



25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COB-2019 - 0482

RANDOM FIELD GENERATION OF THE MATERIAL PROPERTIES IN A PERIDYNAMIC MODEL

Leandro Ferreira Friedrich

Mylena Barcellos

Applied Mechanics Group, Post graduate Program in Engineering, Federal University of Pampa, Alegrete, RS, Brazil.
leandroffriedrich@gmail.com, mylenabsilva@gmail.com

Ignacio Iturrioz

Department of Mechanical Engineering, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil.
ignacio.iturrioz@ufrgs.br

Abstract. Simulations based on the peridynamic theory are a promising approach to understand the processes involved in the fracture of different materials. In the case of nonhomogeneous materials, the randomness of the material properties is an important aspect because it can vary the mechanical behavior and the fracture location. This work presents a proposal to simulate 3D random fields characterized by arbitrary probability distributions. The Specific fracture energy (G_f), which is directly related to the critical stretching of peridynamic bonds, is defined as a 3D scalar random field with Weibull probability distribution and correlation length. A plate on traction load illustrates the proposed approach. In this case, the correlation length of the random field is kept constant when the discretization changes. The results show an independence of the fracture pattern for the level of discretization used. Other results contribute to provide a flexibility in the calibration of the peridynamic model response allowing that the fracture patterns and the global material behavior to change.

Keywords: Peridynamic theory, random field generation, fracture mechanics

1. INTRODUCTION

The process of damage in structures is a main topic in the solid mechanic area. Two of the most important researchers, Lemaitre (1992) and Kachanov (2013), have proposed models of damage that are mainly applied to ductile materials. On the other hand, Peridynamics (PD) presents a promising approach to simulate the initiation, evolution and interaction of damage in any material. It is a nonlocal theory that takes into account the long-range forces between material points in a certain neighborhood, the horizon δ (Rädel *et al.* 2017). PD constitutive models depend on finite deformation vectors, as opposed to classical constitutive models, which depend on deformation gradients (Pablo *et al.* 2016). Heterogeneous materials constitute a very large part of our natural environment as well as a substantial fraction of man-made objects: glasses, polymers and amorphous materials are among the vast array of examples (Herrmann, 1990). On a larger scale, porous media, composites and suspended solids can also be mentioned (Birck *et al.* 2016). However, to simulate nonhomogeneous materials, with PD, we must change the distribution of properties on the volume. For this, we need to generate random fields of material properties.

Some authors have already addressed the distribution of properties within PD models. Rädel *et al.* (2017) uses a stochastic distribution for generate elastic properties of material by incorporating a statistical nature in the damage initiation, obtaining as results the reduction of mesh dependence. At Silling *et al.* (2007) random fluctuations are introduced in the critical stretch by means of a Weibull statistical distribution. In the work done by Bobaru (2007), the authors introduced randomness in fiber directions using Gaussian distribution and more recently, Cabral *et al.* (2019) considered the stochastic nature of material properties through the introduction of a random toughness field using a Weibull probability density distribution. For each material point is assigned a critical stretching value and the bond was a mean value between this material points.

This paper presents a methodology for modeling heterogeneous materials in peridynamics. Using a technique originally proposed and used by Miguel *et al.* (2009) in the modeling of wind speed in structures, the stochastic nature of material properties is introduced in the model considering toughness as a random field with a Weibull density of probability and applying the concept of correlation length, that can vary in the 3 cartesian directions (x, y and z), what

makes each PD bond becomes different. The implementation is tested on a plate subject a traction load and some parameters of the proposed methodology are evaluated and discussed.

2. PERIDYNAMICS

Different types of loads, boundary conditions and the scale of analysis can affect the materials modelling. To minimize these influences, several approaches have been developed. Among them, the Molecular Dynamics (MD) used to model the behavior of atoms and molecules on a nano scale problem and the Classical Continuum Mechanics (CCM) that analyzes macro scale problems by means, mainly, of finite element method (FEM). However, when discontinuities are present in the body as cracks, the approaches mentioned fail and the nonlocal continuum mechanics are necessary, such as PD theory.

Peridynamics is a nonlocal theory of continuum media able to eliminate the mathematical inconsistency present in the classical continuum media theory by substituting the spatial derivatives by force integrals in the material points in the spatial domain. The PD theory divides a continuum media into material points occupying volume in space, Figure 1. The most important point of the PD formulation is that each material point has its behavior governed by the interaction with the points located in its neighborhood. In other words, the PD theory is about the interaction forces between material points within a given neighborhood (Oterkus *et al.* 2018).

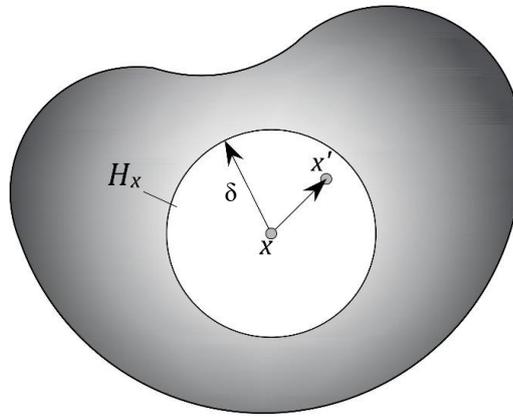


Figure 1. A scheme with the main parameters used in the PD (Silling and Askari, 2005).

There are many PD formulations since its creation; however, the bond-based theory (Silling, 2000) satisfies the requirements for the aim of this work, despite its limitations, and results in a computational advantage because of its simple application. The term “bond” refers to the interaction between the material points at x and x' . The Equation 1 presents the bond-based PD equation of motion,

$$\rho(x)\ddot{u}(x,t) = \int_{H_x} f(u' - u, x' - x)dV' + b(x,t) \quad (1)$$

where ρ is the material density, $b(x, t)$ is the body force acting at point x and H_x is the space of the material points near the point x , Figure 1. $f(u' - u, x' - x)$ is a pairwise force density vector function and u is the displacement of the material point at x ; f contains all the constitutive properties of the material and for a linear elastic isotropic solid, for instance, can be expressed as Equation 2.

$$f(u' - u, x' - x) = cs \frac{y' - y}{|y' - y|} \quad (2)$$

where $y = x + u$ is the position of the material point in the deformed configuration. The bond constant c is the peridynamic material parameter and can be expressed in terms of the material constants of CCM (Oterkus *et al.* 2018). That is, for a linear isotropic material, the bond constant is given by:

$$c = \frac{12E}{\pi\delta^4} \quad (3)$$

where E is the elastic modulus of material. In the Equation 2, s is the bond stretch expressed on the following form,

$$s = \frac{|y' - y| - |x' - x|}{|x' - x|} \quad (4)$$

The PD formulation is based on the assumption of pairwise interactions in which the force density vectors exerted by the point x on the point x' of the material and are equal in magnitude and parallel to the relative position vector in the deformed state (Oterkus *et al.* 2018).

In the PD model damage is introduced by means of bond breaking. The bonds lose their load capacity when a limit s_0 is reached, as it is shown in Figure 2. w_0 is the work required to break an individual bond and could be represented as the area under the f - s bond law, in Figure 2. ξ is the relative position vector.

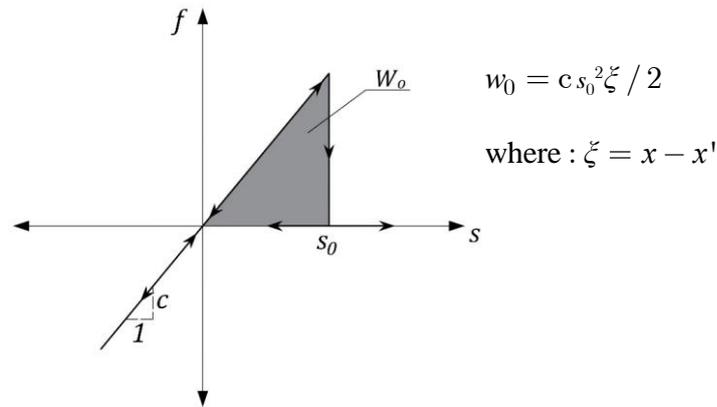


Figure 2. The uniaxial constitutive law to simulate damage (Silling, 2000).

The energy per unit of fracture area required to separate the body can be expressed in the Equation 5, for a 3D case,

$$G_f = \frac{\pi c s_0 \delta^5}{10} \quad (5)$$

and thus, the critical stretch s_0 can be determined by the material fracture energy,

$$s_0 = \sqrt{\frac{5G_f}{6E\delta}} \quad (6)$$

In order, to compute the fracturing process is applied a local characteristic function φ to identify the connection state of each bond, as follows

$$\vartheta(\eta, \xi, t) = \begin{cases} 1, & s \leq s_0 \\ 0, & s > s_0 \end{cases} \quad \text{and} \quad \varphi = 1 - \frac{\int_{H_x} \vartheta dV_\xi}{\int_{H_x} dV_\xi} \quad (7)$$

Therefore, if a particle has no broken bonds its local damage is $\varphi = 0$, on the other hand if the point is completely disconnected from the rest of the body its value is $\varphi = 1$.

3. 3D RANDOM FIELD GENERATION IN PERIDYNAMICS

This work presents a methodology to generate a random toughness field regardless of discretization used in the PD model. The methodology presented here was originally proposed by Miguel *et al.* (2009) and more recently implemented in a version of Discrete Element Method (DEM) and designated Lattice Discrete Element Method (LDEM) by Puglia *et al.* 2010. The random properties are equally distributed along the correlation length (l_{cor}). The

approach consists of dividing the domain formed by the material points into prismatic regions that have their sides formed by the correlation lengths, which can be different in the three cartesian directions (l_{cx} , l_{cy} , l_{cz}) (Puglia *et al.* 2010), see Figure 3. The line of axes X_G , Y_G and Z_G represents the global coordinate system used to reference the global model. Each PD bounds i of the system is referenced by a new coordinate system X_{Gi} , Y_{Gi} and Z_{Gi} , and by the local coordinate system within the prism x_i , y_i and z_i .

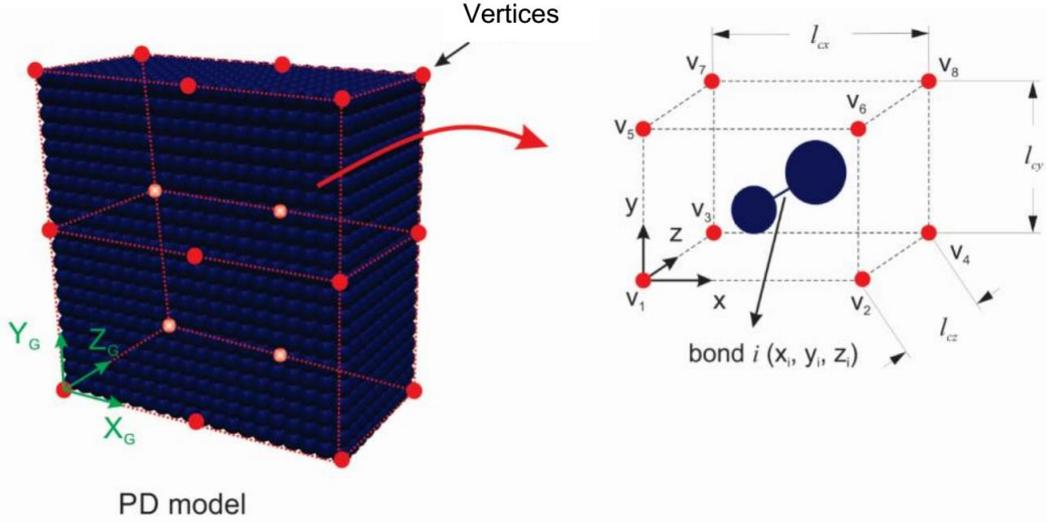


Figure 3. Vertices location and the correlation length in the domain of the PD model and the detail of the centroid location of the one PD bond.

At each vertices of these prisms ($V_1 \dots V_8$) are assigned random values with uncorrelated probabilities distributions. Subsequently, to determine the value of the random field corresponding to each bond i inside the prism a three-dimensional (3D) interpolation is performed. In the present implementation, the spatial localization of the PD bonds i is characterized by the coordinates of its barycenter (x_i , y_i , z_i). The 3D interpolation is given as follow,

$$\begin{aligned} \varphi_v(x_i, y_i, z_i) = & V_1 + \frac{V_2 + V_1}{l_{cx}} x_i + \frac{V_3 - V_1}{l_{cy}} y_i + \frac{V_5 - V_1}{l_{cz}} z_i + \frac{V_4 - V_3 - V_2 + V_1}{l_{cx}l_{cy}} x_i y_i + \\ & \frac{V_6 - V_5 - V_3 + V_1}{l_{cx}l_{cz}} x_i z_i + \frac{V_7 - V_5 - V_3 + V_1}{l_{cy}l_{cz}} y_i z_i + \frac{V_8 - V_7 - V_6 + V_5 - V_4 + V_3 + V_2 - V_1}{l_{cx}l_{cy}l_{cz}} x_i y_i z_i \end{aligned} \quad (8)$$

where $\varphi_v(x_i, y_i, z_i)$ is the interpolated random value for bond i of coordinates x_i , y_i , z_i . V_k ($k = 1, 2, 3, \dots, 8$) is the values of the random field at the vertices. The randomness of the material properties in V_k considers that the G_f is a random field with a distribution of Type III (Weibull), characterized by Equation 9.

$$F(G_f) = 1 - \exp[-(G_f / \beta)^\gamma] \quad (9)$$

where β and γ are the scaling and shape parameters, respectively. The mean μ and the standard deviation sd are related to the scale and shape parameters by means of the Equation 10.

$$\mu = \beta[\Gamma(1 + 1 / \gamma)] \quad sd = \beta[\Gamma(1 + 2 / \gamma) - \Gamma^2(1 + 1 / \gamma)]^{1/2} \quad (10)$$

where Γ is the Gamma function. Since G_f and s_0 are directly related (Equation 6), it is possible to prove that approximately $CV_{s_0} = 0.5 CV_{G_f}$, see Hahn and Shapiro (1967).

4. PLATE SUBJECT TO TENSILE LOAD

The elemental PD code published by Madenci and Oterkus (2014) was used as basis to make the simulations and new implementation here presented. To analyze the behavior of the generated random field, was used a plate, as PD model, with the dimensions shown in Figure 4. The Figure 4 also details the boundary conditions regions and the

mechanical properties of the material used. For all cases was considered a fixed correlation length in the three principal directions with $l_{cx} = l_{cy} = l_{cz} = 0.1$ m and $\delta = 3.015 dx$, where dx is the distance between material points.

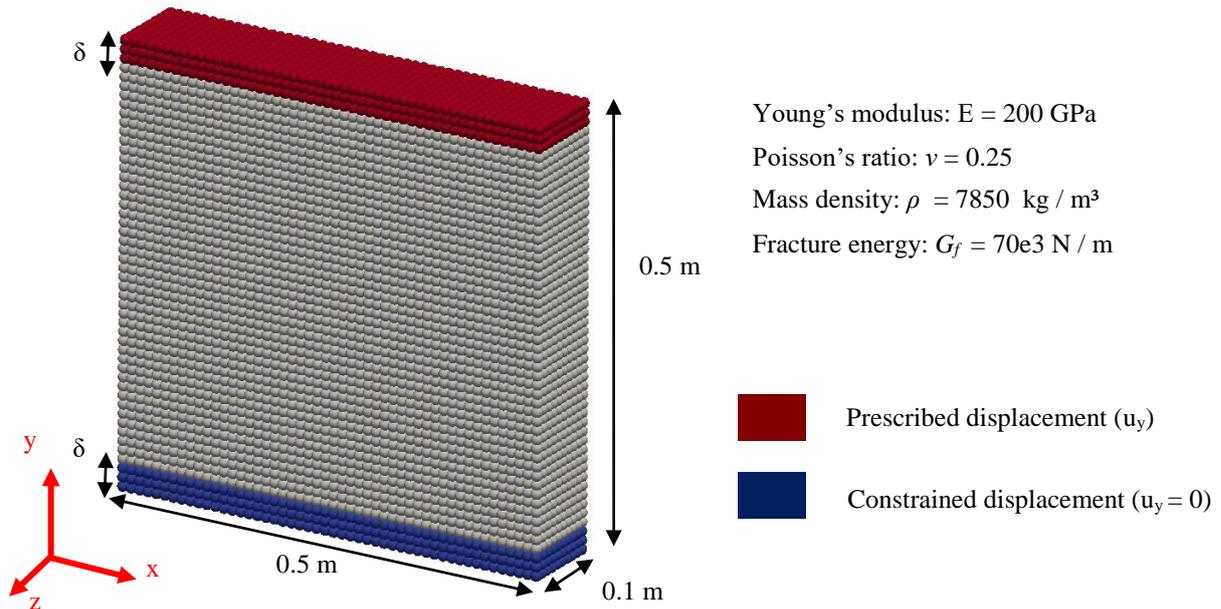
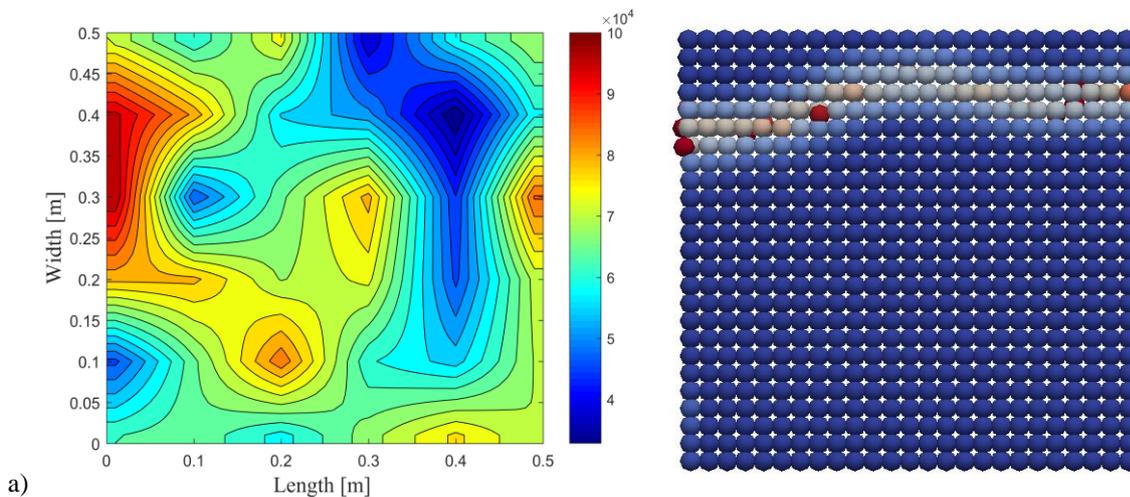


Figure 4. Plate used as PD model.

The prescribed displacement, in all cases, was applied slowly to disregard the effects of inertia on the body. In order to guarantee this condition, it was verified that in the loading process the kinetic energy remains much lower than the other energies involved in the process (elastic and damage).

With the methodology previously described in Section 3, was generated a value of G_f for each pole. From these G_f values, was obtained the critical stretching (s_0), for each pole too. Thus, the 3D interpolation for each bond (Equation 8) of the PD model can be determined. The coefficient of variation (CV) controls the fluctuation of the random field of G_f and consequently of s_0 . For the analyzed cases, we consider CV_{G_f} equal to 50%.

To analyze the independence between the discretization and the final configuration of rupture, we evaluated three discretization levels $dx = 0.005, 0.01$ and 0.02 m. Figure 5 presents the generated fields G_f and the final configuration of rupture for each discretization used. The color map represents the distribution of G_f on the outer face of the plate at $z = 0.1$ m. It is possible to note that in all cases the G_f random field are very close, only with small details of difference. One of the advantages of working with the proposed methodology is that the patterns of the fracture will always be the same, as it is possible to see in Figure 5. To change this state it is necessary to change the seed of generation of random numbers obtained from G_f .



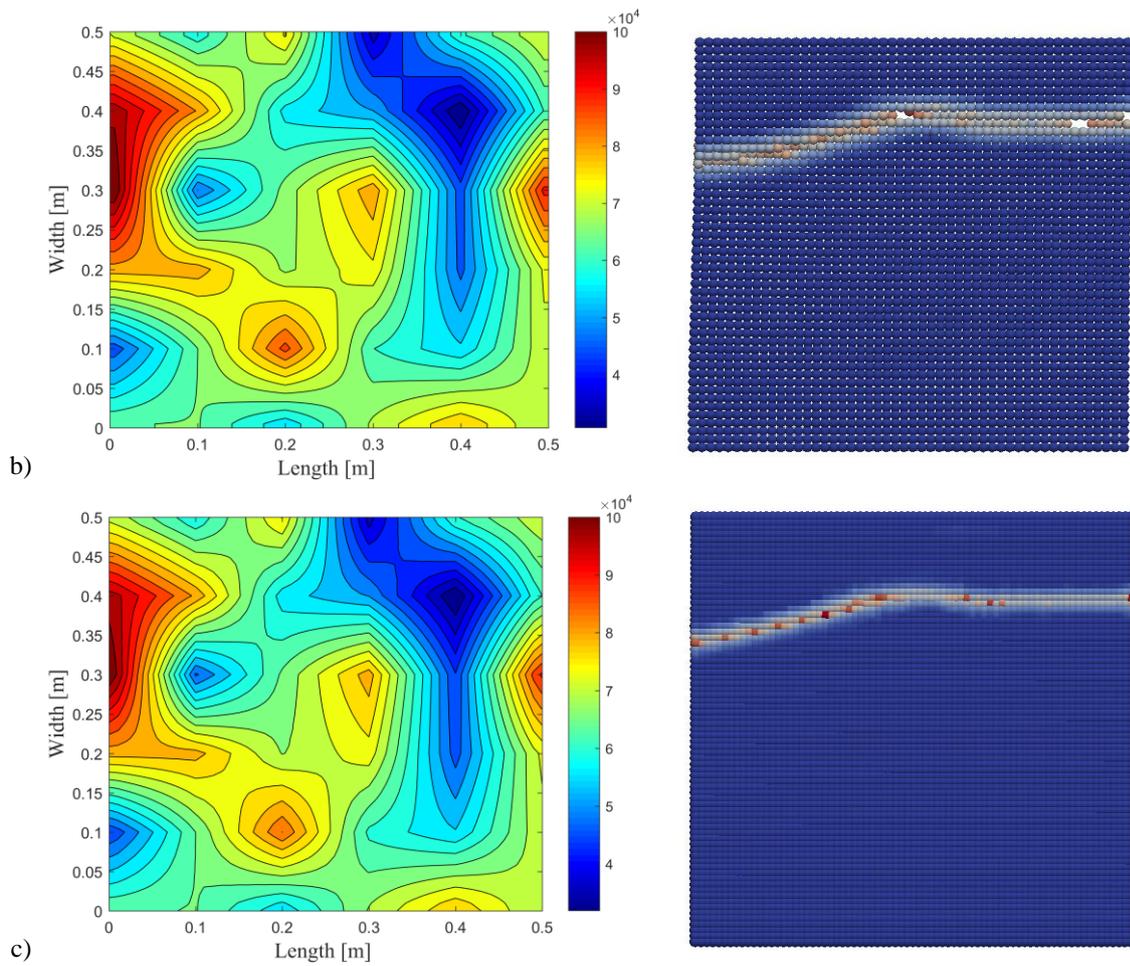


Figure 5. G_f random field and final configuration of rupture for: a) $dx = 0.02$ m; b) $dx = 0.01$ m; c) $dx = 0.005$ m.

Still in the Figure 5, it is possible to see that the higher the level of discretization the clearer the cracks patterns. The final fracture configuration occur in the same region and have the same shape. It is clear in the comparison that there is total independence of the random field generated and the discretization used.

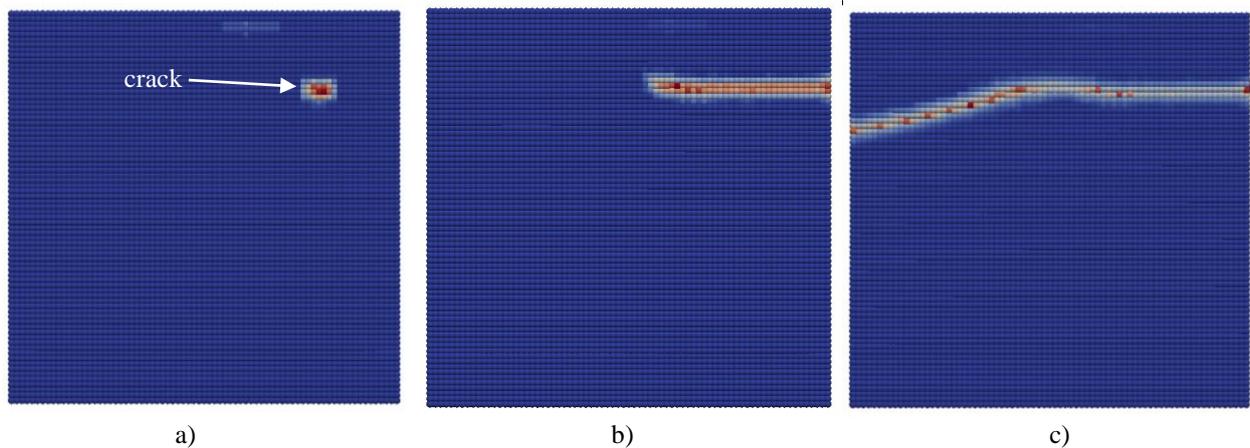


Figure 6. Plate damage evolution for $dx = 0.005$ m, a) initiation; b) unstable propagation; c) rupture.

The Figure 6 shows the nucleation and evolution of plate damage under traction for discretization level $dx = 0.005$ m. When we observe the G_f random field distribution map, it is possible to directly relate the points where the values of G_f are smaller than the initial location of the damage. Figure 6a shows a crack formed in the highlighted region.

Subsequently, in Figure 6b, it naturally propagated in the correct direction until it reached the edge of the plate. Finally, the crack propagated unsteadily in the other direction until it reached the final configuration of rupture (Figure 6c).

The Figure 7 shows the change in the shape of the fracture patterns, when three different seeds are used to change the distribution of the G_f random values of the poles. The plate has the same conditions as described in Figure 4, but the thickness is considered equal $\delta = 3.015 dx$ and the correlation length in z direction follows this change $l_{cz} = \delta$. With the proposed methodology, it is possible to generate different fracture patterns, which can help to equalize the model with different experimental fracture patterns. The Figure 7 show that the weak points, in blue in the color map, remain the points of initiation of propagation of the cracks.

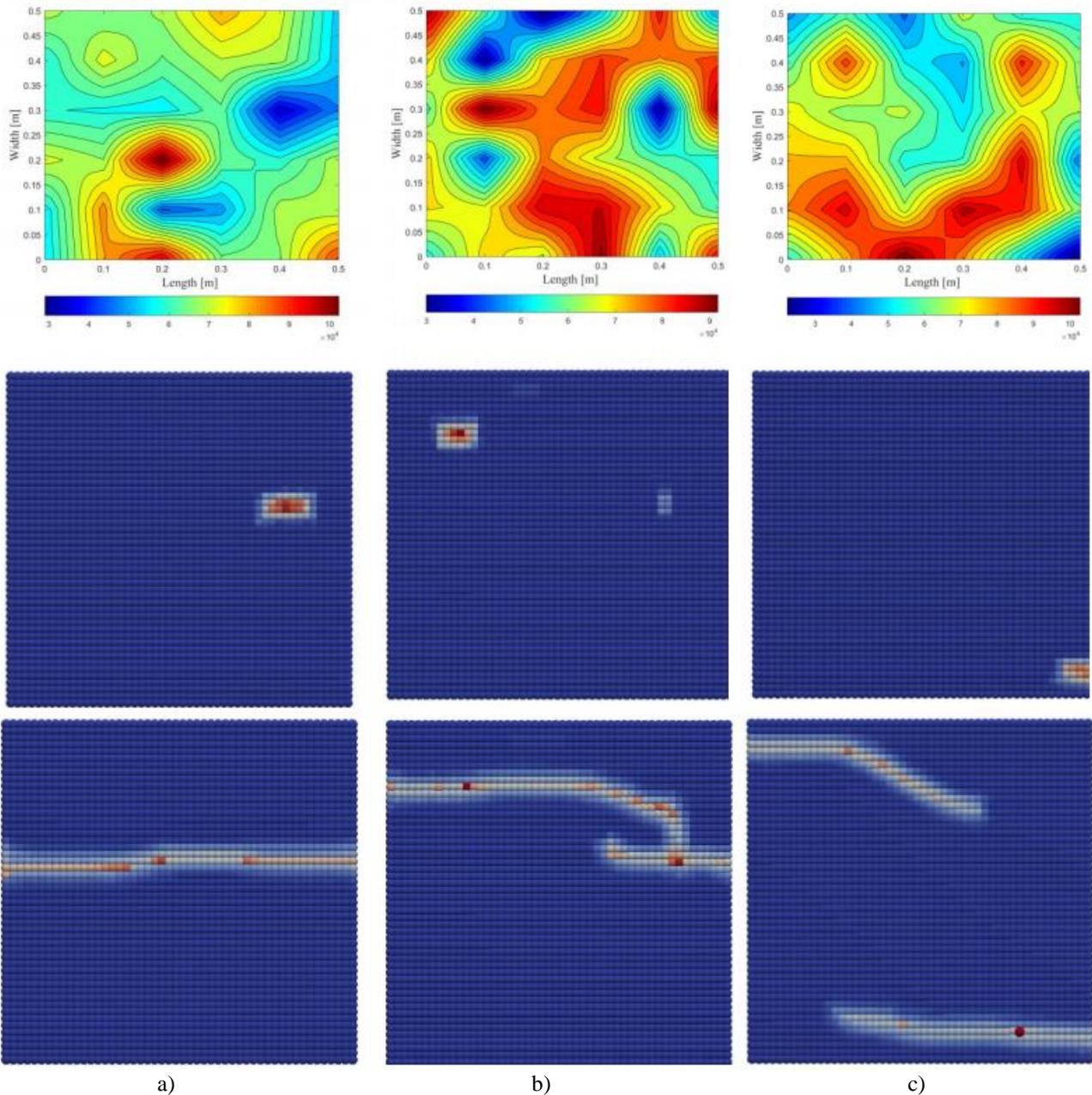


Figure 7. G_f random field, crack initiation and final configuration of rupture for three different seeds: a) seed 1, b) seed 2, and c) seed 3.

The Figure 8 shows the comparison of the force vs. displacement curves between the three situations simulated (seed 1, seed 2 and seed 3) and showed, previously, in Figure 7. The small oscillation in the curves are due to happen simulations without any damping and a small number of output time steps. Observing Figure 8 it is possible to note a small difference between the results for the three analyzed situations. There is a maximum difference of 10% in relation

to the peak load force and the maximum displacement. The result is consistent since we are testing three different situations (seeds) and since randomness changes the distribution of material properties.

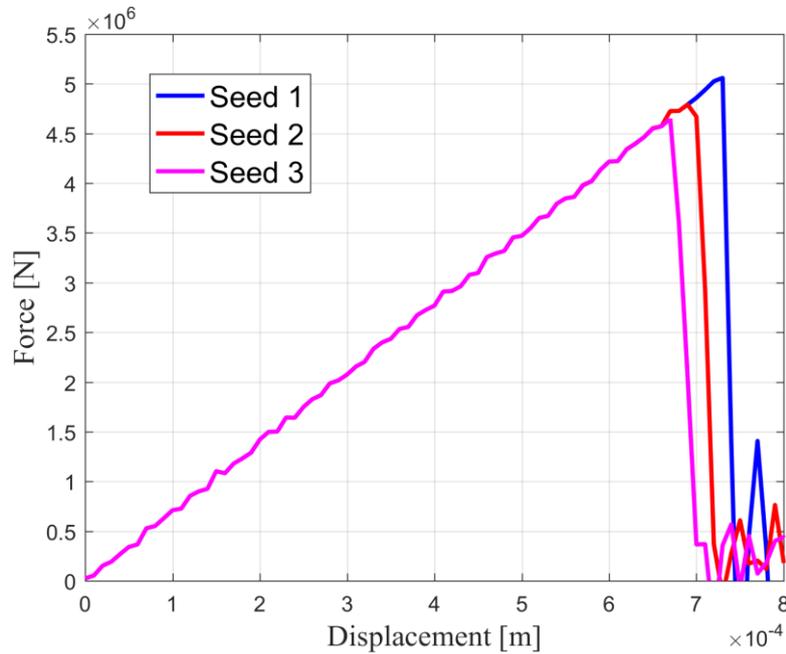


Figure 8. Comparison force vs. displacement for the three seeds simulated in Figure 7.

The Figure 9 shows the influence of different coefficients of variation of G_f (CV_{G_f}), on the same plate from the previous example. The Figure 9a shows the distribution of G_f . The CV_{G_f} changes the float limits of G_f but the shape of the field remains the same, which means that the seed adopted was the same for all the cases. Figure 9b also shows the behavior of the force vs. displacement curves for the analyzed CV_{G_f} cases. Note that the CV completely changes the loading curves. Since the G_f fluctuations are larger, the material changes its behavior and loses the capacity to withstand larger loads.

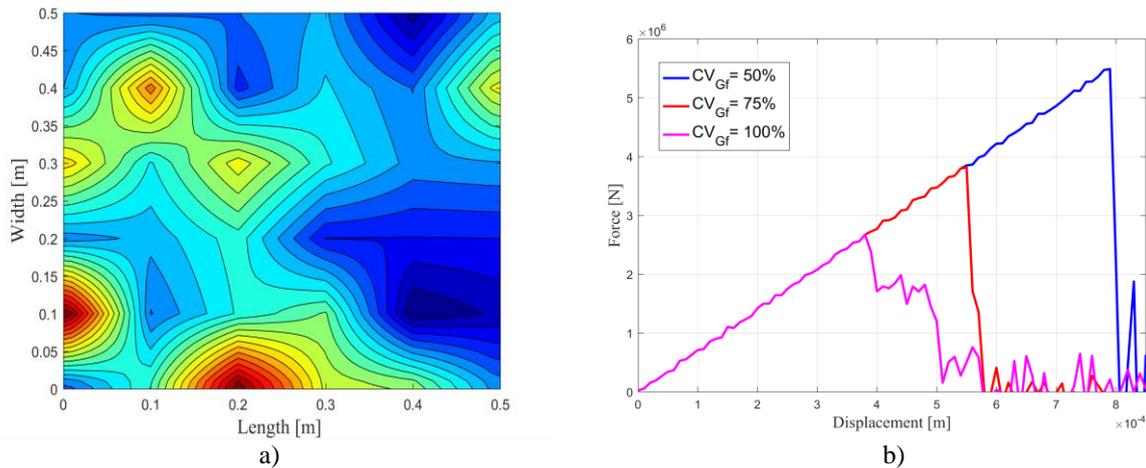


Figure 9. a) G_f random field and b) force vs. displacement for different values of CV_{G_f} .

Figure 10 presents the final rupture configuration for the three cases analyzed in Figure 9b. It is also important to note that as the CV_{G_f} increases, the crack path is altered and as CV_{G_f} increases the crack go through the "weaker" region. Then the propagation happens with two competitive tendencies: the level of stress and the distribution of the G_f random field, both over the plate (Figure 10a).

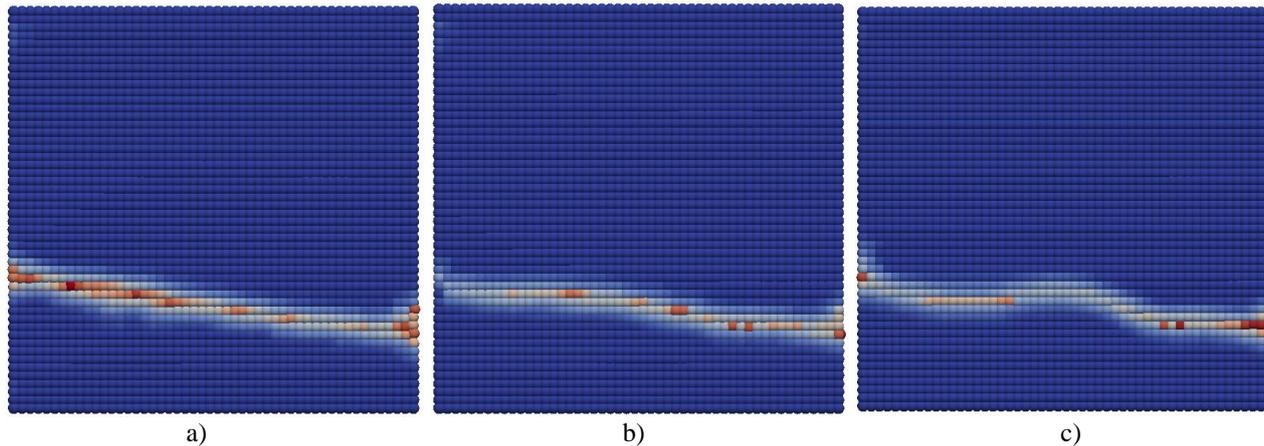


Figure 10. Final rupture configuration for different values of CV_{Gf} : a) 50%, b) 75% and c) 100%.

5. CONCLUSIONS

In the present work, was described a procedure to simulate 3D scalar random fields with Weibull probability distribution and linear correlation length functions. The proposed method is applicable in the spatial determination of the random properties in nonhomogeneous materials in general. The PD theory is seen as a effective tool for the simulation of nonhomogeneous materials. The method implemented is inspired in the methodology presented (H) by Puglia *et al.* 2010. Therefore, from this paper is possible to conclude that:

- The methodology enables simulate the same material up to reach the rupture and in its post-peak behavior when using different levels of discretization.
- The link between the random field generated and the crack configuration was presented, and was pointed out that the crack nucleation and propagation depends of the two factors: the spatial distribution of the stress and the G_f random field distribution.
- These results will be useful to improve the representation, making it more realistic, of the behavior up to collapse of materials.

6. ACKNOWLEDGEMENTS

The authors would like to acknowledge the support obtained from academic development program (ADP) - UNIPAMPA, which made the research possible and a continuous professional improvement.

7. REFERENCES

- Birck, G., Iturrioz, I., Lacidogna G. and Carpinteri A., 2016. "Damage process in heterogeneous materials analyzed by a lattice model simulation". *Engineering Failure Analysis*, vol. 70, pp. 157–176.
- Bobaru, F., 2007. "Influence of van der Waals forces on increasing the strength and toughness in dynamic fracture of nanofibre networks: A peridynamic approach". *Modelling and Simulation in Materials Science and Engineering*.
- Cabral, N.R., Invaldi, M.A., D'ambra, R.B. and Iturrioz, I., 2019. "An alternative bilinear peridynamic model to simulate the damage process in quasi-fragile materials". *Engineering Fracture Mechanics*. Vol. 216.
- Hahn, G. J. and Shapiro, S. S., 1967. *Statistical Models in Engineering*. John Wiley and Sons.
- Herrmann, H.J. and Roux, S., 1990. "Statistical Models for the Fracture of Disordered Media", Elsevier Science Publishers B.V, North Holland.
- Javili, A., Morasata, R., Oterkusb, E. and Oterkus, S., 2018. "Peridynamics Review". *Mathematics and Mechanics of Solids*.
- Kachanov, L., 2013. "Introduction to Continuum Damage Mechanics", *Mechanics of Elastic Stability*, Springer, Netherlands.
- Lemaitre, J., 1992. "A Course on Damage Mechanics", Springer, Berlin Heidelberg.
- Madenci, E. and Oterkus, E., 2014. "Peridynamic theory and its applications". Springer, New York.

- Miguel, L.F.F., Fadel, M.L.F., Kaminski, J.Jr., Riera, J.D. and Menezes, R.C.R., 2009. "Model uncertainty in the assessment of PES wind loads in transmission line design". Proceedings of the International Seminar on Modeling and Identification of Structures Subject to Dynamic Excitation – emphasis of Transmission Lines, Bento Gonçalves, Brasil.
- Pablo, S., Qiang, D., and Parks, M.L. 2016. "On the consistency between nearestneighbor peridynamic discretizations and discretized classical elasticity models", *Computer Methods in Applied Mechanics and Engineering*, pp. 698–722.
- Puglia, V.B., Iturrioz, I., Riera, J.D. and Kostasky, L., 2010. Random field generation of the material properties in the truss-like discrete element method. In: CILAMCE-MECOM 2010, Buenos Aires, Argentina. Mecánica Computacional, 2010. XXIV: 6793-2801. Santa Fé, Argentina: Asociación Argentina de Mecánica Computacional (AMCA).
- Rädel, M, Bednarek, A-J, Schmidt, J, Willberg, C., 2017. Peridynamics : Convergence & Influence of Probabilistic Material Distribution on Crack Initiation. 6th ECCOMAS Them. Conf. Mech. Response Compos.
- Silling, S.A. and Askari, E., 2005. "A meshfree method based on the peridynamic model of solid mechanics". *Computers & Structures*, vol. 83, pp. 1526–35.
- Silling, S.A., 2000. "Reformulation of elasticity theory for discontinuities and long-range forces". *Journal of the Mechanics and Physics of Solids*, vol. 48, pp. 175–209.
- Silling, SA, Demmie, P, Warren, T.L., 2007. "Peridynamic Simulation of High-Rate Material Failure". *ASME Applied Mechanics and Materials Conference*, Austin, TX. June 2007.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.