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Force Estimation Using the Augmented Kalman filter algorithm

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Abstract. *In several engineering applications, the knowledge of state variables and input forces are required to predict the dynamic behavior of mechanical systems. However, in real applications, it is often difficult to measure directly the physical quantities associated with the state variables and input forces. The Augmented Kalman Filter algorithm can be applied to overcome this limitation. In this approach, the unknown forces are included in the state vector of the system allowing for the estimation of input forces and states, simultaneously. For this aim, a reduced number of measurements is required. In the present contribution, the Augmented Kalman Filter algorithm is applied to a multi-degree-of-freedom (multi-DoF) mass-spring-damper system. The numerical results show that the states and an equivalent set of input forces can be satisfactorily estimated using a reduced number of measurements.*

Keywords: *Augmented Kalman Filter, state estimation, force estimation, mass-spring-damper system.*

1. INTRODUCTION

In many applications of condition monitoring and vibration control, the estimation of vibration responses and input forces are required. Nevertheless, in real systems, the measurement of all states and input forces is not feasible. This limitation can be associated with many reasons, such as the cost of the sensors, the technical constraints regarding the position of the sensors and the aggressive conditions of the environment (Naets *et al.*, 2015).

Input forces can be estimated indirectly by using the available responses of the structure and extracted information from its mathematical model (Hwang *et al.*, 2009). Generally, this problem is solved as an inverse problem involving the identification of inputs from the structural responses. However, this approach assume that a representative model of the system is known, which is difficult in various real problems (Lourens *et al.*, 2012). Consequently, stochastic methods have been applied for to estimate input forces, such as the Augmented Kalman Filter (AKF) algorithm (Álvarez-Briceño and de Oliveira, 2018). This method provides a practical and efficient state and force estimation procedure.

In the conventional KF algorithm, only the states are estimated while the inputs are assumed to be known. Petersen *et al.* (2018) applied the KF algorithm to estimate the full-field response of a bridge. Álvarez-Briceño and de Oliveira (2018) applied the AKF algorithm to a structure under air and water environments. The authors demonstrated that the method is robust enough to estimate the associated input forces for both environmental conditions. Other applications of the AKF algorithm can be found in Hwang *et al.* (2009) and Berg and Miller (2011).

In the present contribution, the AKF algorithm is applied to a four degree of freedom mechanical system considering some of the states as known information. For this aim, the conventional KF algorithm as well as the AKF algorithm are presented in detail in Section 2. Additionally, the modal modeling and state space methodology are reviewed. The current problem and results are discussed in Section 3. Finally, the conclusions are presented in Section 4.

2. METHODOLOGY

2.1 State space equations

Mechanical systems can be modeled by using a set of second-order differential equations, as given by Eq. (1).

$$[M]\{\ddot{\xi}\} + [C]\{\dot{\xi}\} + [K]\{\xi\} = [b]\{f\} + [b_d]\{f_d\} \tag{1}$$

where $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices, respectively, and $\{\xi\}$ is the vector of generalized physical coordinates. The terms $\{f\}$ and $\{f_d\}$ represent the forces and unknown disturbances, respectively, which are applied to the mechanical system at positions described by the matrices $[b]$ and $[b_d]$.

A common approach in structural dynamics and control is to use a modal representation, which often leads to compact and computationally efficient models. Thus, an n x p modal matrix $[\Psi]$ is used to convert the physical model into the modal domain. In this case, n is the number of degrees-of-freedom (DoFs) and p is the number of vibration modes used to represent the system. This approach enables the derivation of a wholly defined modal model, which is a decoupled representation of the system represented by Eq. (1), as given by Eq. (2).

$$\{\ddot{\eta}\} + 2[\Lambda][\Omega]\{\dot{\eta}\} + [\Omega]^2\{\eta\} = [\Psi]^T[b]\{f\} + [\Psi]^T[b_d]\{f_d\} \tag{2}$$

where $\{\eta\}$ is the vector of modal coordinates. Λ and Ω are defined as:

$$[\Omega] = \begin{bmatrix} \ddots & & \\ & \omega_n & \\ & & \ddots \end{bmatrix}, \quad [\Lambda] = \begin{bmatrix} \ddots & & \\ & \zeta_r & \\ & & \ddots \end{bmatrix} \tag{3}$$

where ω_n are the n undamped natural frequencies and ζ_r are the n viscous damping ratios.

The relationship between the physical and modal domains is given by:

$$\{\xi\} = [\Psi]\{\eta\} \tag{4}$$

The system of equations presented in Eq. (2) may be described by ordinary differential equations, in which the time is an independent variable. Thus, an nth-order differential equation can be expressed by a first-order vector-matrix state space differential equation, which is more convenient from a computational point of view (Ogata and Yang, 2002). The state space representation of a mechanical system is given by:

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\} + [B_1]\{w\} \tag{5}$$

$$\{y\} = [H]\{x\} + \{v\} \tag{6}$$

where $\{x\}$ is the state vector defined as $\{\eta, \dot{\eta}\}^T$, $\{u\}$ is the input vector, and $\{w\}$ is the disturbance (also called process noise) vector. $[A]$, $[B]$, and $[B_1]$ are the state, input, and input disturbance matrices, respectively. $\{y\}$ is the measured outputs vector, $[H]$ is the output matrix, and $\{v\}$ is the noise vector. The matrices $[A]$, $[B]$, and $[B_1]$ are given by Eq. (7).

$$[A] = \begin{bmatrix} 0 & \mathrm{I} \\ -\Omega^2 & -2\Lambda\Omega \end{bmatrix} \quad [B] = \begin{bmatrix} 0 \\ \Psi^T b \end{bmatrix} \quad [B_1] = \begin{bmatrix} 0 \\ \Psi^T b_d \end{bmatrix} \tag{7}$$

Equations (5) and (6) represent a continuous time modeling. However, this state space model can be sampled at regular time intervals Δt to obtain discrete-time equations. Thus, Eq. (5) and Eq. (6) are modified as given by Eq. (8) and Eq. (9), respectively.

$$\{x\}_{k+1} = [\Phi]\{x\}_k + [\Gamma]\{u\}_k + [\Upsilon]\{w\}_k \tag{8}$$

$$\{y\}_k = [H]\{x\}_k + \{v\}_k \tag{9}$$

where $[\Phi]$, $[\Gamma]$, and $[\Upsilon]$ are the matrices $[A]$, $[B]$, and $[B_1]$ in discrete-time, respectively. The subscript k refers to the k^{th} time step. These discrete-time terms can be obtained as indicated by Eq. (10) (Berg and Miller, 2011).

$$[\Phi] = e^{A\Delta t}, \qquad [\Gamma] = \left[\int_0^{\Delta t} e^{At}dt\right]B, \qquad [\Upsilon] = \left[\int_0^{\Delta t} e^{At}dt\right]B_1 \tag{10}$$

The discrete-time system matrices can be found by using a numerical approach. In the present contribution, the Matlab® command $c2d$ was used to obtain the discrete-time system matrices.

2.2 Kalman Filter

As presented by Naets *et al.* (2015), the KF algorithm is based on the propagation of the covariance of the state estimates (P). Thus, the KF is an recursive solution to the minimization problem of the trace of this covariance matrix. In addition, the KF equations can be split into two groups, namely *time update* equations and *measurement update* equations. Time update equations are responsible for projecting forward (in time) the current state and error covariance permits to obtain the estimates for the next time step. On the other hand, the measurement update equations are responsible for the feedback. In this case, a new measurement is incorporated into a previous estimate to obtain an improved next estimation. Time update equations can be called as predictor equations. During measurement, update equations can be called as corrector equations. The final estimation algorithm is a predictor-corrector algorithm, as shown in the scheme shown by Fig. 1 (Bishop *et al.*, 2001).

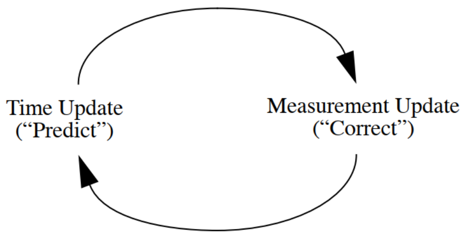


Figure 1. KF algorithm scheme.

Therefore, the idea behind this algorithm is to calculate the best state estimate $\{\hat{x}\}_{k+1}$ by combining a previous estimate $\{\bar{x}\}_k$ with the current measurement $\{\bar{y}\}_k$. It is performed based on the covariances of the prior estimate $[G]_k$ and the current measurement $[R_v]$.

Revisiting the discrete-time state space formulation given by Eq. (8) and Eq. (9), the discrete KF algorithm may be organized in two-steps, namely measurement update and time update (Álvarez-Briceño, 2018).

- Measurement update

$$\{\hat{x}\}_k = \{\bar{x}\}_k + [P]_k[H]^T[R_v]^{-1}(\{y\}_k - [H]\{\bar{x}\}_k) \tag{11}$$

$$[P]_k = [G]_k - [G]_k[H]^T([H][G]_k[H]^T + [R_v])^{-1}[H][G]_k \tag{12}$$

- Time update

$$\{\bar{x}\}_{k+1} = [\Phi]\{\hat{x}\}_k + [\Gamma]\{u\}_k \tag{13}$$

$$[G]_{k+1} = [\Phi][P]_k[\Phi]^T + [\Upsilon][R_w][\Upsilon]^T \tag{14}$$

The initial conditions for $\{\bar{x}\}$ and $[G]$ must be assumed for initialization. $[R_w]$ is the process noise covariance matrix. According to Lourens *et al.* (2012), a constant value is used on the diagonals of $[R_w]$ and $[R_v]$ matrices.

2.3 Augmented Kalman Filter

The discrete KF algorithm can be designed such that the input vector is included in the state vector, resulting in the AKF algorithm. However, the dynamics associated with $\{\dot{u}\}$ present in the state vector is unknown. This lack of knowledge can be modeled by setting $\{\dot{u}\} = 0$ and then adding a random vector $\{z\}$. Thus, $\{u\}$ is assumed as constant. This discrete-time state representation is presented in Eq (15) (Berg and Miller, 2011).

$$\begin{Bmatrix} x_{k+1} \\ u_{k+1} \end{Bmatrix} = \begin{bmatrix} \Phi & \Gamma \\ 0 & \mathrm{I} \end{bmatrix} \begin{Bmatrix} x_k \\ u_k \end{Bmatrix} + \begin{bmatrix} \Upsilon & 0 \\ 0 & \Delta t\, b \end{bmatrix} \begin{Bmatrix} w_k \\ z_k \end{Bmatrix} \tag{15}$$

The matrix $[H]$ depends on the measurements that are used by the AKF, as given by Eq. (16) (Lourens *et al.*, 2012).

$$[H] = \begin{bmatrix} S_d\Psi - S_a\Psi\Omega^2 & S_v\Psi - 2S_a\Psi\Lambda\Omega & S_a\Psi\Psi^T b \end{bmatrix} \tag{16}$$

where S_a, S_v, and S_d are selection matrices for acceleration, velocity and displacement, respectively. S_a, S_v, and S_d are Boolean vectors, in which the locations of the measurements are specified. In the case of displacement measurement, the corresponding equation is given by

$$\{y\}_k = \begin{bmatrix} S_d\Psi & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \{v\}_k \tag{17}$$

The AKF algorithm can be implemented by substituting the matrices presented in Eq (11-14) by those presented in Eq (18).

$$\Phi^* = \begin{bmatrix} \Phi & \Gamma \\ 0 & \mathrm{I} \end{bmatrix}, [\Gamma]^* = 0, [\Upsilon]^* = \begin{bmatrix} \Upsilon & 0 \\ 0 & \Delta t\, b \end{bmatrix}, [R_w]^* = \begin{bmatrix} R_w & 0 \\ 0 & R_z \end{bmatrix}, [H]^* = \begin{bmatrix} S_d\Psi & 0 & 0 \end{bmatrix} \tag{18}$$

where $[R_z]$ is the input covariance matrix.

3. NUMERICAL RESULTS

The AKF algorithm was verified numerically using a four degrees of freedom (DoF) mechanical mass-spring-damper system subjected to sinusoidal loads, as presented in Fig. 2. The four masses of this system have $m = 1$ kg, the spring's stiffness are $k = 500$ N/m, and all damping coefficients are $c = 1$ Ns/m. The resulting continuous time equations were discretized with a time-step Δt = 10 ms. In this application, only the displacements of the first and fourth DoFs (x_1 and x_4, respectively) are considered as known information. Therefore, these measurements are used to estimate the unknown states and the associated input forces.

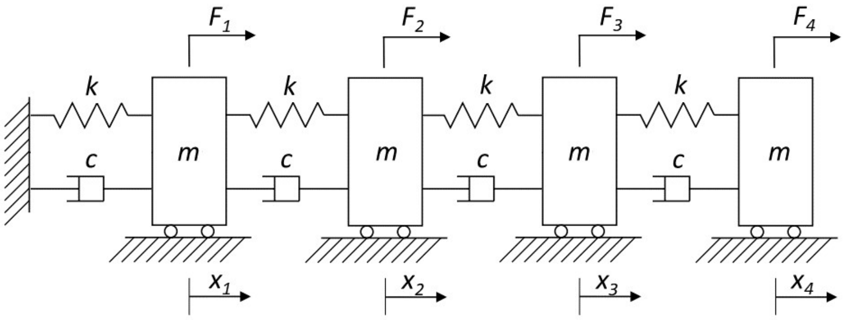


Figure 2. Schematic model of four DoF system.

The process noise covariance matrix $[R_w]$ contains information about the uncertainty on the system model. Therefore, null covariance values are assumed. Additionally, $[R_v]$ (sensors covariance) and $[R_z]$ (input forces covariance) are considered as unknown. Therefore, an optimization procedure is performed to find ideal covariance values that are able to minimize the difference between the measured and the estimated displacements by using the AKF algorithm, as shown in Eq. (19). In this case, the Differential Evolution (DE) optimization algorithm was used (Viana *et al.*, 2008).

$$F_f = \sum_{k=1}^{N} \frac{||\{y\}_k - \{\hat{x}\}_k||}{||\{y\}_k||} \tag{19}$$

where F_f is the objective function, N is the number of known displacements (N = 2), $\{y\}$ and $\{\hat{x}\}$ represent the measured and estimated displacements, respectively.

Table 1 presents the obtained gains through the optimization process. R_v represents the covariance of the sensors and R_{zi} are the input forces covariances ($i = \{1, 2, 3, 4\}$) associated with the four input forces (see Fig. 2).

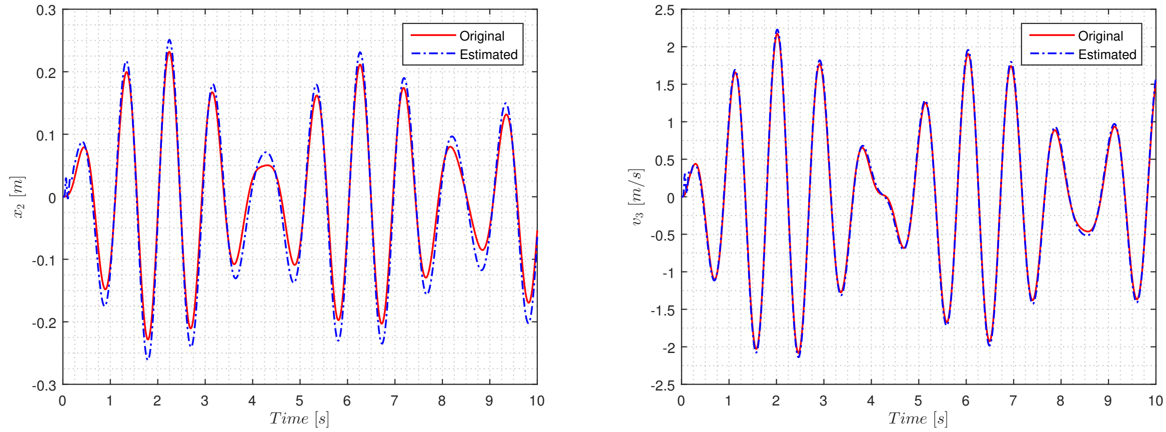
Table 1. Gains obtained by optimization

Parameter	Gain Obtained
R_v	2.5919x10^3
R_{z1}	1.3149x10^9
R_{z2}	4.4674x10^9
R_{z3}	2.5427x10^5
R_{z4}	3.3224x10^5

Figure 3(a) presents the original and estimated displacements of the second DoF (see x_2 in Fig. 2) while Fig. 3(b) presents the original and estimated velocities of the third DoF (associated with x_3 in Fig. 2). Note that a good prediction of this unknown state was obtained, thus evidencing the efficiency of the AKF algorithm. It is worth mentioning that these results were determined after tuning the gains of the AKF.

Figures 4(a) and 4(b) show the original and estimated forces applied to the second and third DoFs (F_2 and F_3), respectively. It can be observed that the estimated forces are different from the applied ones.

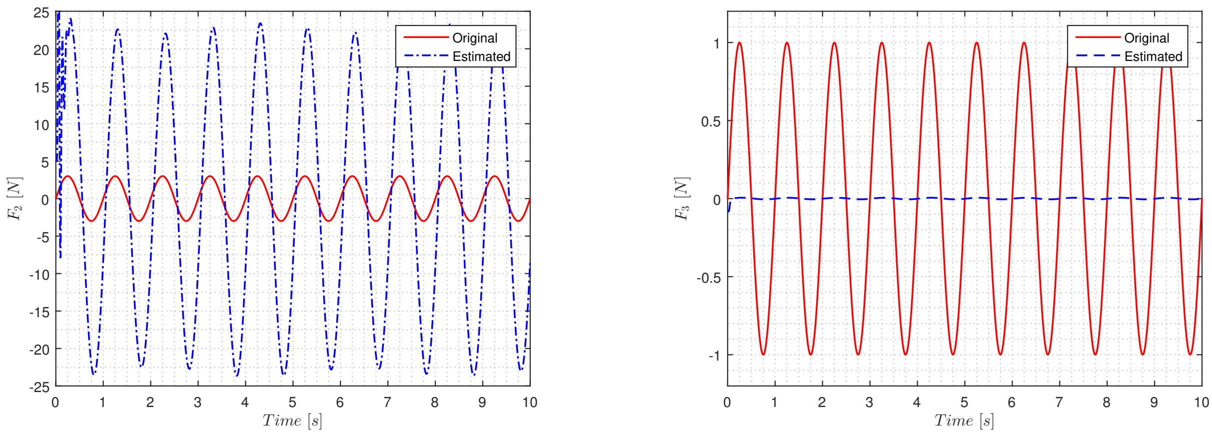
However, when the estimated forces are applied to the mass-spring-damper system, equivalent responses are obtained. Figures 5(a) and 5(b) present the original and equivalent displacements of the second DoF and the velocity of the third



(a) Displacement of the second DoF

(b) Velocity of the third DoF

Figure 3. Comparison between the original and estimated states.



(a) Force applied in the second DoF

(b) Force applied in the third DoF

Figure 4. Comparison between original and estimated forces.

DoF, respectively, for comparison purposes. Note that the estimated vibration responses and the ones determined by applying the estimated forces into the mass-spring-damper system are similar to the original vibration responses (see Fig. 3). Thus, the estimated forces can be considered as equivalent forces.

4. CONCLUSIONS

In this paper, the AKF algorithm was used to estimate unknown states and inputs of a four DoF mass-spring-damper system. The obtained results demonstrated that the AKF algorithm was able to estimate the states satisfactorily and finds a set of inputs able to generate vibration responses similar to the original ones. Nevertheless, the AKF algorithm needs to be well tuned. In this sense, an optimization method was performed in the present contribution. Due to its ability to incorporate modeling errors into the identification process, the AKF algorithm contrasts significantly with deterministic methods in which the accuracy of the model bounds the accuracy of the solution. Therefore, the AKF algorithm represents an useful tool for the estimation of states and force, being able to be considered as a solution for many engineering problems. It is believed that the present work provides a base for future investigations. Therefore, an experimental validation of the technique conveyed is scheduled.

5. ACKNOWLEDGEMENTS

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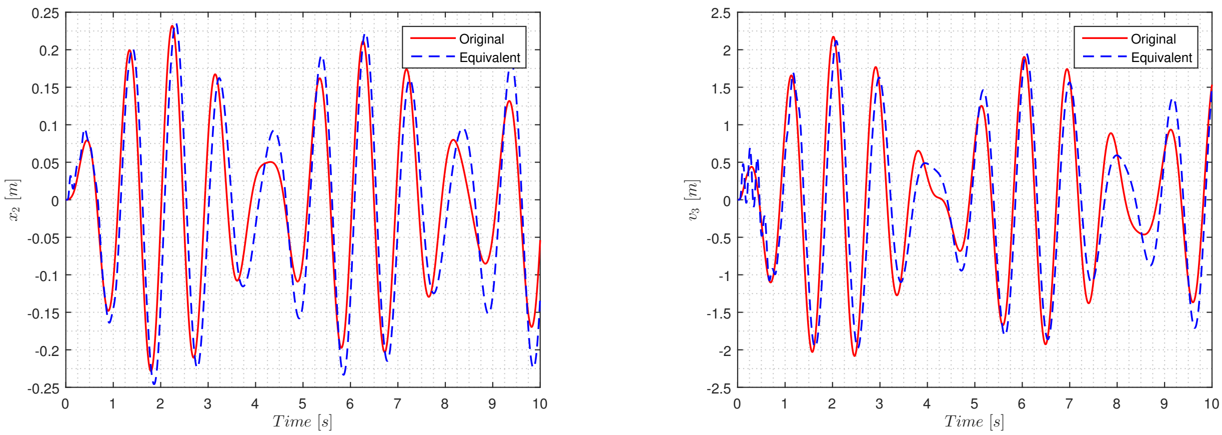

(a) Displacement of second DoF (b) Velocity of third DoF

Figure 5. Comparison between original and equivalent states.

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