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# ACTUATOR AND SENSOR PLACEMENT FOR CLOSED-LOOP CONTROL OF CONVECTIVE INSTABILITIES. PART 2: 3D TOLLMIEN-SCHLICHTING WAVES

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**Abstract.** *This work deals with the characterization of the closed-loop control performance aiming at the delay of transition in a 3D Blasius boundary layer. The objective of this study is to explore how the sensor-actuator placement affects the optimal control problem, formulated using linear quadratic gaussian (LQG) regulators through reduced order models. This will be carried out by evaluating errors of the optimal estimator at positions where control gains are significant through the  $\gamma$  metric, analysing why some choices of sensor-actuator placement are more effective than others for flow control.*

**Keywords:** *3D Blasius boundary layer, Closed-loop control, Sensor-actuator placement*

## 1. INTRODUCTION

The positioning of sensors and actuators in a 3D Blasius boundary layer includes a degree of freedom in the problem, as they are displaced in a plane and not in a line, as in the 2D case. The usual straight lines of sensors and actuators not necessarily present the best performance, and a systematic investigation of different positionings is proposed through the  $\gamma$  metric. The high dimensionality of the problem begins to pose serious constraints on the investigation and becomes a even more critical aspect of the problem.

## 2. METHODS

### 2.1 Model Reduction

A model reduction is often necessary for the application of optimal control for fluid systems, due to the high dimension of the problem (Rowley and Dawson, 2017). The Navier-Stokes equations for the 3D boundary layer are given by

$$\begin{aligned} \frac{\partial q}{\partial t} &= -(U \cdot \nabla)q - (q \cdot \nabla)U - \nabla p + Re^{-1}\nabla^2 q \\ \nabla \cdot q &= 0 \end{aligned} \quad (1)$$

where  $q = (v_x, v_y, v_z)'$  is the velocity,  $p$  the pressure,  $U$  the free-stream velocity and  $Re$  the Reynolds number. The linearized system can be written in state-space form using the Orr-Sommerfeld and Squire equations (Schmid and Henningson, 2001; Jovanovi and Bamieh, 2005; Kim and Bewley, 2007), and thus the pressure does not need to be absorbed in the state  $q$ .

As the order of the linearized version of the Navier-Stokes equations is at least  $10^5$  (Semeraro *et al.*, 2011), the linear control techniques are not only costly computationally, but provide controllers of equivalent order, which in practice are of no interest for real time applications. This imposes the necessity for model/controller reduction, a widespread need in flow control.

The model reduction is dependent on the choice of actuator (through  $B_u$ ) and sensor ( $C_y$ ), and, in principle, each choice of sensor/actuator configuration may lead to a different reduced-order model. We circumvent this issue, at least partially, by using the controllability or proper orthogonal decomposition (POD) modes (Zhou *et al.*, 2004; Rowley, 2005;

Taira *et al.*). The set of POD modes depends only on  $B = [B_d \ B_u]$ , and is thus the same for a fixed actuator position; all the different sensor positions can be explored with a single set of modes for a specific actuator position. In this sense, the POD model reduction is preferred here over the balanced POD (Zhou *et al.*, 2004; Rowley, 2005) eigensystem realization algorithm (ERA) (Ma *et al.*, 2011) model reduction approaches, as the latter methods would require a new reduced order model for each sensor and actuator positioning.

The first step is to obtain the controllability gramian

$$P = \int_0^{+\infty} e^{At} B B^T e^{A^T t} dt \quad (2)$$

provided by the solution of the Lyapunov equation

$$AP + PA^T + BB^T = 0 \quad (3)$$

Performing the singular value decomposition on  $P$  leads to

$$P = U \Sigma U^T = \left[ \dots \mid \phi_v \mid \dots \right] \begin{bmatrix} \ddots & & \\ & \sigma_{cv} & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \frac{\phi_v^T}{\sigma_{cv}} \\ \vdots \end{bmatrix} \quad (4)$$

where  $\phi_{*v}$  are the controllability modes, which form an orthonormal basis that is optimal in representing the state of the system forced with white noise (Farrel and Ioannou, 1993; Dergham *et al.*, 2013). The order of the reduced model  $r$  must be chosen such that  $r \leq \text{rank}(\Sigma)$ . We can then define a transformation matrix  $T$  by

$$T = U(:, 1:r) \quad (5)$$

where  $U(:, 1:r)$  are the first  $r$  columns of matrix  $U$ . A reduced order model of order  $r$  is then obtained as

$$\begin{cases} \dot{\tilde{q}}(t) = T^T A T \tilde{q}(t) + T^T B_u u(t) + T^T B_d d(t) \\ z(t) = C_z T \tilde{q}(t) \\ y(t) = C_y T \tilde{q}(t) + n(t) \end{cases} \quad (6)$$

where  $\tilde{q} = T^T q$ .

For high order systems, as usually is the case in flow control, the solution of the Lyapunov equation (3) directly is too costly computationally. Then, the POD modes can be obtained using a snapshot method using impulse responses, as described in (Rowley, 2005; Taira *et al.*).

### 3. Results

The simulations of the 3D Blasius boundary layer are being carried out with the SIMSON pseudo-spectral solver (Chevalier *et al.*, 2007). The size of the domain used was  $l_x \times l_y \times l_z = 1000 \times 30 \times 240$ , with a fringe of length  $L_f = 200$ . Non-dimensional variables are used, considering the free-stream velocity and the inflow displacement thickness as reference velocity and length, respectively. With these definitions, the Reynolds number is 1000. The discretization in the horizontal axis was done through a Fourier series expansion with  $n_x = 768$  modes, and in the normal axis through a Chebyshev expansion with  $n_y = 101$  and  $n_z = 96$  as the highest order of the polynomials. The disturbance  $B_d$  was placed at  $(x_d, y_d, z_d) = (35, 0, 0)$ , the actuator at  $(x_u, y_u, z_u) = (400, 0, 0)$  and the sensor at  $(x_y, y_y, z_y) = (300, 0, 0)$  (Fig. 1). The actuators are localized volume forcings, as described in Semeraro *et al.* (2013) and the sensors are localized stress sensors.

Using snapshots of the impulse responses from  $B_u$  and  $B_d$ , a reduced order model is being obtained through the POD model reduction (Sec 2.1). The first POD mode is shown in Fig. 2. Due to the high order of the system, it is a computationally and time demanding procedure, that must be taken completely until the reduced order model is obtained and finally verified if represents the full order system properly. As Fig. 3 shown, no stable reduced order model has been obtained so far, unabling the calculation of the LQR and LQE gains for the 3D boundary layer. Nevertheless, it shows a consistent trend showing that a higher number of modes may eventually lead to a stable model.

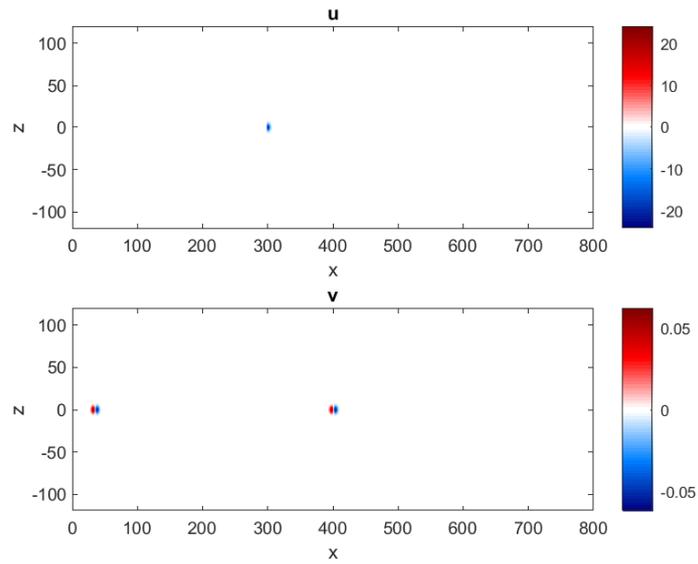


Figure 1: The components  $u$  and  $v$  of the sensors and actuators at  $y = 0.0074$ . The component along  $w$  is identically zero. View from above.

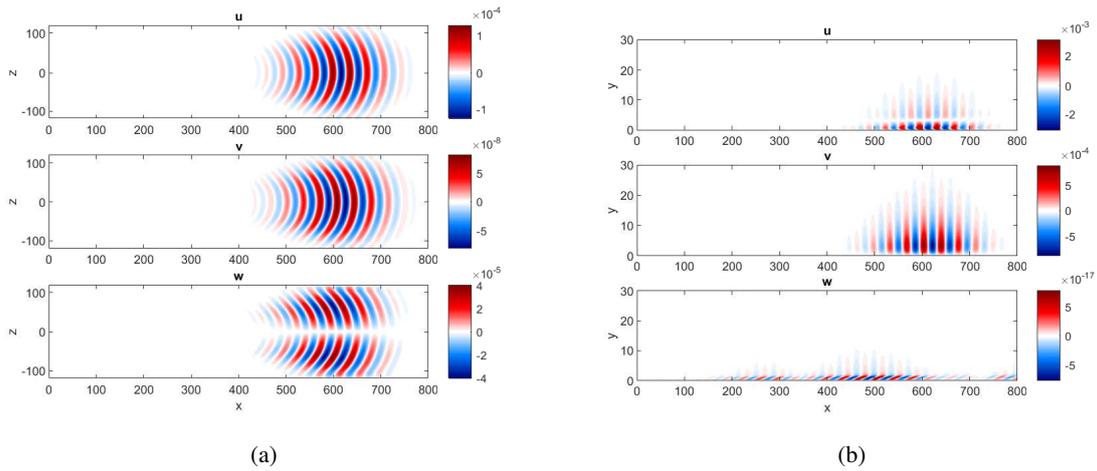


Figure 2: The first POD mode at (a)  $y = 0.0074$ , view from above (b)  $z = 0$ , view along the spanwise direction.

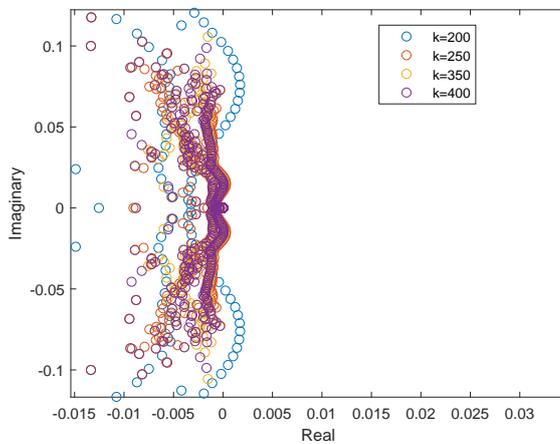


Figure 3: Eigenspectrum of the reduced order model matrix  $T^T A T$  for  $k$  POD modes.

#### 4. ACKNOWLEDGEMENTS

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