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NUMERICAL SIMULATION OF FLOWS IN CARBONATE RESERVOIRS USING A STOKES-BRINKMAN MODEL AND LOCALLY CONSERVATIVE METHODS

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Abstract. *Recently, due the Brazilian Pre-Salt exploitation there has been an increase in the study of carbonate, karst reservoirs. These geological formations contain vugs, cavities and caves, which are voids of different length scales. Traditionally, the mathematical model that represents the flow of fluids in oil reservoirs is based on the Darcy's equation, however, this model may not represent well situations in which it is necessary to deal with carbonate (karst) reservoirs due to the presence of free flow regions. In the Stokes-Brinkman model there is a smooth transition between the free flow region and the Darcy´s flow regime. In this paper, we implement a finite difference numerical discretization to solve Stokes-Brinkmam equations assuming single phase flow in heterogeneous carbonate karstic reservoirs. In order to verify the accuracy of our implementation we solve some simple but representative problems found in literature.*

Keywords*: Finite Difference Method, Carbonate Reservoir, Stokes Brinkman Model*

1. INTRODUCTION

There is a high degree of uncertainty associated with the parameters that lead to the exploration of a field (Nikravesh, 2004; Sancevero, 2007). The productivity guarantee of a reservoir is directly linked to the way these uncertainties are mitigated. For this, chemical, physical, petrophysical and geological properties of rock and fluid must be measured and / or estimated with minimum errors. These properties will be fundamental to compose a geological model that will allow to choose the best decisions to obtain satisfactory results in the production of a certain reservoir.

Based on the Petroleum Resources Management System, in order to consider a rock as a reservoir, the rock must have interconnected voids that allow the accumulation and circulation of recoverable oil and /or gas and in economically viable quantities. Carbonates and sandstones are the main types of reservoir rock, accounting for about 90% of proven reserves in the world (Spadini, 2008). Modern estimates point out that more than 60% of the oil and 40% of the gas world's reserves can be found in carbonate (karst reservoirs) reservoirs.

The phenomena of tectonization and karstification generate many macro-fractures and vugs in the rock formations, in many cases their geometries and size have dimensions much larger than the intergranular space of the pores. In this way, when analyzing the flow of fluids in rocks that have in their composition these structures, we need to be attentive to the impacts of these structures provide in the flow of fluids in these formations (Mylroie, 2012; Huang 2010, 2008; Yan, 2013).

Esteban and Klappa (1983) define karst as a set of diagenetics, or a superposed agglomeration of under exposed carbonate bodies, formed and modeled by the dissolution and migration of calcium carbonate in waters with specific properties. James and Choquette (1988) extend the definition of karst to include all structures with diagenetic characteristics, on macroscopic and sometimes microscopic scales, that are produced during the chemical dissolution, and the associated modification of a set of superposed carbonates, as can see in Figure (1).

Figure 1. Carbonate rock outcropping with a continuous multiple system with presence of vugs, cavities and fracture lines. Available from Wu (2006).

In the specific case of the Brazilian pre-salt basins, in addition to the natural dissolution in the carbonate diagenesis process, other factors contributed to the current configuration of these reservoirs. During the natural process of magma elevation and the opening of the continents, a series of tectonic effects influenced the elevation of the hydrothermal fluids, such as carbon dioxide, which was imprisoned by the impermeability of the salt, contributing even more to the corrosion of the environment (Correa, 2013). This scenario allowed the formation of typical structures of these reservoirs, giving them the ability to accumulate huge quantities of oil not common in other types of reservoirs.

In general, naturally fractured karstic reservoirs present multiple challenges for numerical simulations, one of the major problems of these structures is due to their different scales, for example, we can immediately separate in two scales, a region that presents the porous medium (Darcy flow), with porosity and permeability as well defined and often more uniform, and a free flow region (Stokes flow). The presence of vugs, which are connected by discrete fracture network, can significantly increase both the porosity and permeability of the porous media, but differences between these structures in many cases is not well represented only with these two divisions (Hornung, 1997).

From these considerations a variety of configurations began to be adopted in order to find the model that best represents each situation of flow, respected the reservoir and its singularities. As examples, double porosity and unique permeability by Ng and Aguilera (1999), Ozkan and Raghavan (1991); triple porosity and unique permeability by Pulido et al (2006), Nie et al. (2011); triple porosity and double permeability by Camacho-Velázquez et al. (2005); Some approaches have been developed to model the flow of fluid in naturally fractured karst reservoirs. One of the best known is the Continuous Multiple Model, which in general associate multiple fractures, vugs and porous media at a high and unique permeability and porosity value (Bai et al., 1993; al., 2006; Wu et al, 2006).

Astrid et al. (2009) assert that the most accepted and used model is the Darcy-Stokes model. In this model, the Darcy, Navier-Stokes and mass conservation equations are used. In this approach two domains are separated and each equation are used in a different domain. Darcy's law associated with mass conservation in the porous subdomain and Stokes in the free-flowing subdomain. However, this model presents a problem because in carbonate reservoirs these domains are not well defined, i.e vugs, caves, fractures and the porous medium are interconnected throughout the reservoir in different scales and without clear definitions of beginning and ending. In addition, the free-flow domains contain particles in suspensions that can fill the void space of the porous medium and this phenomenon is not easy to predict.

Finally, we also have the Stokes-Brinkman's equations to model the fluid flows in these reservoirs. These equations associated with the mass conservation equation provide an approach that avoids some of the problems encountered in previous models. This model has its origin in the Darcy and Stokes equations, and can portray the flow in both the rocky matrix and in media with high porosities such as vugs and caves. With this configuration we were able to avoid some problems found in the Darcy-Stokes system, because a unified approach is used with only one moment equation to describe in both domains. The Stokes-Brinkman equation can be shown to be equivalent to those of Darcy and Stokes, since certain parameters are correctly selected in the corresponding flow regions (Gulbransen, 2009, 2010, Ligaarden, 2010). In this paper, we present a single-phase stationary flow model for fluid transport in naturally fractured carbonate karst reservoirs. This model consists of the Stokes-Brinkman equation, finite differences for the solution of the proposed stationary flow model, which provides a smooth transition from standard multiple-porosity/permeability reservoir simulators.

2. MATHEMATICAL MODEL

To find Brinkman's equations (1949) we sought to understand the influences of viscous forces in a fluid particle involved in a porous mass. When we observe that the Darcy equation did not represent the flow of this particle in regions with high permeability, we tried to associate this specific case with the stokes equations. By means of these two equations and taking into account the fluid flow around this particle and the influences of others represented by the porous mass it is possible to arrive at Equation (2) which indicates the balance between the forces acting on a volume of fluid, in other words, a balance of forces that includes the pressure gradient, the divergent viscous tensions and the damping force caused by the porous mass.

In the present paper, in order to model the one phase fluid flow in a karst reservoir, we will use the Stokes-Brinkman model. In this case, we use the mass balance equation and the Brinkman equation. The conservation of mass can be written as:

$$\frac{\partial}{\partial t}(\phi\rho)+\nabla.(\rho\vec{v})=\dot{q} \quad (1)$$

where we can define a single-phase fluid flow in porous media and a free-flow region assuming ϕ and k as the porosity and permeability value respectively, ρ the fluid density, v the velocity vector, t is time, $\dot{q}$ as the mass injection (+) or production (-) rate per unit volume, and ∇ is the divergent operator.

In the Sotkes-Brinkman model (Brinkman, 1949), we combine Darcy's equation (Darcy, 1856), and Stokes's equation (Fox, 1998), into the Stokes-Brinkman Equation (2), which is given by:

$$\vec{v}+\frac{\boldsymbol{k}}{\mu}(\nabla p-\rho\vec{g}-\mu^{*}\Delta\vec{v})=0 \quad (2)$$

where p is the pressure, g is earth gravity, μ^* is called the effective fluid viscosity, and **k** is the permeability tensor of the rock matrix. Equations (1) and (2) constitute the Stokes-Brinkman system, which unifies fluid flow in nonporous and porous regions. It is possible to say that Equation (2) can be made mathematically equivalent to Darcy's equation and Stokes' equation by appropriate assignments of **k** and μ^* values. We can represent the different flow regions, if we set $\mu^*=0$ in porous media or if we choose large k values (ideally let $\boldsymbol{k}\to\infty$) and set $\mu^*=\mu$ in the nonporous regions. Effective viscosity must be calculated when the presence of particulate matter in a fluid medium changes the value of the dynamic viscosity, according to Einstein (1911), by Brinkman (1949)

$$\mu^*=\mu(1+2.5\phi) \quad (3)$$

However, most authors consider the effective viscosity equal to the dynamic viscosity, disregarding the effect of the suspended particles. Compared to the former differential treatment, this uniform approximation only introduces a small perturbation into the numerical solutions, because $\mu^*\Delta\vec{v}$ is normally several orders of magnitude smaller than the other terms on the left side of Eq. (2) in typical porous media (Gulbransen, 2009).

3. NUMERICAL FORMULATION

Having chosen the Brinkman model in the whole domain and aiming to study the coupled fluid flow behavior, one must solve two equations for two unknown variables: pressure and the components of a velocity field. We cannot obtain an explicit expression of the velocity vector *v* as a function of the pressure *p* from the Stokes-Brinkman equation (Equation 2). Therefore, we solve for both pressure and velocity at the same time.

Following the scheme proposed in He (2015), to solve the problem, we have applied the standard finite difference method for the discretization of the transient flow model in cartesian coordinates using a uniform and structured mesh. In our strategy we have adopted a scheme in which the pressures were associated to the centroids of the blocks and the velocities in the faces between them as show in Figure (2).

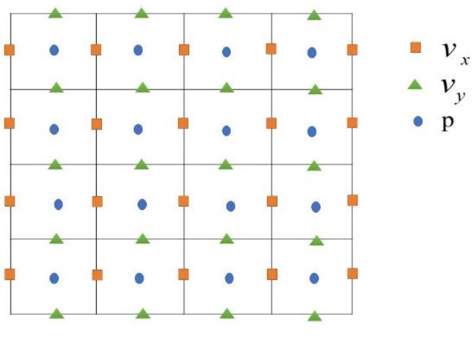


Figure 2. Schematic representation of pressure and velocities in a structured mesh

The mass conservation equation (Eq.1) is scalar, and for a grid block with indices (i), it can be discretized as:

$$\left[\frac{\rho^{n+1}v^{n+1} - \rho^{n}v^{n}}{\Delta t}\right]+\left[\frac{(\rho v_x)_{i+1/2} - (\rho v_x)_{i-1/2}}{\Delta x}\right]^{n+1}+\left[\frac{(\rho v_y)_{j+1/2} - (\rho v_y)_{j-1/2}}{\Delta y}\right]^{n+1}+\dot{q}^{n+1}=0 \tag{4}$$

where v_x and v_y are the velocities in the corresponding x and y directions, q is the mass injection / production. The subscript n denotes the nth time level. In these subscripts, 1/2 means that the velocities are defined at the block interfaces rather than block centers.

The Stokes-Brinkman equation (Eq. 4) can be discretized for each grid block, as:

$$(v_x)_{i+1/2}+(\frac{k_x}{\mu})_{i+1/2}\left[\left(\frac{p_{i+1}-p_i}{\Delta x}\right)-\mu^*\left(\frac{v_{x,i+3/2}-2v_{x,i+1/2}+v_{x,i-1/2}}{\Delta x^2}\right)\right]-\left(\frac{k_x\mu^*}{\mu}\right)_{i+1/2}\left(\frac{v_{x,i+1/2,j+1}-2v_{x,i+1/2}+v_{x,i+1/2,j-1}}{\Delta y^2}\right)=0 \tag{5}$$

$$(v_y)_{j+1/2}+(\frac{k_y}{\mu})_{j+1/2}\left[\left(\frac{p_{j+1}-p_j}{\Delta y}\right)-\mu^*\left(\frac{v_{y,i+1,j+1/2}-2v_{y,j+1/2}+v_{y,i-1,j+1/2}}{\Delta x^2}\right)\right]-\left(\frac{k_y\mu^*}{\mu}\right)_{j+1/2}\left(\frac{v_{y,j+3/2}-2v_{y,j+1/2}+v_{y,j-1/2}}{\Delta y^2}\right)=0 \tag{6}$$

where k_x and k_y are the permeability values have been estimated by the simple harmonic average as follow

$$k_{1/2}=\frac{2k_l k_{l+1}}{k_l+k_{l+1}} \tag{7}$$

where $k_{1/2}$ represents interface permeability and k_l and k_{l+1} the permeability of neighboring blocks, (in both directions x and y). In equations (5) and (6) we do not consider the gravity term, and centered finite difference were adopted.

The Stokes-Brinkman equation defines a nonlinear relationship between the velocity vector and pressure. If we discretize the reservoir into n_x and n_y grid blocks in the subscripted directions, and impose no flow boundary conditions on all the four boundaries, then we will have a total of pressure variables

$$Tot_{pres}=n_x n_y \tag{8}$$

and to velocities we have:

$$Tot_{vel_x}=n_x n_y-n_y \tag{9}$$

$$Tot_{vel_y}=n_x n_y-n_x \tag{10}$$

From these equations we can deduce that each pressure variable is associated to Equation (4), as well as the velocities v_x and v_y associated with Equations (5) and (6) respectively.

In order to take into account compressibility effects, we use (Yaws, 1998):

$$\rho=a\cdot b^{-(1-T/T_c)^n}\cdot e^{c_o(p-p_{ref})} \tag{11}$$

where a, T_c, n and b are fluid-specific constants, T is the reservoir temperature, C_o is the compressibility of the oil and $p_{ref.}$ the reference pressure. The discretized equations can then be recast into a system of nonlinear residual functions and solved with the Newton's method (Nocedal and Wright, 2006) by constructing the Jacobian matrix and iteratively solving for pressure and velocities. The nonlinear system can be arranged into the following residue equation:

$$res(x)=0 \tag{12}$$

and *res* is the residue vector of which each entry is the left-hand side of one of Equations (4-6) at whichever block center or interface the corresponding independent variable is defined, and x is the unknown vector formed by p, v_x and v_y:

$$\mathrm{x}^T=\begin{bmatrix} p_1 & p_2 & \cdots & p_n & v_{x1/2} & v_{x3/2} & \cdots & v_{xn} & v_{y1/2} & v_{y3/2} & \cdots & v_{yn}\end{bmatrix} \tag{13}$$

The Jacobian matrix J is composed of the derivatives of the Stokes-Brinkman and Mass Conservation in relation to each of its unknown variables, so we have:

$$J(x)=\begin{bmatrix} \frac{\partial res_1}{\partial x_1} & \frac{\partial res_1}{\partial x_2} & \cdots & \frac{\partial res_1}{\partial x_n} \\ \frac{\partial res_2}{\partial x_2} & \frac{\partial res_2}{\partial x_2} & \cdots & \frac{\partial res_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial res_n}{\partial x_n} & \frac{\partial res_1}{\partial x_1} & \cdots & \frac{\partial res_n}{\partial x_n} \end{bmatrix} \tag{14}$$

Finally, with all the equations, the residuals and the Jacobian matrix, it is possible to find the vector x compose by pressure and velocity values in an iterative way by the follow expression:

$$x^{k+1}=x^k-J^{-1}(x^k)res(x^k) \tag{15}$$

where k represents the iterations counter the convergence criterion consider the number of iterations $k \le 7$ or the L_1 norm of residuals $\left\| res(x) \le 10^{-8} \right\|$

The workflow with the Newton-Raphson simulation using stokes-brinkman equations can be seen at Figure (3).

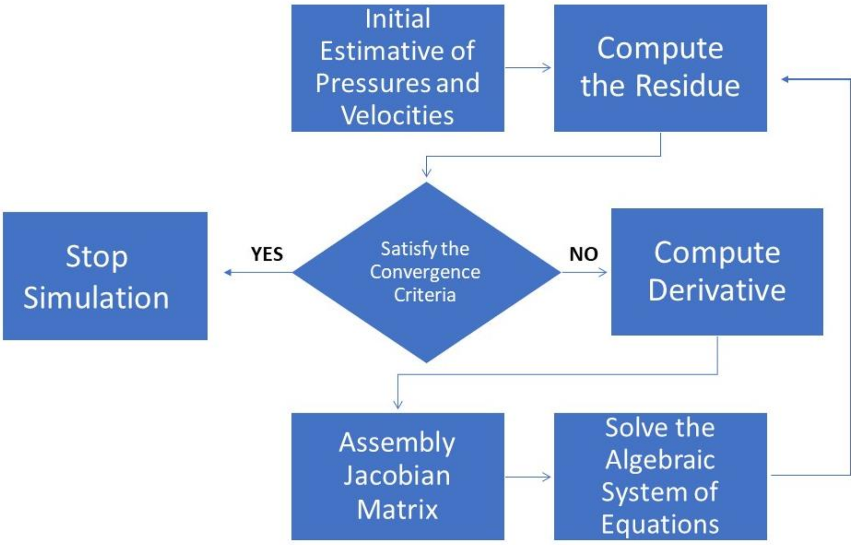


Figure 3. Simulation workflow for the transient Stokes-Brinkman model

4. NUMERICAL EXAMPLE

Figures (4 – 6) are shows meshes that were used to simulate the cases studied, all with injection and production well at opposite ends represented by red and blue dots in the first and last blocks, respectively. In Figure (4a) we have a one-dimensional mesh composed of 100 blocks representing a homogeneous reservoir of five hundred meters.

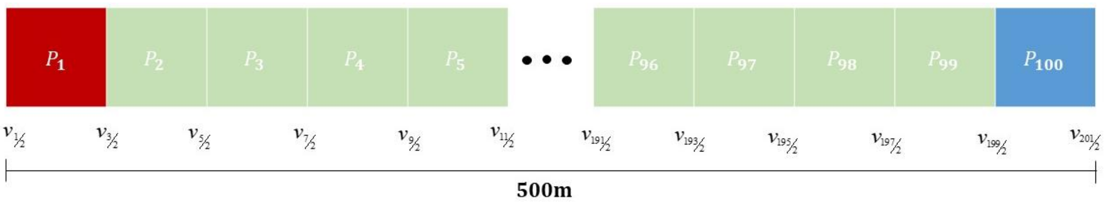


Figure 4. Unidimensional computational mesh

The other simulations were performed in the uniform structured 2D mesh with 2500 blocks (50x50) shown in Figure (5), which represents 1/4 of five-spot in homogeneous and heterogeneous media.

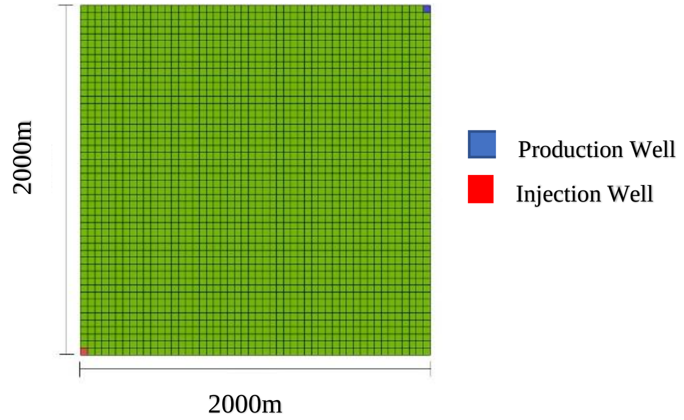


Figure 5. Bidimensional computational mesh.

The cases representing the heterogeneities were simulated in the meshes illustrated in Figures (6a), (6b) and (6c), which always has the same size and configuration of wells.

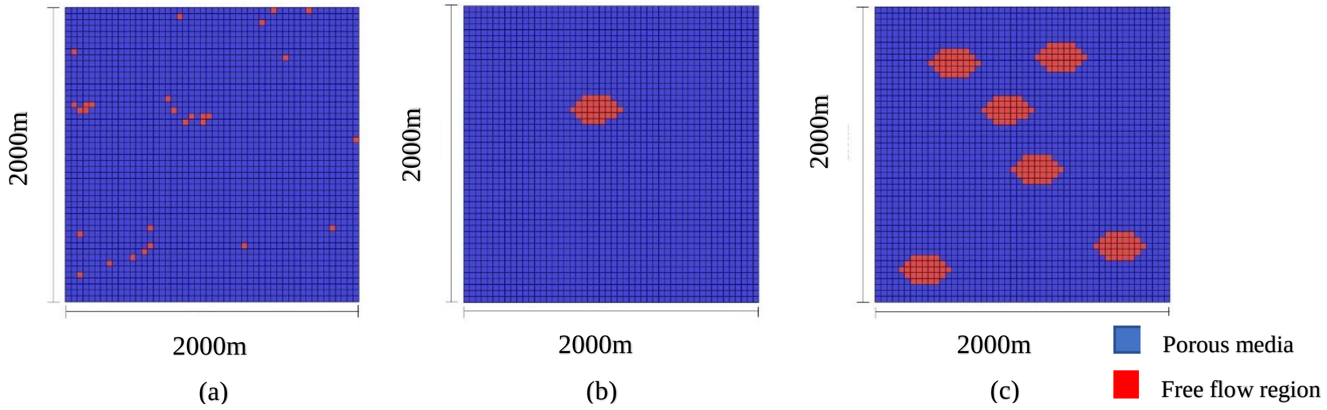


Figure 6. 2D computational grids for the heterogeneous reservoir

The insertion of the heterogeneities arranged in Figure 6a was generated in a random manner, considering that there is no specific law that provides for the formation of vugs in carbonate reservoirs, and these were each represented by a mesh element. However in figures 6b and 6c we tried to represent the karst cavities with the ellipsoidal shape, as is commonly represented in the literature. The arrangement of the caves in Figure 6 (c) was also randomly generated.

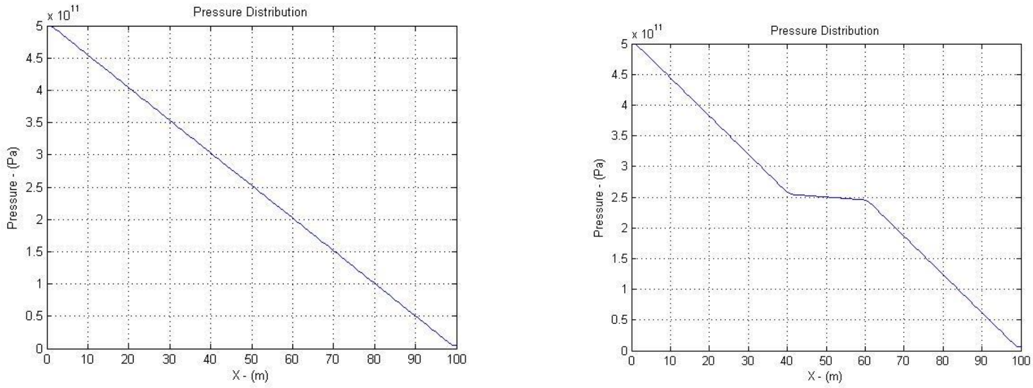
Table 1 presents the values defined for the physical properties of fluids and rock, for the cases analyzed, which were taken from He (2015). In all simulated cases we considered no flow and prescribed pressure at wells boundary conditional.

Table 1. Fluid and rock properties and constants used for simulation.

Properties	**Value**
a	2.2807×10^{-1} kg/m^3
b	2.5476×10^{-1}
T_c	2.9568×10^{-2} K
n	2.6940
C_o	1.0×10^{-8} Pa^{-1}
P_{ref}	1.0×10^{5} Pa
ϕ	1.5×10^{-1}

k	1.5 x 10^{-14} m^2
μ	0.2708 Pa . s
$\mu*$	0.2708 Pa . s

In order to analyze the behavior of the flow in regions with different permeabilities, initially, a simplified one-dimensional, compressible, stationary and single-phase case was run. The idea was to analyze the pressure behavior in the interfacial regions before blocks with significant variation of permeability. Figure 7 (a) shows that in the case where we have a homogeneous reservoir with permeability equal 15md, the pressure differential between the injection well and the production well decreases linearly, typical behavior for the Darcy flow in these described conditions. On the other hand, in Figure 7 (b) we can see that between the blocks with different permeability, we have a change in the pressure behavior, in which case the permeability of the blocks 40 to 60 was considered 1000md, while the remainder of the blocks 15md. The pressure remains practically constant in the free-flow region, obtaining an almost imperceptible reduction. And when leaving this region, it drops again linearly.



(a) (b)

Figure 7. Pressure distribution along the homogenous (a) and heterogeneous (b) one-dimensional reservoir.

This case allows us to observe that the Stokes-Brinkman equations are well sensitive to flows with significant variation of permeabilities. In the following cases, pressure fields of Darcy flows were simulate using two-dimensional, incompressible, stationary and single-phase cases.

In the first case which represents a homogeneous reservoir with permeability of 15mD. The solution is show in Figure (8), it can be observed that the pressure profile has a clearly defined radial flow patter characteristic. The cases were simulated using the mesh represented by Figure 5, with the properties of Table 1 assuming no flow in the boundaries and a production and injection pressure of 2.9502x10^7 Pa and 3.1327x10^7 Pa. Note that the natural logarithm of the pressure is rather than the pressure itself.

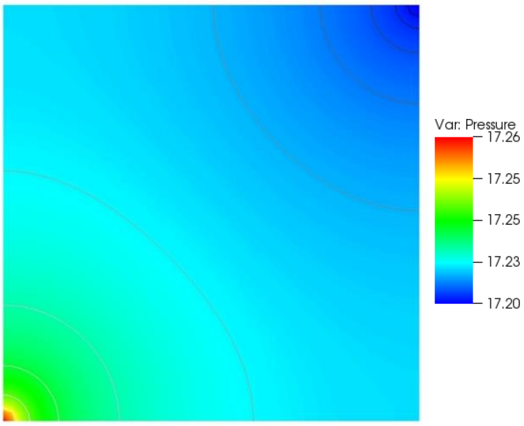


Figure 8. Pressure distribution along the homogenous bidimensional reservoir.

Figure 9 shows the influence of the heterogeneities inserted in the reservoir according to the distribution seen in Figure 6 (a), its permeabilities are set at 1,000mD while the rock matrix has 15mD of permeability. As can be seen, even with a significant variation in permeability, these apertures are disposed in an isolated manner, they do not strongly influence the pressure field, however, as they agglomerate and form even larger cavities, they can distort the pressure profile, giving a different characteristic to the flow of the fluid in those regions.

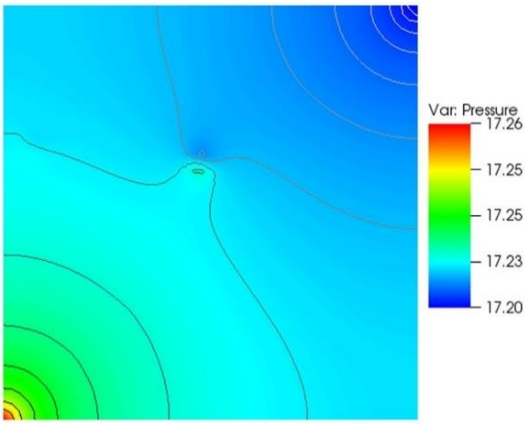


Figure 9. Pressure distribution along the heterogeneous bidimensional reservoir with multiples vugs.

Figure 10 shows the influence of a single karst structure with a permeability of 1,000mD, representing a cave (Figure 6b), while the rest of the reservoir has only a carbonate matrix of 15mD permeability. As can be seen in the vicinity of this region, the pressure profile changes abruptly, and the whole area with the highest permeability has almost the same pressure. This indicates that this carbonate structure has a significant influence on the flow in this reservoir.

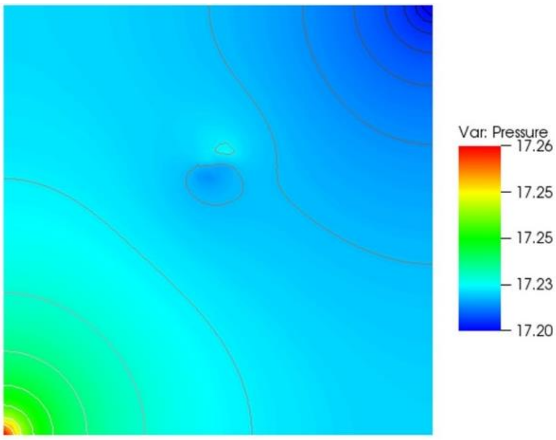


Figure 10. Pressure distribution along the heterogenous reservoir with a unique cave

Finally, we decided to analyze the influence of multiple cavities in the pressure field. Thus the distribution presented in Figure 6 (c), under the same flow conditions as previous cases, was and realized that the pressure profile changes completely considering each of the carbonate structures and their this permeabilities. These results can be observed as shown in the Figure (11).

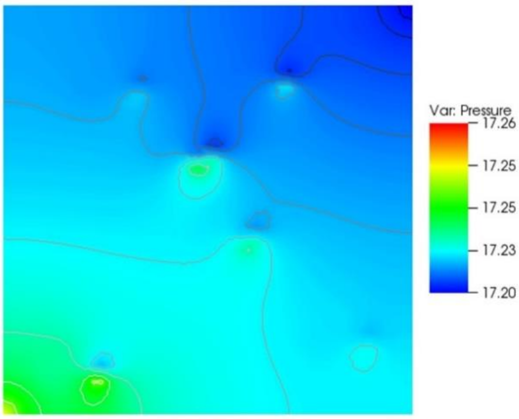


Figure 11. Pressure distribution along the heterogenous reservoir with a multiple cave

All the cases studied originated in the cases simulated by He (2015) and all the values obtained, as well as the values of maximum and minimum were close to that of the author.

5. CONCLUSIONS

In view of the observed behavior, it has been possible to perceive that the Stokes Brinkman equations are sensitive to large heterogeneities, representing consistently the flow in regions of different permeability coexisting in the same medium. Besides that, the presence of carbonate structures with high permeabilities, when agglomerated or of large size, modify the pressure differential and this consequently brings about changes in the flow of fluids in porous media. This leads us to conclude about the necessity of the proper its representation, closer of the karstic structures in order to reality obtain good results to in the exploration and production of an oil field.

6. ACKNOWLEDGEMENTS

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