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EXTRACTION OF TRANSVERSE AND IN-PLANE DISPLACEMENTS AND STRESSES FROM GENERALIZED FINITE ELEMENT METHOD BASED ON REISSNER-MINDLIN MODEL

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Abstract. *This paper reports the development of extraction procedures for stresses and displacements, post-processed from the Finite Element (FEM) models of laminated composite plates based on the Reissner-Mindlin first order model. The procedures start with the stresses obtained directly from the Generalized FEM results, obtained by constitutive equations. These distributions are modified by imposition of the following general local equations: (a) local satisfaction of the local equilibrium equations; (b) boundary conditions on both laminate faces; (c) equivalence with the constitutive shear stresses resultants; (d) stress continuity at layers interfaces. (e) transverse and in-plane displacements are obtained from the three-dimensional local stress-strain relations for anisotropic layers. Numerical comparison with three dimensional exact solutions shows that the transverse stresses obtained across thickness are effective for non-symmetric laminates. The displacements obtained across the thickness are satisfactory.*

Keywords: *Layered composite structures, Stress recovery, Post-processing, Reissner-Mindlin model , Generalized finite element method*

1. INTRODUCTION

The numerical analysis of structural laminated composite plates continues to present an important challenge in the accurate estimate of localized stress distribution, mostly the transverse components. This situation is aggravated by the fact that composite laminates show several failure modes whose simulation demands precise stress estimates at geometric stress concentrations, stress singularities. Since the laminate is composed by several layers stacked, the interfaces are regions prone to failure. The transverse resistance of the set is usually smaller than the in-plane resistance. Therefore, it is crucial a reliable evaluation of the transverse stresses.

Considering a plate or shell in its three-dimensional character, the obvious formulation to obtain numerical solution which approximate the three-dimensional solid mechanics solution is by a Finite Element model (FEM) based on solid elements. However, even today, this strategy is too time consuming to be useful in everyday engineering. Most of the analysis are done in commercial codes, which are based on first order bending models, of type Reissner-Mindlin. This model considers uniform shear strain deformations across thickness and discontinuous transverse shear stresses, which are constant in each layer. When one considers the exact three-dimensional exact relations, it is clear that the transverse deformations should be discontinuous and the stresses should be variable in each layer and continuous across layers. Also, these stresses should satisfy the force conditions on both faces of the laminate, which also is not satisfied by the Reissner-Mindlin model. In spite of all these shortcomings, this model is widely used by its low cost and approximations that are satisfactory for thin laminates, in overall quantities like transverse displacements, first vibration modes, in-plane stress components and shear stress resultants.

This discrepancies of the Mindlin model are well known and there are several procedures proposed to obtain improved results from those directly obtained from the Generalized Finite Element Method (GFEM) results. The most used is the one due to Chaudhuri (1986), which consists in using the in-plane local equilibrium equations, to integrate across the laminate thickness a continuous distribution of the transverse shear stresses.

Noor *et al.* (1990), has a very intricate iterative procedure to extract all stress components. The procedure is usually implemented in cases of absence of inertia and body forces. In most cases, the transverse integration starts with null stresses at the bottom face of the laminate, or a prescribed nonzero value. At the end of the integration, the integrated values do not satisfy the force boundary conditions at the top face.

Most authors concentrate in techniques to recover transverse stresses, based on the local equilibrium equations. The objective of the present investigation is the attempt to obtain improved displacement and stresses that satisfy approximately, other general equations of 3D continuum mechanics, that is, besides local equilibrium in domain, but also on the boundary of the faces, interlayer continuity. Also, we seek to extract estimates of transverse normal stresses and displacement across the laminate thickness, which are not provided by the direct GFEM computation.

With this regard, the GFEM provide a very opportune setting to perform the extraction, because of its easy way of doing in-plane enrichment of the function basis used in the generalized displacement functions of the Mindlin model. Even in problems without any discontinuity of data or solution, the basis can be easily enriched to obtain polynomial of higher orders, using a fixed mesh of nodes and elements. This allows, in a straightforward way, all the differentiations necessary in all stress recovery processes available. It is worth noting that commercial codes, uses low order finite elements, usually linear or quadratic. In these cases, the element basis does not allow the differentiations necessary to most of the recover processes. Therefore, it becomes necessary to perform a previous step, which consist in computing stress distributions defined over a patch of elements, with order higher than the one of the elements, using least square methods to minimize errors, or other procedure. This new stress distribution is, next, used in the extraction procedures. On the other hand, in GFEM, the basis can be of polynomials of reproducibility of degrees 3 or 4, which allows a straightforward way to extract the stresses and displacements.

Present paper outlines the main points of the formulation to extract transverse normal and shear stresses in laminates, processing results obtained from GFEM Mindlin kinematic model. Next it shows some comparative results with three-dimensional exact elastic solution for a benchmark problem.

2. BIBLIOGRAPHIC

2.1 Generalized Finite Element Method – GFEM

The GFEM is a numerical variation of the Finite Element Method in which enrichment of the basis of approximation is performed straightforwardly, without need to change the mesh partition of node quantity or positions. The mesh (Fig.1) is used to generate geometric modeling, to easy the numerical integration process, and to define a Partition of Unity (PoU), usually with the usual linear Lagrangian functions. The PoU is enriched by an arbitrary set of enrichment functions. In this work are the following:

Linear enrichment, $p = 1$: $L_{i\alpha} = [1, \overline{x}, \overline{y}]$;

Quadratic enrichment, $p = 2$: $L_{i\alpha} = \left[1, \overline{x}, \overline{y}, \overline{x}^2, \overline{xy}, \overline{y}^2\right]$;

Cubic enrichment, $p = 3$: $L_{i\alpha} = \left[1, \overline{x}, \overline{y}, \overline{x}^2, \overline{xy}, \overline{y}^2, \overline{x}^3, \overline{x}^2\overline{y}, \overline{xy}^2, \overline{y}^3\right]$;

Quartic enrichment, $p = 4$: $L_{i\alpha} = [1, \overline{x}, \overline{y}, \overline{x}^2, \overline{xy}, \overline{y}^2, \overline{x}^3, \overline{x}^2\overline{y}, \overline{xy}^2, \overline{y}^3, \overline{x}^4, \overline{x}^3\overline{y}, \overline{x}^2\overline{y}^2, \overline{xy}^3, \overline{y4}\,]$ where

$$\overline{x} = \frac{x - x_\alpha}{h_\alpha}, \qquad \overline{y} = \frac{y - y_\alpha}{h_\alpha}, \tag{1}$$

where x_α, y_α are coordinates of node $\mathbf{x}_\alpha$ and h_α is a representative ray of the cloud ω_α. These are chosen from polynomials, in smooth regions of the solution, or special ones to cope with many different types of singularities.

Therefore, in the last 20 years, GFEM has become gradually the preferred tool to model problems like crack opening, moving discontinuity in plastic band, discontinuity associated with internal boundaries in multi-phase materials. Usually, the enrichment functions are chosen from parts of the analytic solutions of the boundary value problem at hand and multiplied by the PoU nodal functions (Mendonça *et al.*, 2011). A more complete review can be seen in Torres (2008).

In the present paper, tests have been conducted using three node triangular elements, with linear PoU, and polynomial enrichment.

2.2 Reissner-Mindlin model

The Reissner-Mindlin model, also known as First-Order Shear Deformation Theory (FSDT), considers constant shear across the thickness of homogeneous isotropic plates, initially proposed by Reissner (1945) and Mindlin (1951). Subsequently they were extended to composite materials by (Yang *et al.*, 1966) and modified by Whitney and Pagano (1970).

Because this theory requires only class C^0 approximation functions, computational implementation is facilitated and is widely used in commercial software and academic applications. However, in traditional FEM implementations, there may be numerical pathologies such as shear locking for thin plates, which can be mostly avoided by GFEM.

The hypothesis of the theory considers that any segment initially straight and normal to the undeformed reference surface of the plate remains straight and inextensible, but not necessarily normal to the deformed reference surface, allowing some degree of approach to transverse shear (Mendonça, 2019).

Since it is a first-order theory, the displacements are assumed to vary linearly along the thickness. The Mindlin's kinematic model can be summarized by the following displacement relations and the linear strain-displacement relations in Eq. $(2)_1$ and the three equations of local equilibrium for infinitesimal deformations are given by Eq. $(2)_3$

$$\begin{array}{llll} u(x,y,z) = u^o(x,y) + z\theta_x(x,y), & \varepsilon_x = \frac{\partial u}{\partial x}, & \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, & \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + b_x = 0, \\ v(x,y,z) = v^o(x,y) + z\theta_y(x,y), & \varepsilon_y = \frac{\partial v}{\partial y}, & \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}, & \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + b_y = 0, \\ w(x,y,z) = w(x,y), & \varepsilon_z = \frac{\partial w}{\partial z}, & \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, & \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + b_z = 0. \end{array} \quad (2)$$

2.3 Usual Post-processing

Low order plate theory, such as the Reissner - Mindlin model, calculates the field of transverse stresses from the constitutive equations. As Mindlin model is based on hypotheses that allow only an approximation of constant shear transverse tensions along the thickness, it is possible to use the equilibrium equations to calculate these stresses more precisely. Initially Pagano (1969, 1970) calculated the cross shear stresses, considering Kirchhoff's hypotheses, by the integration process and compared them with the exact solution obtained from the theory of three-dimensional linear elasticity. The results are satisfactory within Kirchhoff's hypotheses. Due to the fact that it has few analytical solutions to determine the transverse tensions and restricted to problems with loads and simple boundary conditions, studies using the Finite Element Method began to be developed in order to obtain solutions of more complex problems.

Chaudhuri (1986) used the first two equilibrium equations to calculate the variation of the cross shear stresses along the thickness of thick laminates from the in-plane stresses of an FEM model using the Layerwise Constant Shear Angle Theory - LCST. These equations result in:

$$\begin{array}{lcl} \tau^i_{xz}(z) - \tau^i_{xz}(-H/2) & = & -\int_{-H/2}^{z} \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + b_x \right) dz, \\ \tau^i_{yz}(z) - \tau^i_{yz}(-H/2) & = & -\int_{-H/2}^{z} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + b_y \right) dz, \\ \sigma^i_z(z) - \sigma^i_z(-H/2) & = & -\int_{-H/2}^{z} \left(\frac{\partial \tau^i_{xz}}{\partial x} + \frac{\partial \tau^i_{yz}}{\partial y} + b_z \right) dz. \end{array} \quad (3)$$

The previous works did not mention the normal transverse stress. Engblom and Ochoa (1986) used quadrilateral elements to obtain the in-plane stresses and the equilibrium equations to obtain the shear and normal transverse stresses. And obtained efficient solutions for thin and moderately thick laminates from the Kirchhoff-Love plate model.

Chaudhuri (1986) presented a method to estimate, more precisely, the shear stresses. This procedure is commonly called stress recovery and part of the integration of local equilibrium differential equations, according Eq. (3). The accuracy of the process depends on the quality of the in-plane stresses. Some typical situations occur, for example, in a linear problems, with a symmetric laminate subjected only to transverse loads, the normal resultant forces are null, both in exact and in GFEM approximate solutions. In this case, the GFEM solution will result in correct values of $\tau^i_{xz}(H/2) = 0$ at the integration process. However, in asymmetric laminates subjected only to transverse loads, or in symmetric ones under in-plane loads, the GFEM equilibrium, in general, is not satisfied, and $\tau^i_{xz}(H/2)$ will be incorrect.

In addition to these authors, many others have published methods for determining interlaminar stresses from low order models. Noor *et al.* (1990, 1994); Noor and Malik (1999, 2000) presented several corrector-predictor procedures for the accurate determination of global, as well as detailed, static and vibrational response characteristics of plates and shells. All cases use the FSDT for mechanical and thermal loads.

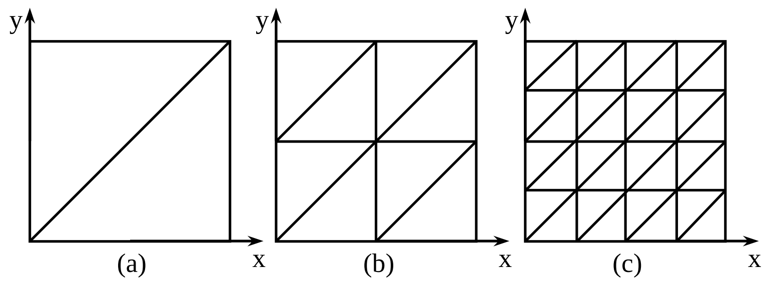


Figure 1. Examples of meshes used in the laminated plate problem, with mesh indices M= $1, 2$ and 4, and coordinate axis.

3. FORMULATION

In this chapter we present the formulation used to obtain the in-plane and transverse displacement through thickness by the integration method and post-process to correction the integrated transversal stresses, based on GFEM results obtained for the linear problem of laminate bending based on the Reissner-Mindlin.

3.1 In-plane and Transverse Displacement Through Thickness

An estimate can be obtained post-processing the entire set of stress components available: the in-plane components σ and the extracted transverse ones, $\tau^i_{xz}(z)$, $\tau^i_{yz}(z)$ and $\sigma^i_z(z)$, computed at a given point (x, y). The staring point is the general 3D local stress-strains relations for an anisotropic layer, $\varepsilon = \mathbf{S}\,\sigma$, this is

$$\left\{\begin{array}{c}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{array}\right\} = \overline{\mathbf{S}}\left\{\begin{array}{c}\sigma_x\\ \sigma_y\\ \sigma^i_z\\ \tau^i_{yz}\\ \tau^i_{xz}\\ \tau_{xy}\end{array}\right\}. \tag{4}$$

Where matrix $\overline{\mathbf{S}}$, is the material flexibility matrix, and for orthotropic material:

$$\overline{\mathbf{S}} = \mathbf{T}^T\mathbf{S}\mathbf{T} \tag{5}$$

Now we integrate the strain-displacement relation according Eq. 6

$$u^i(z) = \int_{-H/2}^{z}\left(\gamma^i_{xz} - \frac{\partial w^i}{\partial x}\right) dz, \qquad v^i(z) = \int_{-H/2}^{z}\left(\gamma^i_{yz} - \frac{\partial w^i}{\partial y}\right) dz, \qquad w^i(z) = \int_{-H/2}^{z}\varepsilon_z\, dz. \tag{6}$$

After integration until the upper surface, $z = +H/2$, we take the value at the reference surface, $u_o = u^i(0)$, $v_o = v^i(0)$ and $w_o = w^i(0)$, and perform a translation to correct the function, that is, compute

$$\delta u = u_o - u_{GFEM}, \qquad \delta v = v_o - v_{GFEM}, \qquad \delta w = w_o - w_{GFEM}, \tag{7}$$

$$u^i(z) = u^i(z) - \delta u, \qquad v^i(z) = v^i(z) - \delta v, \qquad w^i(z) = w^i(z) - \delta w. \tag{8}$$

where u_{MEF}, v_{MEF} and w_{MEF} are the GFEM value of displacement, which is associated with the reference surface. The index ^{i} refers to the results obtained by the integration method.

3.2 Correction of the Integrated Transverse Stresses

Let us consider a cubic polynomial in z, $f(z) = a + b\,z + c\,z^2 + d\,z^3$, and restrict $f(z)$ such that it satisfies the following conditions: $f(-H/2) = 0$ and $f(H/2) = A$. The coefficients are restricted such that the polynomial takes the form $f(\mathbf{x}, z) = A(\mathbf{x})\,(1/2 + z/H) + c\,(z^2 - H^2/4)$, where $A(\mathbf{x})$ is supposed to depend on the position $\mathbf{x} = (x, y)$ over the reference surface. Consider the following problem: given $\tau_{xz}(z)$ and $\tau_{yz}(z)$ computed direcly from the constitutive and kinematic equations, and given $\sigma^i_z(z)$, $\tau^i_{xz}(z)$ and $\tau^i_{yz}(z)$ computed by integration of the local motion equations following the Chaudhuri and Seide (1987) procedure. Find the adjusted functions:

$$\sigma_z(z) = \sigma^i_z(z) + f_z(z), \qquad \tau_{xz}(z) = \tau^i_{xz}(z) + f_x(z), \qquad \tau_{yz}(z) = \tau^i_{yz}(z) + f_y(z), \tag{9}$$

where $f_x(z)$, $f_y(z)$ and $f_z(z)$ are the adjusted functions and take the form

$$f_x(z) = A_x\left(\tfrac{1}{2} + \tfrac{z}{H}\right) + c_x\left(z^2 - \tfrac{H^2}{4}\right), \; f_y(z) = A_y\left(\tfrac{1}{2} + \tfrac{z}{H}\right) + c_y\left(z^2 - \tfrac{H^2}{4}\right), \; f_z(z) = A_z\left(\tfrac{1}{2} + \tfrac{z}{H}\right). \tag{10}$$

Impose the following conditions at the top surface of the laminate:

$$\left\{\begin{array}{l}\sigma_z(H/2) = q^s_z,\\ \tau_{xz}(H/2) = q^s_x,\\ \tau_{yz}(H/2) = q^s_y,\\ \int_{-H/2}^{H/2}\{\tau_{xz}, \tau_{yz}\}\; dz = \{Q_x, Q_y\},\\ \left.\frac{\partial \tau_{xz}}{\partial x}\right|_{H/2} = \frac{\partial q^s_x}{\partial x},\\ \left.\frac{\partial \tau_{yz}}{\partial y}\right|_{H/2} = \frac{\partial q^s_y}{\partial y},\end{array}\right. \tag{11}$$

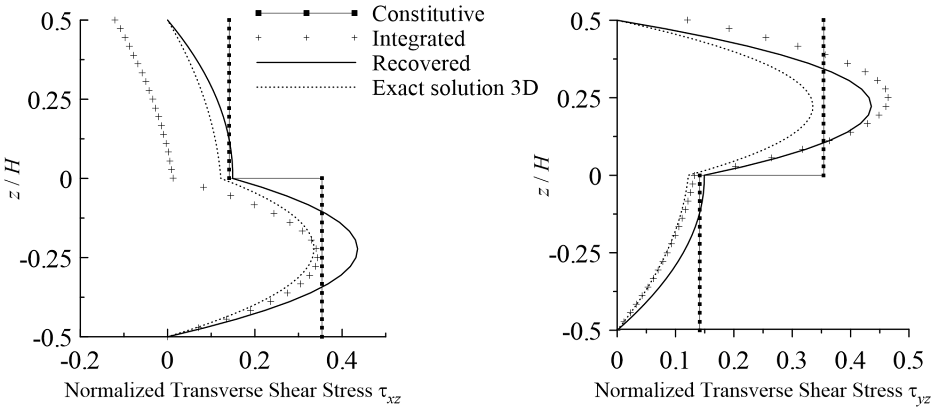
where q^s_x, q^s_y and q^s_z are the known values of the distributed loads at the cartesian directions applied on the upper surface of the laminate, that is, at coordinates $(x, y, H/2)$. Also, Q_x and Q_y are the shear forces at the point, computed from the GFEM results by the constitutive equations. Therefore, the adjusted functions in Eq. (9) are defined by the constants A_x, c_x, A_y, c_y and A_z.

4. RESULTS

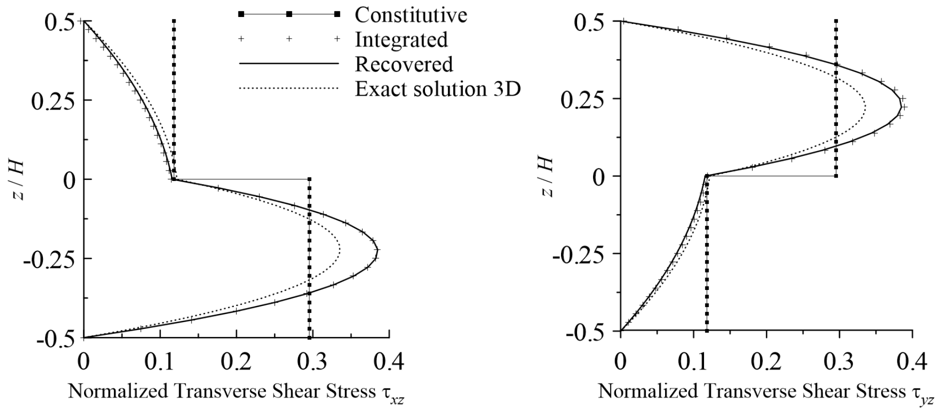
In this section numerical results are compared with the analytical solution of Pagano (1970). The aspects investigated are the following: (a) effectiveness of the foregoing computational procedure for calculating the transverse and normal stresses and (b) to assess the effectiveness of the post-process to extract the transverse and in-plane displacement.

4.1 Numerical studies

The problems analyzed consist of a laminated plate simply supported with the following geometry: sides $a = b$, thickness H. The material constants are: $E_1 = 25E_2$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $E_2 = 6.89$ GPa, and $\nu_{12} = \nu_{23} = 0.25$. And a transverse distributed load is applied with a double sinusoidal variation in the x, y plane, defined as $q(x, y) = q_0 \sin(\pi x/a) \sin(\pi x/b)$. Two problems are considered, the first an asymmetric orthotropic laminated plate oriented to $[0°/90°]$ and the second case a symmetric orthotropic laminated plate oriented to $[0°/90°/0°]$ both with respect to the x axis.



(a) Results of normalize transverse stress recovery $\tau_{xz}(0; b/2) = \tau_{xz}(0; b/2)\ H/q_0 a$ and $\tau_{yz}(a/2; 0) = \tau_{yz}(a/2; 0)\ H/q_0 b$ along the thickness. Laminate square plate, asymmetric $[0°/90°]$. Aspect ratio $a/H = 100$. Mesh index M8, degree of enrichment function p2.



(b) Results of normalize transverse stress recovery $\tau_{xz}(0; b/2) = \tau_{xz}(0; b/2)\ H/q_0 a$ and $\tau_{yz}(a/2; 0) = \tau_{yz}(a/2; 0)\ H/q_0 b$ along the thickness. Laminate square plate, asymmetric $[0°/90°]$. Aspect ratio $a/H = 100$. Mesh index M16, degree of enrichment function p4.

Figure 2. Results of transverse stress recovery and displacements.

For the asymmetric case $[0°/90°]$, two meshes are compared. First example with mesh M$= 8$ and enrichment functions are of degree $p = 2$, and the second M$= 16$ and $p = 4$. Figure 2a presents the results of transverse stresses obtained for an orthotropic asymmetric laminated plate oriented to $[0°/90°]$ with respect to the x axis, for a case M8 p2. In transverse shear stresses (τ_{xz}, τ_{yz}) the process of recovery was effective because it satisfies the boundary conditions on both laminate faces. However the behavior in transverse normal stress σ_z, despite satisfying the boundary conditions at $z = +H/2$, is not correct. This is because the recovery process requires higher order derivatives, so we must use enrichment functions with a minimum degree of $p = 3$. In the case with mesh M16 and enrichment functions p4 (Fig. 2b), the error of the integrated stresses are smaller but they do not satisfy the boundary conditions on the top surface ($z = +H/2$), and there is a need to apply the recovery process, which also results in good results.

Figure 3 show the behavior of normal transverse stress σ_z for p4 M16 model, along the x, y surface $z = +H/2$. It is notable that the stresses obtained by the integration process, shown with the index *i*, are not correctly integrated in the surface $z = +H/2$.

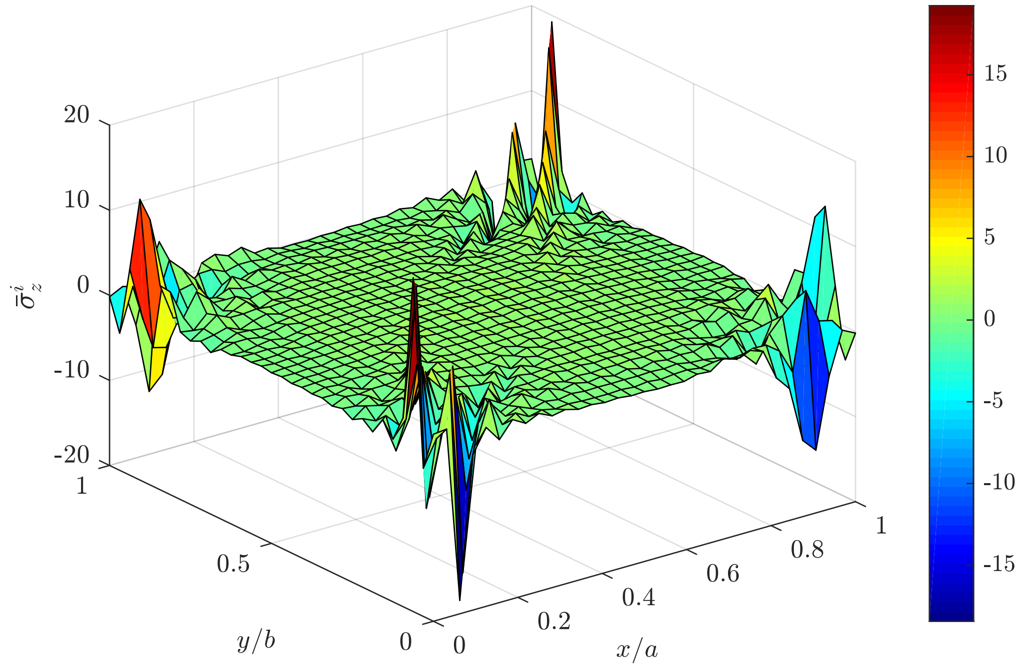


Figure 3. Results of normalized transverse normal stresses recovery $\sigma_z(x;y;+H/2) = \sigma_z(x;y;+H/2)\ /q_0$. Laminate square plate, asymmetric $[0°/90°]$. Aspect ratio $a/H = 100$. Mesh index M16, degree of enrichment function p4.

In Fig. 4 the in-plane and transverse displacement are show for a symmetric orthotropic plate oriented to $[0°/90°/0°]$ with respect to the x axis. The distribution along the thickness is more accurate than FSDT results, when compared with 3D solution. It is noted that, although the errors of the constitutive curve are smaller, its behavior is physically incorrect. The recovery curve shows an excellent qualitative behavior, and presents errors originating mainly from a translation. As a consequence, the derivatives of w with respect to z appear to be very well approximated.

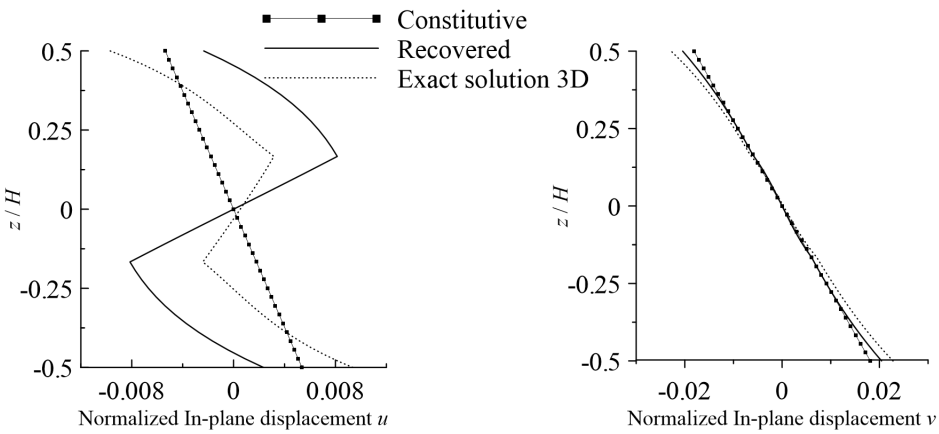


Figure 4. Results of in-plane displacements recovery $u(0;b/2) = u(0;b/2)\ E_2H^2/q_0a^3$ and $v(a/2;0) = v(a/2;0)\ E_2H^2/q_0a^3$ along the thickness. Laminate square plate, symmetric $[0°/90°/0°]$. Aspect ratio $a/H = 4$. Mesh index M16, degree of enrichment function p4.

5. CONCLUSIONS

A simple method computational, based on post-processing of the displacement and transverse stresses results within GFEM calculations based on FSDT has been demonstrated. The basic idea consists in integrated the constitutive equations of Mindlin model, and find the adjusted functions to corrected the stresses. The computational procedure proposed was shown to be able, in numerical experiments, to improve the accuracy of the transverse stresses and displacements.

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