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DESIGN AND ANALYSIS OF ACTIVE CONTROL TECHNIQUES FOR STICK-SLIP SUPPRESSION IN ROTARY DRILLING SYSTEMS

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Abstract. *Vibrations induced by stick-slip are a harmful phenomenon that can reduce effectiveness of the drilling processes and cause damage to drillstring components. A possible way to overcome this problem is to apply active control techniques. This work intends to design, analyze and compare a traditional Proportional-Integral (PI) controller and an alternative Optimal Static Output Feedback (OSOF) controller applied to the drilling processes. To implement the OSOF controller, a translated modal form is developed, which transforms the problem of tracking the desired system velocity into a problem of asymptotic stabilization about the origin. Simulations are performed for the nominal model and cases in which an uncertainties range is considered for the friction parameters of the bit-rock interaction. Preliminary results indicate that the OSOF controller proposed present better performance and lower sensitivity to parameter variations.*

Keywords: *stick-slip, oil well drilling, torsional vibration control*

1. INTRODUCTION

The processes of oil and gas exploration consists on three basic steps: prospecting, drilling and extraction. Due to the rising demand for oil products, the areas along the earth's surface became intensively explored, and there was a need to search for deeper wells that presented more technological difficulties for their extraction. Deeper oil wells are also more susceptible to failure, especially in the drilling processes (Vromen, 2015). The large financial losses to the oil industry and the great theoretical and practical challenges posed by the drilling processes have motivated several researches in this area, which have mostly considered the application of control techniques to mitigate or suppress vibrations (Patil and Teodoriu, 2013; Saldivar *et al.*, 2016; Zhang *et al.*, 2017).

The usual structure of a drilling system consists of a top drive at surface, which transmits a torque through the drill-strings to the drill-bit. The drill-bit is part of the Bottom Hole Assembly (BHA), which is also composed of stabilizers and drill collars. The latter are made of thick-walled tubes in order to prevent underbalancing of the whole mechanism. A schematic representation of this system is depicted in Fig. 1.

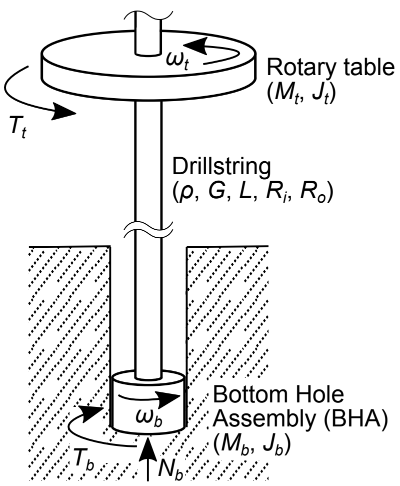

Figure 1. Schematic representation of a drilling rotary system.

The general dynamics of this mechanism involves lateral, axial and torsional vibrations, besides a nonlinear interaction between the drill-bit and rock formation that may cause coupling of these modes (Saldivar *et al.*, 2016). In addition, the interactions between bit-rock, drillstring-fluid and drill collars-wellbore have parameters whose nature is stochastic or unmeasurable. The representation of all the phenomena described is difficult to capture by physical and computational models, and in cases where the representation is reliable, the complexity of the model makes the synthesize of control laws unfeasible. For these reasons, the models employed for control consider vibrations in different directions in decoupled form (Liu, 2015; Monteiro and Trindade, 2017).

A harmful type of phenomenon that can occur due to torsional vibrations is the stick-slip. During the stick-slip, the bit speed fluctuation can vary from zero in the stick phase to more than twice that of the rotary table during the slip phase (Chen *et al.*, 2002), which may cause fatigue failure on drill collars, damage to the drill bit, and slow down the drilling process (Elsayed *et al.*, 1997). Field measures also shows that stick-slip can occur up to 50% of the drilling process (Henneuse, 1992). These factors motivated several researchers to propose different control techniques to reduce or eliminate stick-slip in the drilling process (Patil and Teodoriu, 2013; Zhu *et al.*, 2014).

In this paper, it is considered the mitigation of the stick-slip phenomenon using active control techniques. An optimal static output feedback control (OSOF), which is based on Cruz Neto (2018), is developed and compared to a traditional Proportional-Integral (PI) control. In order to implement the OSOF controller, which is based on the linear quadratic regulator (LQR), a modal form with a translation of the desired configuration to the origin is considered, such that the problem of tracking the desired bit velocity is transformed into a regulation problem. For the PI controller, the control gains are chosen such that a criterion given by the average deviation from the drill-bit target angular velocity is minimized. Finally, a sensitivity analysis is done to evaluate and compare both controllers considering stability and performance when parameters of the bit-rock interaction are allowed to belong to a certain uncertainties range.

2. DRILLING SYSTEM MODEL

The drillstring is assumed to behave as a circular shaft with constant geometric and physical properties. The rotary table and BHA are assumed to be rigid bodies attached to drillstring ends as indicated in Fig. 1, also with constant properties. The values of these properties were taken from (Monteiro and Trindade, 2017) and are indicated in Tab. 1.

Table 1. Geometrical and material parameters considered for the drilling system.

Property	Value
Drillstring mass density, ρ (kg m^{-3})	8010
Drillstring shear modulus, G (GPa)	79.6
Drillstring length, L (m)	3000
Outer radius of drillstring, R_o (m)	0.0635
Inner radius of drillstring, R_i (m)	0.0543
Driving table rotary inertia, J_t (kg m^2)	500
BHA rotary inertia, J_b (kg m^2)	394

This system was modeled using ten finite elements with Hermite cubics as interpolation functions, in order to improve convergence. The equations of motion were transformed into a modal form, adding a damping factor of 1% for every mode. Given the hypothesis that the sensors do not change the mass and stiffness of the system, a continuous model was constructed using the approximate eigenfunctions in order to simplify sensors placement in the OSOF controller design. These functions were calculated as in the Rayleigh-Ritz method since, except for convergence and embedding properties of mass and stiffness matrices, the FEM can be treated as a Rayleigh-Ritz method (Meirovitch, 1997). The non-regularized dry friction model given in Eq. (1) was considered for the bit-rock interaction, with T being the torque transmitted to the BHA by the drillstring, a_1, a_2 and β the dry friction parameters and the remaining parameters indicated in Fig. 1. An illustration of the dry friction as a set valued map of the bit angular velocity is presented in Fig. 2. The normal force N_b was assumed to be equal to the weight-on-bit (WOB). For this study, the values considered for the WOB were 80 kN and 120 kN. The dry friction parameters associated with these WOB were taken from (Monteiro and Trindade, 2017).

$$T_b = \begin{cases} T, & \text{for } |\omega_b| \leq \delta \text{ and } |T| \leq a_2 N_b, \\ a_2 N_b sgn(T), & \text{for } |\omega_b| \leq \delta \text{ and } |T| > a_2 N_b, \\ \left[a_1 + (a_2 - a_1)e^{-\beta|\omega_b|}\right] N_b sgn(\omega_b), & \text{for } |\omega_b| > \delta \end{cases} \tag{1}$$

These assumptions were used to construct the following state space model:

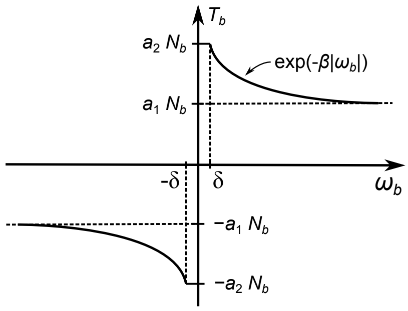

Figure 2. Illustration of the dry friction function.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_c T_t + \mathbf{B}_r T_b$$
$$\mathbf{x} = \begin{bmatrix} \boldsymbol{\eta} \\ \dot{\boldsymbol{\eta}} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\boldsymbol{\Lambda} & -\mathbf{D} \end{bmatrix}, \quad \mathbf{B}_c = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\phi}(0) \end{bmatrix}, \quad \mathbf{B}_r = \begin{bmatrix} \mathbf{0} \\ -\boldsymbol{\phi}(L) \end{bmatrix} \tag{2}$$

in which $\boldsymbol{\eta} \in \mathbb{R}^n$ is the vector of modal displacements, $\boldsymbol{\Lambda} \in \mathbb{R}^{n\times n}$ is a diagonal matrix of system eingenvalues or natural frequencies squared, $\mathbf{D} \in \mathbb{R}^{n\times n}$ is a diagonal matrix of damping and $\boldsymbol{\phi} \colon \mathbb{R} \to \mathbb{R}^n$ is the vector of approximated eigenfunctions.

3. CONTROLLER DESIGN

The controller objective is to maintain the whole system rotating at a constant speed ω_{ref} and to avoid oscillations. This desired equilibrium configuration is defined in terms of systems states as $\mathbf{x}_{eq} = [\boldsymbol{\eta}_{eq}\ \dot{\boldsymbol{\eta}}_{eq}]^\mathsf{T}$. In order to reach this configuration, the applied torque is decomposed into a feedforward constant component $\tilde{u}$, inducing $\mathbf{x}_{eq}$, and a feedback component u to avoid oscillations, such that $T_t = \tilde{u} + u$. The constant parameters $\mathbf{x}_{eq}$ and $\tilde{u}$ can be obtained from the equilibrium condition of Eq. (2):

$$\boldsymbol{\Lambda}\boldsymbol{\eta}_{eq} + \mathbf{D}\dot{\boldsymbol{\eta}}_{eq} - \boldsymbol{\phi}(0)\tilde{u} + \boldsymbol{\phi}(L)T_b(\omega_{ref}) = \mathbf{0} \tag{3}$$

in which T_b is evaluated at the angular velocity $\omega_b = \omega_{ref}$ using Eq. (1). Since the modal displacement associated with the rigid body mode cannot be determined using Eq. (3) and it also does not affect the equations of motion, it can be eliminated from state space equations. Next, the change of coordinates

$$\boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{\xi}_1 \\ \boldsymbol{\xi}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\eta}' - \boldsymbol{\eta}'_{eq} \\ \dot{\boldsymbol{\eta}} - \dot{\boldsymbol{\eta}}_{eq} \end{bmatrix} = \mathbf{x} - \mathbf{x}_{eq} \tag{4}$$

in which $\boldsymbol{\eta}'$ and $\boldsymbol{\eta}'_{eq}$ are vectors of modal displacements without the rigid body mode, is proposed to characterize the state error. In these coordinates, Eq. (2) can be rewritten as:

$$\dot{\boldsymbol{\xi}} = \mathbf{A}'\boldsymbol{\xi} + \mathbf{B}'_c u + \mathbf{B}'_r \left[T_b\left(\boldsymbol{\phi}^\mathsf{T}(L)\boldsymbol{\xi}_2 + \boldsymbol{\phi}^\mathsf{T}(L)\dot{\boldsymbol{\eta}}_{eq}\right) - T_b\left(\boldsymbol{\phi}^\mathsf{T}(L)\dot{\boldsymbol{\eta}}_{eq}\right)\right] \tag{5}$$

in which $\mathbf{A}'$ is the matrix $\mathbf{A}$ without the first column and row, and $\mathbf{B}'_r$ and $\mathbf{B}'_c$ are the vectors $\mathbf{B}_r$ and $\mathbf{B}_c$ without the first element. In this form, the problem of tracking the desired velocity ω_{ref} is transformed into a problem of asymptotic stabilization about the origin. This form will be particularly interesting for the design of the OSOF controller, since it relies on a quadratic regulation problem.

3.1 Proportional-Integral control

The controller proposed here to maintain the desired velocity ω_{ref} is a simple proportional-integral (PI) that uses only cinematic information of the rotary table for feedback. Equation (6) describes the form of this controller in physical coordinates.

$$u = k_p(\omega_{ref} - \omega_t) + k_i(\omega_{ref} t - \theta_t) \tag{6}$$

The controller parameters k_i and k_p are chosen such that the cost function defined in terms of the average deviation from the drill-bit target velocity is minimized:

$$J = \frac{1}{\Delta t} \int_0^{\Delta t} \frac{|\omega_b - \omega_{ref}|}{\omega_{ref}} \mathrm{d}t \tag{7}$$

The optimal control parameters k_i and k_p are dependent on system initial conditions and the time Δt considered for simulation. In this paper, the values for system initial conditions and time simulation were chosen such that it was possible to observe the stick-slip phenomenon and the suppression of vibrations. The particle swarm optimization method was used to find the optimal values of control parameters with respect to the cost function given in Eq. (7).

3.2 Optimal Static Output Feedback control

The optimal static output feedback control (OSOF) is a suboptimal controller based on the linear quadratic regulator (LQR). Based on the LQR problem of need for measurability of all state variables and the LQG problems of unguaranteed stability margins, Levine and Athans (1970) suggested a technique that still consists on minimizing the LQR quadratic cost function but with the constraint of using only measured signals for feedback. The problem with this technique is that the optimal control gains are dependent on the dynamical system initial conditions. Recently, Cruz Neto (2018) proposed a simultaneous optimization of control gains and sensors locations together with an alternative solution to the initial conditions dependence problem:

$$\min_{(\mathbf{K},\boldsymbol{\alpha})} \max_{\mathbf{x}_0} \frac{\mathbf{x}_0^{\mathsf{T}} \mathbf{P}_o(\mathbf{K},\boldsymbol{\alpha}) \mathbf{x}_0}{\mathbf{x}_0^{\mathsf{T}} \mathbf{P}_l \mathbf{x}_0} \tag{8}$$

in which $\mathbf{x}_0$ is the system initial condition, $\mathbf{K}$ is the control gain, $\boldsymbol{\alpha}$ represents sensors locations and $\mathbf{P}_o$ and $\mathbf{P}_l$ are solutions of following Riccati and Lyapunov equations:

$$\mathbf{A}^{\mathsf{T}}\mathbf{P}_l + \mathbf{P}_l\mathbf{A} + \mathbf{Q} - \mathbf{P}_l\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathsf{T}}\mathbf{P}_l = 0 \tag{9}$$

$$\mathbf{A}_c^{\mathsf{T}}\mathbf{P}_o + \mathbf{P}_o\mathbf{A}_c + \mathbf{Q} + \mathbf{C}(\boldsymbol{\alpha})^{\mathsf{T}}\mathbf{K}^{\mathsf{T}}\mathbf{R}\mathbf{K}\mathbf{C}(\boldsymbol{\alpha}) = 0 \tag{10}$$

in which $\mathbf{Q}$ and $\mathbf{R}$ are the usual weighting matrices of LQR control, $\mathbf{A}_c$ is the closed loop matrix and $\mathbf{C}(\boldsymbol{\alpha})$ is the output matrix. This optimization consists on approaching performances of the LQR and OSOF controllers for any initial condition. Details on its solution can be found in (Cruz Neto, 2018). Since this formulation should be applied for linear systems, a linearization of Eq. (5) is required. The number of sensors available for optimization must be pre-established, and for the problem studied in this paper two sensors were used. Due to the limitations regarding sensors placement in deep oil wells, the range for sensors locations was set preliminarily at 10% of the drillstring length. The feedback control action using only OSOF control is described as:

$$u = \mathbf{K}\mathbf{C}(\boldsymbol{\alpha})\boldsymbol{\xi} \tag{11}$$

in which the form of the matrix $\mathbf{C}(\boldsymbol{\alpha})$ depends on the type of sensors used. For the problem studied here, it was considered that both angular displacements and velocities could be obtained from the signals of each sensor, such that the matrix $\mathbf{C}(\boldsymbol{\alpha})$ has the form:

$$\mathbf{C}(\boldsymbol{\alpha}) = \begin{bmatrix} \boldsymbol{\phi}'(\alpha_1) & \boldsymbol{\phi}'(\alpha_2) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\phi}(\alpha_1) & \boldsymbol{\phi}(\alpha_2) \end{bmatrix}^{\mathsf{T}} \tag{12}$$

with $\boldsymbol{\phi}'$ being the vector $\boldsymbol{\phi}$ without the first element.

As the constant feedforward control $\tilde{u}$ to maintain the drilling system at the desired equilibrium is a function of the model parameters, the presence of uncertainties can shift the steady state configuration to an undesired point. In order to deal with this problem, an integral control was added to regulate the error to zero, since it ensures regulation for all parameters perturbations that do not destroy the stability of the closed loop system (Khalil, 2002). The final form of the feedback control is:

$$u = \mathbf{K}\mathbf{C}(\boldsymbol{\alpha})\boldsymbol{\xi} + k_i(\omega_{ref}t - \theta_t) \tag{13}$$

4. RESULTS

For all the simulations performed, it was considered as an initial condition that the entire system was rotating undeformed at a constant speed of 70 rpm and a simulation time of 100 s. Since T_b is upper semi-continuous, bounded, has a closed and convex image, and $[-a_2N_b, a_2N_b]$ is the convex hull of the limits $\omega_b \uparrow -\delta$ and $\omega_b \downarrow \delta$ of T_b, a Runge-Kutta integration method can be used to solve for the system time response (Leine and Nijmeijer, 2004). The optimal values for the control parameters and sensors locations are indicated in Tab. 2, for both values of WOB.

Table 2. Optimal gains and sensors locations for PI and OSOF controllers.

WOB	Controller	Gain	Sensors Locations (m)	J_n	J_u
80 kN	PI	$k_p = 647.7$ $k_i = 178.9$	$\alpha = 0.0$	4.41%	4.79%
	OSOF	$\mathbf{K} = [3447.0 \;\; -2049.4 \;\; 1578.7 \;\; -125.4]$ $k_i = 13.0$	$\alpha_1 = 0.0$ $\alpha_2 = 300.0$	2.33%	2.93%
120 kN	PI	$k_p = 555.7$ $k_i = 121.2$	$\alpha = 0$	7.22%	63.42%
	OSOF	$\mathbf{K} = [3937.3 \;\; -2638.5 \;\; 1538.0 \;\; -66.3]$ $k_i = 77.0$	$\alpha_1 = 0.0$ $\alpha_2 = 299.4$	4.91%	33.57%

It was also evaluated the sensitivity of the controlled system to parametric changes in the bit-rock interaction, in which the parameters a_1 and a_2 were allowed to vary $\pm 25\%$. Fixing the optimal gains found, an optimization using the particle swarm method was carried out in order to find the values of the dry friction coefficients that provided the worst results in terms of the performance criterion given in Eq. (7). It was found that the condition that gives the worst performance was characterized by the largest difference between the static and dynamic friction coefficients. The values of the cost function given in Eq. (7) for the nominal case (J_n) and the case with uncertainties (J_u) are also presented in Tab. 2. The integral control gain k_i was obtained for the OSOF control by an optimization of the cost function (7) for the case with uncertainties.

The rotary table and bit velocities for the nominal case are compared in Figs. 3 and 4 for the controlled systems using OSOF and PI. For the nominal system, the control law given in Eq. (11) was considered for the OSOF controller. A similar behavior is presented for both values of WOB, characterized by a higher acceleration of the rotary table at the beginning of the simulation together with a faster stabilization for the OSOF controller. In addition, after the stick phase the system using OSOF control had almost no overshoot, in contrast with the system using PI control.

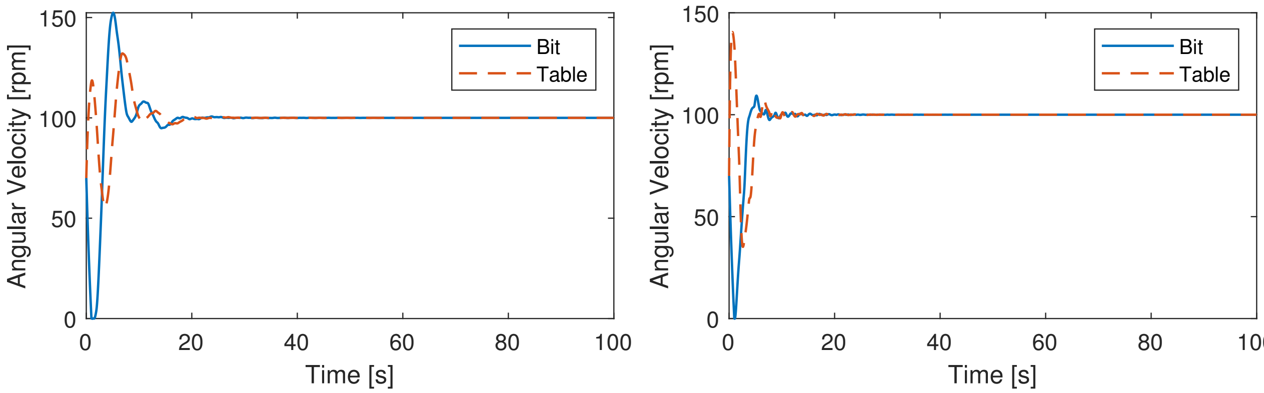

Figure 3. Bit and rotary table velocities for the nominal system using PI (left) and OSOF (right) control with WOB = 80 kN.

The same comparison was also performed for the case with uncertainties, and the results are depicted in Figs. 5 and 6. Regarding the simulation with lower WOB, the difference between the nominal and uncertainties cases for the OSOF controller was negligible, while for the PI controller it was possible to observe a slight increase in the overshoot and settling time. However, for the higher value of the WOB, the system with OSOF control presented a larger overshoot and the time in the stick phase was also increased in comparison to the nominal case. Nevertheless, the controlled system still maintained asymptotic stability, unlike the system with PI control, as for the most severe case of the uncertainties range considered, as indicated in Fig. 6, the PI control entered the regime of stick-slip oscillations.

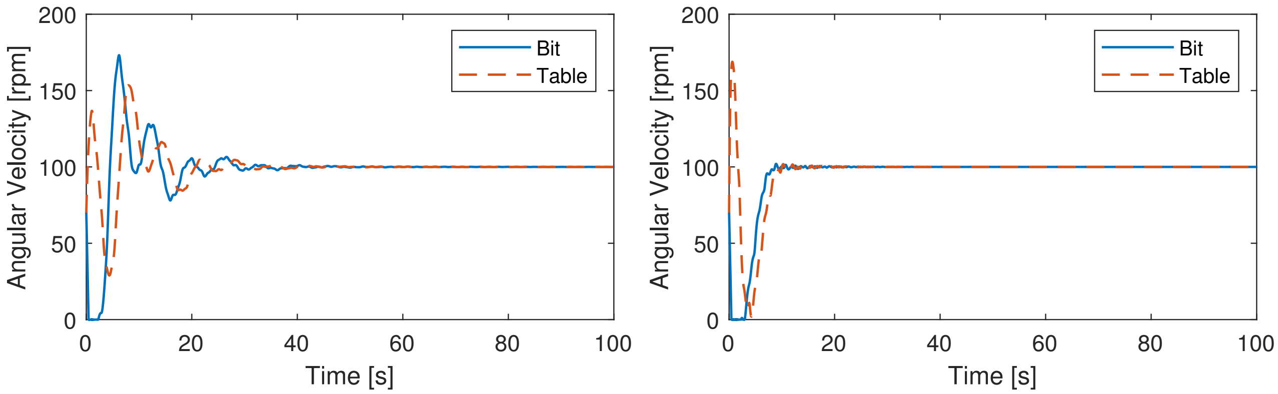

Figure 4. Bit and rotary table velocities for the nominal system using PI (left) and OSOF (right) control with WOB = 120 kN.

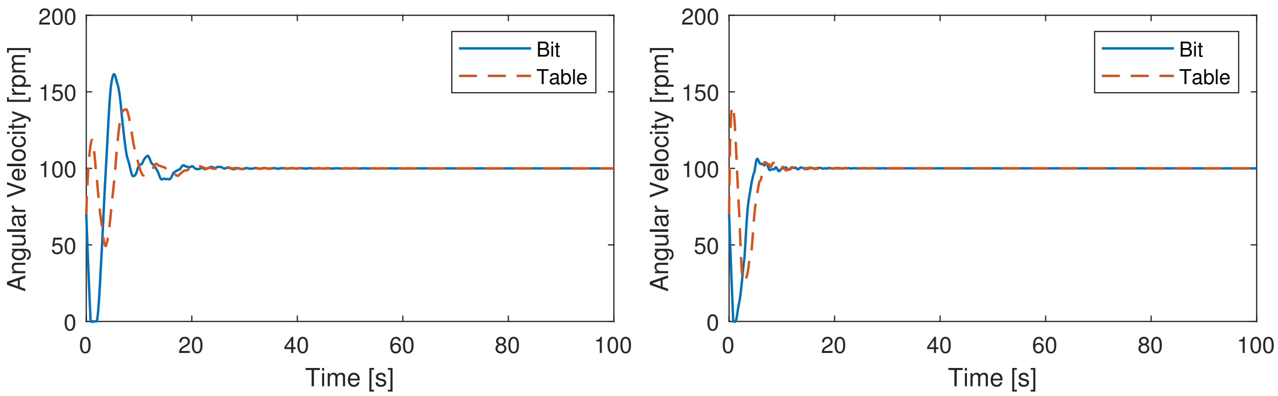

Figure 5. Bit and rotary table velocities for the system with uncertainties using PI (left) and OSOF (right) control with WOB = 80 kN.

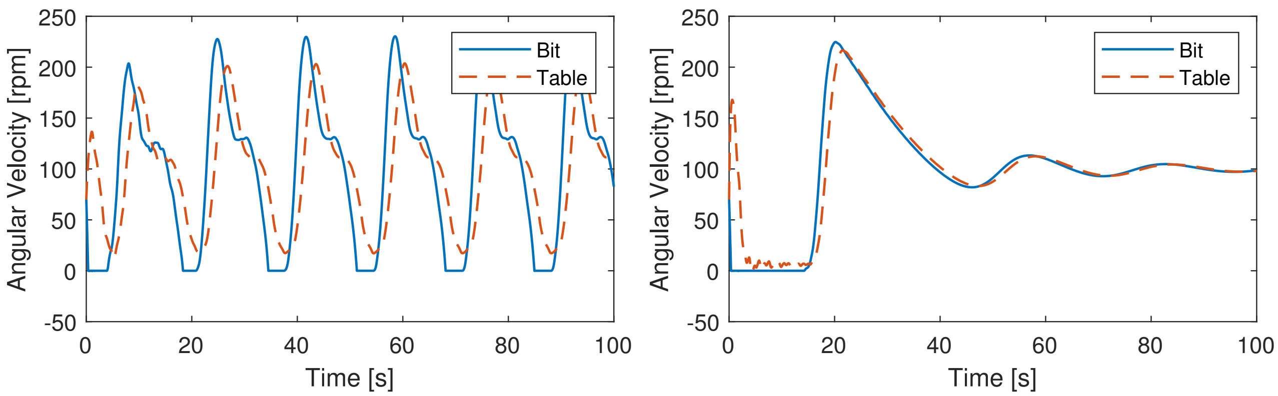

Figure 6. Bit and rotary table velocities for the system with uncertainties using PI (left) and OSOF (right) control with WOB = 120 kN.

5. CONCLUSIONS

In this paper, the problem of active control of stick-slip oscillations in drilling systems was studied. A translated modal form was developed, which transformed the problem of tracking the desired system velocity into a problem of asymptotic stabilization about the origin. This formulation, together with the linearization of the translated system, allowed the synthesis of an OSOF controller for the drilling system. This controller was compared with a traditional PI controller, yielding better performance and lower sensitivity to parameters variations. For smaller values of WOB, the OSOF controller presented almost no overshoot with fast rise time. For the higher values of WOB, the nominal system with OSOF control still exhibited good performance, but in the presence of uncertainties the time in stick phase and the overshoot were largely increased. Further investigation of the controller proposed will be directed to practical implementation and a more detailed analysis of the uncertainties modeling and quantification.

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