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DYNAMICS OF A SYSTEM WITH DRY-FRICTION AND ELECTROMECHANICAL COUPLING

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Abstract. *This work analyzes the nonlinear dynamics of a coupled system with dry-friction. The system is composed of two subsystems, a mechanical and an electromagnetic. The mechanical subsystem is composed of a cart and a disk, whose motion is driven by a DC motor. The coupling between the motor and the disk-cart is made by a mechanism called scotch yoke, so that the motor rotational motion is transformed in horizontal cart motion on a rail. There is dry-friction between the cart and the rail. The friction is modeled as Coulomb's friction. The resulting motion of the cart-disk can be characterized by two qualitatively different and alternate modes, the stick- and slip-modes, with a non-smooth transition between them. In this paper a new analytical approximation of the maximal stick duration is proposed. It should be remarked that the maximal stick duration is usually one of the variables of great interest in systems with stick-slip dynamics. One the advantages of having the analytical approximation we propose is that it allows the observation of the influence of mechanical and electrical variables on the stick duration.*

Keywords: *coupled systems, dry-friction, stick-slip oscillations, stick-duration.*

1. INTRODUCTION

The analysis of electromechanical systems is not a new subject. The interest of analyzing their dynamic behavior is shown by the increasing amount of research in this area. Electromechanical systems present an interesting behavior characterized by the mutual influence between the electrical and mechanical subsystems of the system, that is, the dynamics of the motor is heavily influenced by the mechanical subsystem and the dynamics of the mechanical subsystem depends on the dynamics of the motor (Dantas *et al.*, 2014, 2016; Clerkin and Sampaio, 2017; Manhaes *et al.*, 2018) Each subsystem of the system affects the behavior of the other, i.e., they interact. The coupling varies with the coupling conditions, it is not a functional relation and depends on the initial conditions (Lima and Sampaio, 2016b, 2012) The dynamics of the coupled system is given by an initial value problem comprising a set of coupled differential equations (Lima *et al.*, 2018; Lima and Sampaio, 2018), in which the coupling torque appears as a parametric excitation, i.e., a time variation of the system parameters. The problem becomes even more interesting if it is considered the existence of dry-friction in the electromechanical system. The nonlinearity provided by the friction can induce stick-slip oscillations on the system (Lima and Sampaio, 2015a, 2017a,b, 2013). Depending on the values of the system parameters, the response of the system can be composed of a sequence alternating stick and slip-modes, (Lima and Sampaio, 2015b, 2016a, 2017d,c). The stick and slip-modes have a non-smooth transition (Ding, 2012; Fidlin, 2006; Galvanetto and Bishop, 1999). The interest of analyzing the stick-slip dynamics is reflected by the increasing amount of research in this area (see for instance (Thomsen and Fidlin, 2003; Awrejcewicz and Olejnik, 2007; Hinrichs *et al.*, 1998; Feng, 2003)).

Usually, one of the variables of great interest in systems with stick-slip dynamics is the maximal stick duration. The maximal stick duration is the only variable, important in stick-slip dynamics, considered in this paper. In this paper a new analytical approximation of this variable in an electromechanical system is proposed. This is a new result. With this analytical approximation it is possible to discriminate the influence of the mechanical and the electrical variables in the stick duration.

2. DYNAMICS OF THE ELECTROMECHANICAL SYSTEM WITH DRY-FRICTION

The system analyzed in this paper is composed by a cart-disk whose motion is driven by a DC motor. The motor is coupled to the cart through a pin that slides into a slot machined on a plexiglas plate that is part of the cart, as shown in Fig. 1. The pin hole is drilled off-center on a disk fixed in the axis of the motor, so that the motor rotational motion is transformed into horizontal cart motion over a rail. The eccentricity influences heavily the nonlinearity of the system. Even small eccentricities produces high nonlinearities. The dynamics of a DC motor is given by the following initial value

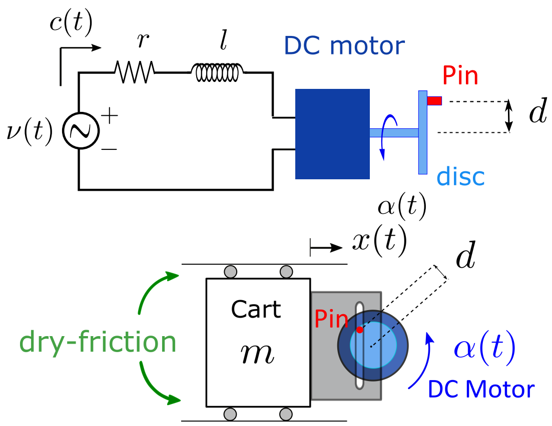

Figure 1. Electromechanical system with dry-friction between the cart and the rail.

problem (IVP). Given a source voltage ν, find $(\alpha,\, c)$ such that, for all $t > 0$,

$$l\dot{c}(t) + r\; c(t) + k_e\dot{\alpha}(t) = \nu(t)\;, \tag{1}$$

$$j_m\ddot{\alpha}(t) + b_m\dot{\alpha}(t) - k_e\; c(t) = -\tau(t)\;, \tag{2}$$

with the initial conditions

$$\dot{\alpha}(0) = \dot{\alpha}_0 \quad , \quad \alpha(0) = \alpha_0 \quad , \quad c(0) = c_0\,, \tag{3}$$

where t is the time, v is the source voltage, c is the electric current, $\dot{\alpha}$ is the angular speed of the motor, l is the electric inductance, j_m is the motor moment of inertia, b_m is the damping ratio in the transmission of the torque generated by the motor to drive the coupled mechanical system, k_e is the motor electromagnetic force constant and r is the electrical resistance. The modulus of the available torque to the coupled mechanical system is τ. The source voltage is considered to be

$$\nu(t) = \nu_0 + \nu_1 \sin\left(\omega_v\, t\right). \tag{4}$$

The mass of the mechanical system is m and the horizontal cart displacement is represented by x. It is considered that the cart is not allowed to move in the vertical direction. Due to the problem geometry, and noting $||\boldsymbol{d}|| = d$, the horizontal motion of the cart and the angular displacement α of the motor are related by the constraint

$$x(t) = d\, \cos\left(\alpha(t)\right). \tag{5}$$

In the model of the coupling between the motor and the mechanical system, it is assumed that the motor shaft is rigid. Thus, the available torque to the coupled mechanical system, $\boldsymbol{\tau}$, can be written as

$$\boldsymbol{\tau}(t) = \boldsymbol{d}(t) \times \boldsymbol{f}(t)\,, \tag{6}$$

where $\boldsymbol{d}$ is the eccentricity of the pin of the motor, considered a vector, and $\boldsymbol{f}$ is the coupling force between the DC motor and the cart. The component of $\boldsymbol{d}$, which is perpendicular to the plane of the cart motion, is always zero, and the others horizontal and vertical components can be calculated from the angular displacement α of the disk. Assuming that there is no friction between the pin and the slot machined on an acrylic plate, the vector $\boldsymbol{f}$ only has a horizontal component, called f, which is the horizontal force that the DC motor exerts in the cart. Thus, the modulus of $\boldsymbol{\tau}(t)$ is

$$\tau(t) = -f(t) d \sin\alpha(t)\,. \tag{7}$$

Since the cart is modeled as a particle, its movement in the horizontal direction satisfies the equation:

$$m\; \ddot{x}(t) = f(t) + f_r(t)\,, \tag{8}$$

where f_r is the dry-friction force between the cart and the rail. The initial value problem to the coupled motor-disk-cart system with dry-friction is: given v, find $(\alpha,\ c)$ satisfying

$$l\dot{c}(t) + r\, c(t) + k_e\dot{\alpha}(t) = v_0 + v_1 \sin(\omega_v\, t)\,, \tag{9}$$

$$\ddot{\alpha}(t)[j_m + m\, d^2 (\sin(\alpha(t)))^2] + \dot{\alpha}(t)[b_m + m\, d^2 \sin(\alpha(t))\cos(\alpha(t))] - k_e c(t) = -f_r(t)\, d\, \sin(\alpha(t))\,, \tag{10}$$

for given initial conditions of electric current, angular velocity and position of the motor. The friction is modeled as Coulomb's, which is a simple model, shown in Fig. 2. This model was chosen because of its simplicity. Different friction models, with hysteresis loops, for example, may leads to different results. The study of different friction models is an interesting research topic and can be investigated in future works.

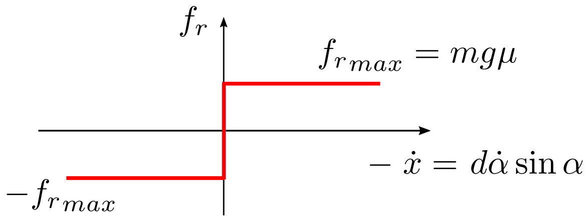

Figure 2. Coulomb dry-friction.

The non-smooth behavior of the dry-friction force can induce in the system stick-slip oscillations. Depending on the values of the system parameters, the response of the system may be composed of a sequence alternating stick and slip-modes. In Fig. 3 it is shown a possible sequence of stick and slip-modes for the interval of analysis $[0, t_a]$.

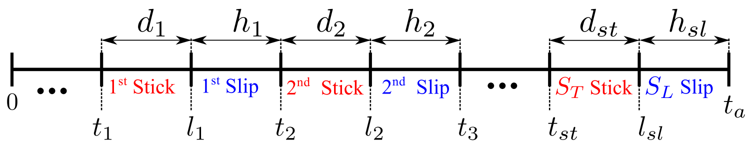

Figure 3. Sequence of sticks and slips in a system response.

During the stick-mode, the disk-cart does not move, so that the angle, describing the angular position of the disk, is constant. The frictional force and the current, however, can vary. There is an internal dynamics of the system in the electromagnetic subsystem. Hence, stick means only no motion of the disk-cart, the mechanical subsystem. The electromagnetic subsystem continues to change its state until it gathers enough power to move the disk-cart again. The stick mode occurs when $\dot{\alpha} = 0$ in an interval and when the frictional force, which satisfies

$$k_e\, c(t) = f_r(t)\, d\, \sin(\alpha(t))\,, \tag{11}$$

is in the interval $-f_{max} \leq f_r \leq f_{max}$, where $f_{max} = \mu m\, g$, g is the gravity and μ is the friction coefficient between the cart and the rail. Equation (11) is obtained considering $\dot{\alpha} = 0$ and $\ddot{\alpha} = 0$ in Eq. (10). Remark that during the stick-mode, the frictional force varies and depends on the angular position of the motor. There is a functional relation between these two variables. Besides, the initial value problem that describes the dynamics of the coupled motor-cart system with dry-friction is reduced to just one differential equation given by

$$l\dot{c} + r\, c = v_0 + v_1 \sin(\omega_v\, t)\,, \tag{12}$$

where the initial condition for the current is the value of it in the beginning of the stick-mode. Observe that during the stick-mode, the sum of the forces that act over the cart is zero (it does not move). The horizontal coupling force between the DC motor and the cart, f, is balanced by the dry-friction force, f_r. This balance lasts until the frictional force, given in Eq. (11), reaches its maximum value, f_{max}. During the stick-mode, the dynamics of the system is governed only by the dynamics of the electrical circuit of the motor. Electrical and mechanical subsystems do not interact during the duration of a stick, the subsystem are decoupled. During the slip-mode, the dry-friction force is

$$f_r(t) = -m\, g\mu\, \mathrm{sgn}(\dot{x}(t)) = -m\, g\mu\, \mathrm{sgn}(-\dot{\alpha}(t)\, d\, \sin(\alpha(t)))\,. \tag{13}$$

3. MAXIMAL STICK DURATION

As explained in the introduction, one of the variables of great interest in systems with stick-slip dynamics is the maximal stick duration. To obtain an analytical approximation to this variable in the electromechanical system analyzed, the starting point is the the initial-value problem that describes the dynamics of the coupled motor-disk-cart system with dry-friction during the stick-phase given by Eq. (12), i.e., the reduced initial-value problem. Calling

- the instant of beginning of a stick as t_1;
- the instant of end of the longer stick as t_2;
- the angle of the disk in the beginning of a stick as $\alpha(t_1) = \alpha^\star$;
- and the current in the beginning of a stick as $c(t_1) = c_0$.

The stick-mode ($t_1 \leq t \leq t_2$) is governed by the following linear initial-value problem:

$$\begin{array}{l} l\dot{c}(t) + \ r\, c(t) = \nu_0 + \nu_1 \sin(\omega_v\, t). \\ c(t_1) = c_0 \end{array} \tag{14}$$

The analytical solution of this initial-value problem ($t_1 \leq t \leq t_2$) is known and it is given by

$$c(t) = \left[\frac{c_0 - \dfrac{\nu_0}{r} - \dfrac{\nu_1\sqrt{r^2+l^2}}{l^2\omega_v^2 + r^2} \sin(\omega_v t_1 + \phi)}{e^{-r/l\; t_1}} \right] e^{-r/l\; t} + \frac{\nu_1\sqrt{r^2+l^2}}{l^2\omega_v^2 + r^2} \sin(\omega_v t + \phi) + \frac{\nu_0}{r}, \tag{15}$$

where $\phi = \arctan \dfrac{-l}{r}$. Since, the variable of interest is the maximal stick duration, the influence of the initial condition (c_1), can be neglected and the longer the stick duration the better the approximation. Doing this, the solution of the IVP during the the stick-mode ($t_1 \leq t \leq t_2$) can be approximated by:

$$c(t) \approx \frac{\nu_1\sqrt{r^2+l^2}}{l^2\omega_v^2 + r^2} \sin(\omega_v t + \phi) + \frac{\nu_0}{r}. \tag{16}$$

Recalling that during the stick-mode

- $f_r \in [-f_{r\,max}, f_{r\,max}]$ and
- $k_e\; c(t) = f_r(t)\, d\, \sin\alpha(t)$.

The stick ends when

$$c(t_2) = \frac{f_{r\,max}\, d\, \sin\alpha^\star}{k_e}. \tag{17}$$

To discover the instant of end of a stick, t_2, and the maximal stick duration $t_2 - t_1$ one has to determine the instant that the current reaches the value $\dfrac{f_{r\,max}\, d\, \sin\alpha^\star}{k_e}$. In this formula appears mechanical and electrical parameters as well as a characteristic of the stick, its angular position.

4. NUMERICAL RESULTS

The maximal stick duration depends on mechanical and electrical variables and geometry of the stick. Among all variables, two of them can be highlighted. They are the angle of the disk in the beginning of a stick, $\alpha(t_1) = \alpha^\star$, and the value of the friction coefficient, μ. To better visualize the influence of these two variables, values to all the others parameters were selected and fixed (they are given in Table 1) and the maximal stick duration was computed for different combinations of $\alpha^\star$ and μ.

The results are shown in Figs. 4 and 5. In the first graph, the values of friction coefficient are $0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5$. It can be observed that with $\mu = 0.1$, the maximal stick duration is 0 for all $\alpha^\star$, which means that there is no stick. When

$l = 1.880 \times 10^{-4}$ H	$k_e = 5.330 \times 10^{-2}$ volts/(rad/s)
$j_m = 1.210 \times 10^{-4}$ kg m^2	$r = 0.307\ \Omega$
$b_m = 1.545 \times 10^{-4}$ Nm/(rad/s)	$m = 5.000$ kg
$\nu_0 = 1.000$ volts	$\nu_1 = 0.500$ volts
$\omega_v = 10.000$ rad/s	$d = 0.010$ [m]

Table 1. Parameter values

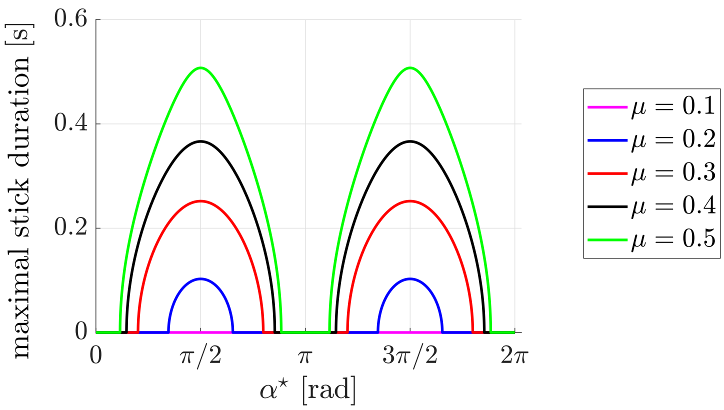

Figure 4. Maximal stick duration as function of the angle of the disk in the beginning of a stick for different values of the friction coefficient.

$\mu = 0.2$, that is possible to have stick. The longest stick happens when $\alpha^\star = \pi/2$ or $\alpha^\star = 3\pi/2$. In the region that $\alpha^\star$ is around 0 or π, there is no stick. As the friction coefficient increases, the region where there is no stick decreases.

In the second graph, the values of friction coefficient are $0.6,\ 0.7,\ 0.8,\ 0.9,\ 1.0$. For these values, an interesting behavior occurs. There are regions of $\alpha^\star$ around $\pi/2$ and $3\pi/2$ that the stick can last forever. This means that if a stick happens when $\alpha^\star$ is in a certain region, the stick will last forever. In other words, the electromagnetic does not gather enough power to move the mechanical part again.

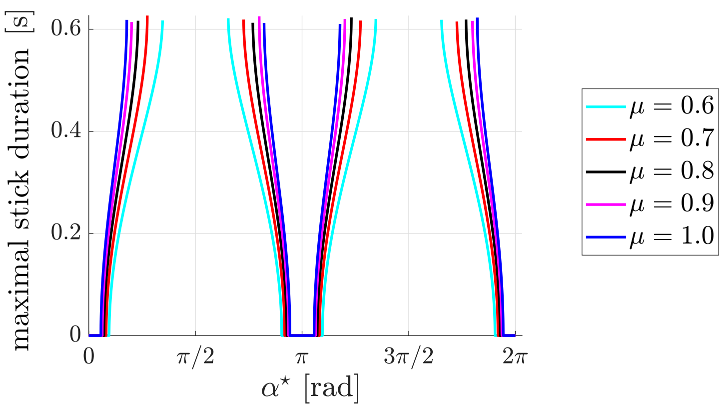

Figure 5. Maximal stick duration as function of the angle of the disk in the beginning of a stick for different values of the friction coefficient.

5. CONCLUSIONS

In this article, it is analyzed the dynamics of an electromechanical system composed by a cart and a DC motor. The coupling between the motor and the cart is made by a mechanism called scotch yoke, so that the motor rotational motion is transformed in horizontal cart motion on a rail. There is dry-friction between the cart and the rail. The resulting motion of the cart can be characterized by stick- and slip-modes, with a non-smooth transition between them. A new analytical approximation for the maximal stick duration, one of the variables of great interest in systems with stick-slip dynamics, is proposed. By the analytical approximation, it is possible to verify that the maximal stick duration depends on the position where the disk sticks. Depending on the system parameter values, a stick does not happen or a stick can last forever. This results are new and appears in the literature for the first time. This problem shows also that an electromechanical system must be described by mechanical and electromagnetic variables, one alone will not do (Lima *et al.*, 2018). During the stick the mechanical subsystem is inactive, but the electromagnetic is not. The motor has a dynamics of its own and it will, in a sense, revive the mechanical subsystem in due time. In other words, once the stick appear, the subsystems uncouples, as the stick finishes, they couple again. If the system were described only by mechanical parameters a new coupling could not take place, of course. One could say that an electromechanical systems is a pair that work so well that the two subsystems may separate only for a time, the maximal stick-duration, not more.

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