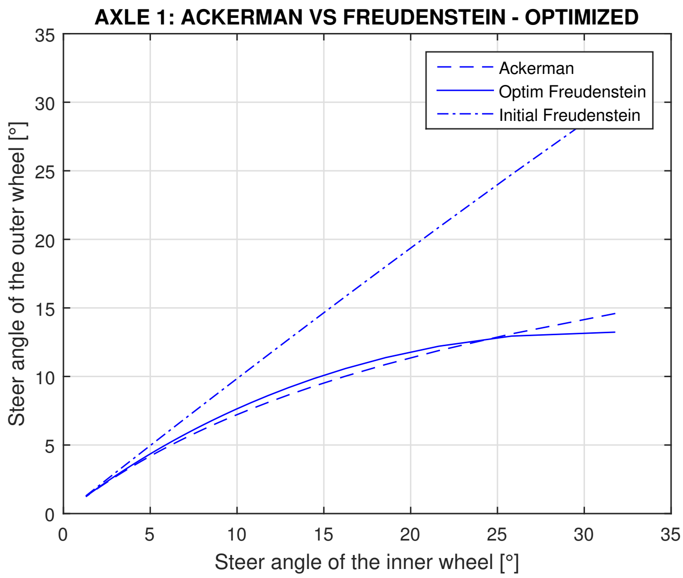

Figure 6: Optimization solution for the front axle.

The optimization solution generates the full-line curves of Fig. 6 and 7. The algorithm converges after 57 iterations to the local minimizer shown in Tab. 1, giving a weighted error of 0.2660.

Table 1: Dimensions of the steering mechanisms

a_1	β_1	a_3	β_3
0.1244 m	57.0833 °	0.1000	50.5726 °

By applying the values of Tab. 1 in the steering mechanisms, there is a strong approximation of the full-lines curves to the Ackermann curves in both Fig. 6 and 7. This represents the reduction of around 90,6% in the error and it means that the behaviour of the steering mechanisms when the vehicle is turning a curve is closer to ideal.

This behaviour is illustrated in Fig. 8, obtained using the dimensions of Tab. 1. Figure 8 shows the variation of the steer angle of the outer wheel for axles 1 and 3, comparing the values obtained by the Ackermann condition and the Freudenstein Equation, as the curve radius changes. The dashed lines correspond to the Ackermann condition and the full lines correspond to results obtained via Freudenstein Equation. It can be verified that the results obtained by the Ackermann condition and by the Freudenstein Equation are very close when computed from the values of Tab. 1. Thus the sizing of the steering systems of the three-axle semi-trailer result in a vehicle able to turn practically slip-free.

6. CONCLUSIONS

The paper presents the procedures for the dimensional synthesis of the steering systems of a three-axle semi-trailer for trucks whose central axle is non-steerable. The steer angles of the four-bar linkage steering systems are calculated according to the Freudenstein Equation and a dimensional synthesis is done via optimization, using the interior-point algorithm, in order to minimize the RMS error between the Ackermann steer angles and the angles obtained by the Freudenstein Equation.

The optimization reduces the RMS error by 90,6%, which means that the behaviour of the steering mechanisms, when

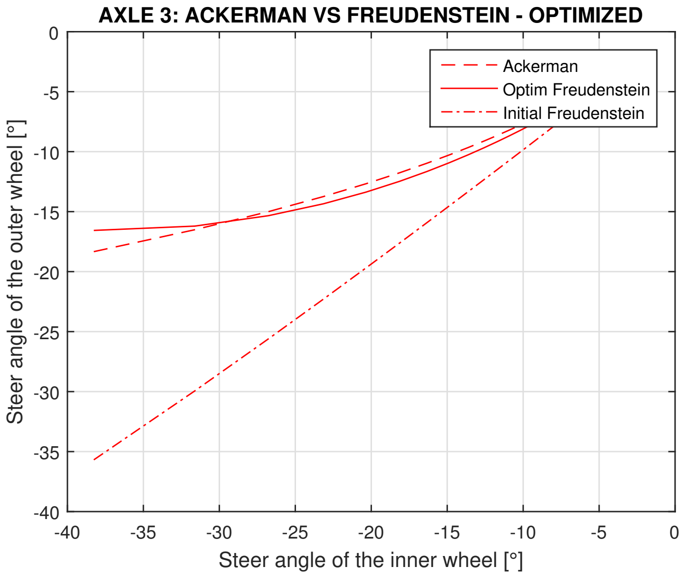

Figure 7: Optimization solution for the rear axle.

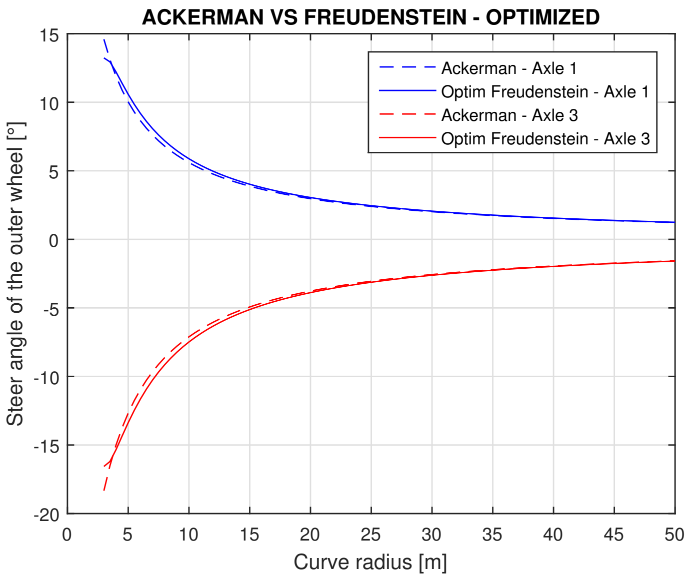

Figure 8: Outer wheel's steer angle defined by Ackermann condition and by the optimization according to curve radius.

the vehicle is turning, is closer to ideal. The algorithm converged to values close to boundary points for both axles, an indication that some progress in performance can still be made. However, dramatic changes in the boundaries could take the problem to values that are not consistent with the real constraints, i.e, even if a smaller error is found, the resulting mechanism could not be achieved in the real world. Therefore, the decision was taken to keep the initially adopted constraints, since the optimized geometry is close to the desired behavior.

Future works include the dimensional synthesis of three-axle semi-trailer steering systems adopting another type of mechanical solution, such as a rack-and-pinion for both the steerable axles, or a combination of a four-bar mechanism and a rack and pinion, for instance. In addition, the same procedure presented here can be assumed to the synthesis of semi-trailer steering systems with a higher number of axles. Finally, the position of the mass center C was considered to be located over the central axles, what might not be a very realistic approach. Thus, the mass distribution of the whole system, including the truck cab and cargo, could be taken into consideration.

7. ACKNOWLEDGEMENTS

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