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ENERGY LOSS IN UNSTEADY TURBULENT FLOWS COMPUTED BY ALTERNATIVE FRICTION MODEL FRAMED ON THEORY OF MIXTURES

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Abstract. *Turbulent fluid transients in pipes occur daily in a diverse range of circumstances in industry Pressure surges during this phenomenon are important to the design and maintenance of the pipe structure. This work argues about an alternative approach to model the unsteady friction framed on the Continuum Theory of Mixtures to compute the pressure fields as well as energy loss of fluid transients. This theory allows the flow to be interpreted as a virtual structured mixture formed by pseudo-constituents, all of them with the same equations of state. The proposed model has its material constants easily determined as they are calculated by the wise use of known steady velocity profiles. Comparison with experimental data makes this model applicable to hydraulic analysis.*

Keywords: *transient flow, turbulent flow, frequency-dependent friction, transient rate of energy loss*

1. INTRODUCTION

Turbulent transient flows are a real concern in pipe design since surge pressures in this phenomenon generate pressures that are greater than the steady-state. Hence, any hydraulic project must take account of the occurrence of it. Another reason for this topic be studied is the promising transient based techniques to leakage detection which needs accurate predictions of the pressure fields (Ghidaoui et al., 2005).

The first and most traditional form of analysis of transient turbulent flow in pipes is to adopt the steady friction equation. However, this methodology does not foresee correctly the rate of energy loss of the phenomenon since it disregards the great difference between the average velocity field in steady and unsteady states. As a consequence, the quasi-steady strategy over-predicts the values of pressure and it does not obtain the particular pressure shaping of the transient flow.

Researchers start to design new approaches to overcoming these drawbacks. In short, two categories of turbulent steady friction have been more highlighted in literature: The Convolution Integral Models, and the Instantaneous Acceleration models. The first follows the same strategy of Zielke (1967) employed to laminar flows. This category is mainly represented by the work of Vardy and Brown (2007). The second used an empirical-heuristic approach to obtain a relationship between the instantaneous acceleration and the wall friction force; this group is represented by Brunone et al. (1995).

This work presents an approach, framed on Theory of Mixtures, to achieve expressions to unsteady friction as well as the rate of energy loss of the fluid flow. This methodology has been proven for laminar flows (Cunha and Rachid, 2005) and, preliminarily, for turbulent flows (Andrade and Rachid, 2018). Comparisons between the pressure fields calculated with this methodology, quasi-steady model and experimental data corroborate the powerful capacity of this model be an available tool for hydraulic analysis.

2. METHODOLOGY

The mainframe of this new approach is the Theory of Mixtures. Then, the first task is to show the main mechanical balances of this theory that drives the development of the unsteady model. The mixture, in this theory, states for a set of n components in which are perfectly blended such that all of them occupies every material point at any instant of time t. Each of these components has its own motion along the centerline of the pipe, which has length L. Assuming that the flow is isotherm and that the constituents do not react, the balances of mass, momentum and second law of thermodynamics for the mixture as a whole and for each constituent in its quasi-one-dimensional form can be expressed as (Drew and Passman, 1998):

$$\sum_{j=1}^{n}\left\{\frac{d}{dt}\int_{a}^{b}\rho_j A dx + \left[\rho_j v_j A\right]_b - \left[\rho_j v_j A\right]_a\right\} = 0, \tag{1}$$

$$\frac{d}{dt}\int_a^b \rho_j A\, dx + \left[\rho_j v_j A\right]_b - \left[\rho_j v_j A\right]_a = 0\,, \tag{2}$$

$$\sum_{j=1}^{n}\left\{\frac{d}{dt}\int_a^b \rho_j A v_j dx + \left[\rho_j A v_j^2\right]_b - \left[\rho_j A v_j^2\right]_a + \int_a^b a_j P dx\right\} = \sum_{j=1}^{n}\left\{\left[t_j A\right]_b - \left[t_j A\right]_a\right\} + \int_a^b \left[b_j \rho_j A - t_j \frac{\partial A}{\partial x}\right] dx, \tag{3}$$

$$\rho_j\left(\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x}\right) = \frac{\partial t_j}{\partial x} - \frac{P}{A} a_j + b_j - m_j\,, \tag{4}$$

$$\sum_{j=1}^{n}\int_a^b \left[\frac{m_j v_j}{\theta} + \frac{a_j v_j \frac{P}{A}}{\theta}\right] \geq 0\,, \tag{5}$$

$$-\rho_j\left(\frac{d^{(j)}\Psi_j}{dt} + \frac{d^{(j)}\theta}{dt}\right) + a_j v_j \frac{P}{A} + t_j^D \frac{\partial v_j}{\partial x} - t_j^S\left(\frac{1}{\rho_j}\frac{d^{(j)}\rho_j}{dt}\right) \geq 0\,. \tag{6}$$

In these equations, $v_j(x,t)$ and $\rho_j(x,t)$ are the velocity and density of each component; P and A are the internal pipe perimeter and cross-sectional area, respectively, and θ is the absolute temperature. While t_j^D and t_j^S stands for deviatory and spherical surface forces per unit of the cross-section area; Ψ is the Helmholtz free energy per unit mass; b_j is body forces per unit of the cross-sectional area; m_j represents the internal interaction force per unit of cross-sectional area exerted by the other constituents on the *j-th* constituent, and a_j is the reactive contact friction force per unit lateral area that acts on the pipe wall-fluid interface.

2.1 Virtual Mixture Model

A model that capture the velocity profile of the transient flows might attain great predictions of pressure, dissipation of the transient flow (Cunha and Rachid, 2005). The mixture in this approach is conceived to allow a description of the velocity field in an insightful manner, forming a concentrically shell-shaped set, as depicted in Fig.1.

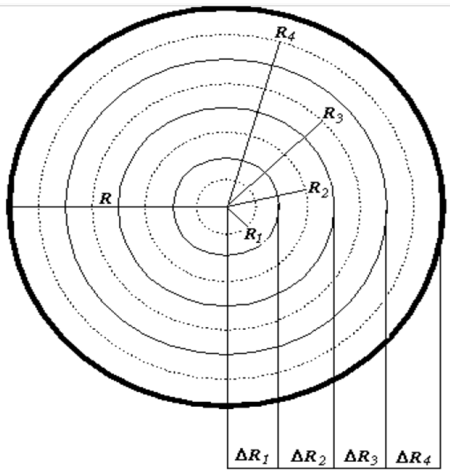

Figure 1. Virtual structured of the mixture for $n = 4$

The governing equations aforementioned are general, and it is a must describe the constitutive equations of pipe and virtual mixture. The forces a_j and m_j are strictly related to the momentum transfer that occurs between the components and the fluid-wall interface. Therefore, they may be modeled as

$$m_j = C_{j,j-1}\left(v_j - v_{j-1}\right) + C_{j,j+1}\left(v_j - v_{j+1}\right), \tag{7}$$

for $j = 1, \ldots, n-1$

$$a_j = \begin{cases} 0, & for\ j = 1, \ldots, n-1 \\ Cv_n, & for\ j = n \end{cases}, \tag{8}$$

in which $C_{j,j+1}$, $C_{j,j-1}$, C are the thermomechanical constants of the model.

Assuming that the gravitational effects may be disregarded, the fluid is barotropic, the pipe has linear small deformations and the flow has a low Mach number; the hyperbolic system of equations is obtained from the equations of mass and momentum for the mixture as a whole (Eqs. (1) and (3)) alongside the momentum for each constituent of the virtual mixture (Eq. (4)). This set of governing equations that characterized the transient flow motion which is given in its local form as

$$\frac{1}{a^2}\frac{\partial p}{dt} + \rho\frac{\partial v}{\partial x} = 0, \tag{9}$$

$$\rho\frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} + \frac{4}{D}\sum_{j=1}^{n} a_j = 0, \tag{10}$$

$$\alpha_j\rho\frac{\partial v_j}{\partial t} + \alpha_j\frac{\partial p}{\partial x} + m_j + \frac{4}{D}a_j = 0, \tag{11}$$

in which a is the wave velocity in the mixture, D is the intern pipe diameter, and α_j is the volumetric fraction occupied by each component $j-th$. Observe that the solution of this system of partial differential equations provides not just the pressure field and flow average velocity as well as the velocity of each constituent. In fact, this last feature that describes the mixture velocity profile which allows the calculation of the unsteady wall shear stress (Eq. 8) and intrinsic energy dissipation. Which is obtained upon substitution of the constitutive equations of the interaction and reactive forces of the constituents, the energy loss per volume unit of the stream may be achieved and it can be expressed as

$$d \equiv \sum_{j=1}^{n} C_{j,j+1}\left(v_{j+1} - v_j\right)^2 + \frac{Cv_n^2 P}{A} \geq 0. \tag{12}$$

2.2 Determination of the material constants

The whole strategy to obtain the material constants of the model is linked with the frozen viscosity hypothesis. This assumption states that the turbulent structured maintains unaltered to the first moments of the transient flow (He and Jackson, 2000). A deep analysis of this assumption unveils that the main actors which control the momentum transfer are conserved in the onset of the flow transients. Such that the material constants of the model may be obtained in a steady state. The use of a steady turbulent profile employed in the steady momentum equation of each constituent (Eq. (11)) allows the determination of analytical expressions of the material constants of the model.

The turbulent velocity profile applied for such a strategy was based on the turbulent viscosity data from the experimental work of Laufer (1954). The present work adopts the latest idealized turbulent viscosity distribution proposed by Vardy and Brown (2007), which was conceived to englobe any pipe wall roughness and Reynolds number. This approach defines the idealized turbulent viscosity distribution with two distinct regions, namely an outer annulus and an inner core. The annular region has the thickness b equal to $0.2\,R$. In the outer region, the viscosity was assumed to vary linearly from the wall, where it assumes the value ν_w, to a maximum value of ν_c at the interface of these regions. The momentum material constants for the core and annulus respectively are given by

$$C_{j,j+1} = \frac{4\rho\nu_c}{R_{j+1}^2 - R_j^2}\left(\sum_{i=1}^{j}\alpha_i\right), \tag{13}$$

$$C_{j,j-1} = \frac{2\nu_w\rho}{b^2\left\{\frac{R_{j+1} - R_j}{(1-\sigma_{cw})b} + \frac{(-4+5\sigma_{cw})}{(1-\sigma_{cw})^2}\ln\left[\frac{\left(\frac{1-\sigma_{cw}}{b}\right)R_j - 4 + 5\sigma_{cw}}{\left(\frac{1-\sigma_{cw}}{b}\right)R_{j+1} - 4 + 5\sigma_{cw}}\right]\right\}}\left(\sum_{i=1}^{j}\alpha_i\right), \tag{14}$$

while the friction material constant is determined by

$$C = \frac{2\nu_w \rho R}{b^2 \left\{ \frac{R - R_n}{(1-\sigma_{cw})b} + \frac{(-4+5\sigma_{cw})}{[(1-\sigma_{cw})]^2} \ln\left[\left(\frac{1-\sigma_{cw}}{b} \right) R_n \ - 4 + 5\sigma_{cw} \right] \right\}} \left(\sum_{i=1}^{n} \alpha_i \right), \tag{15}$$

In which σ_{cw} is the ratio of the turbulent kinematic viscosities in the core and wall ν_c/ν_w.

3. RESULTS

The capacity of the proposed model is measured by comparing its numerical results for normalized pressure and rate of energy dissipation with the forecasted responses of the quasi-steady model aligned with the Adamkowski and Lewandowski (2006) head experimental data. The experimental apparatus used by them is a reservoir-pipe-valve installation depicted in Fig. 2. The reservoir has a constant pressure device from which water flows downstream in the turbulent steady-state regime through a constant diameter steel pipe. The flow has Reynolds Number of 5731 and the pipe has a length $L = 98.11\ m$ and an internal diameter $D = 0.016\ m$. A block valve, initially at the fully-opened position, is installed at the downstream end of this pipe. This initial condition is suddenly altered to a transient phenomenon by a rapid valve closure maneuver taking place in 0.003s. The pressure in the transient conditions was recorded with piezoelectric transducers installed at the middle of the pipe length and at the block valve.

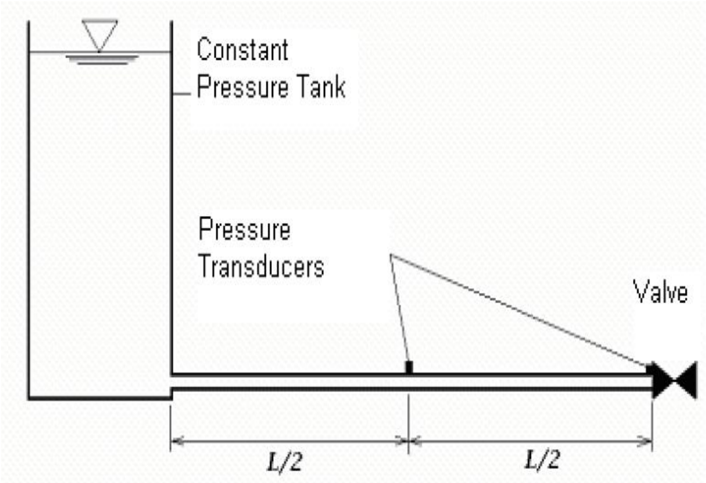

Figure 2. Sketch of the test rig used in the experiments whose data will be used for validating the model.

The approximated solutions of the hyperbolic partial differential system formed by Eq. (9), Eq. (10) and Eq. (11) are achieved by the method of characteristics followed by a house-holder linear system numerical technique within crank-Nicolson approximation for the interaction forces a_j and $m_j's$.

Figure 3 presents the normalized head time histories next to the valve predicted by the present model and by the quasi-steady model whose results were taken from the work of Gonzaga Filho (2017). One can see that the mechanical model developed herein has a great agreement with the experimental data. It may also be noticed through that the quasi-steady model overpredicts the heads and it cannot foresee the shape either phase of the pressure waves. Therefore, one can say that the proposed model seems to be more suitable for computing the heads in the hydraulic analysis than the quasi-steady model.

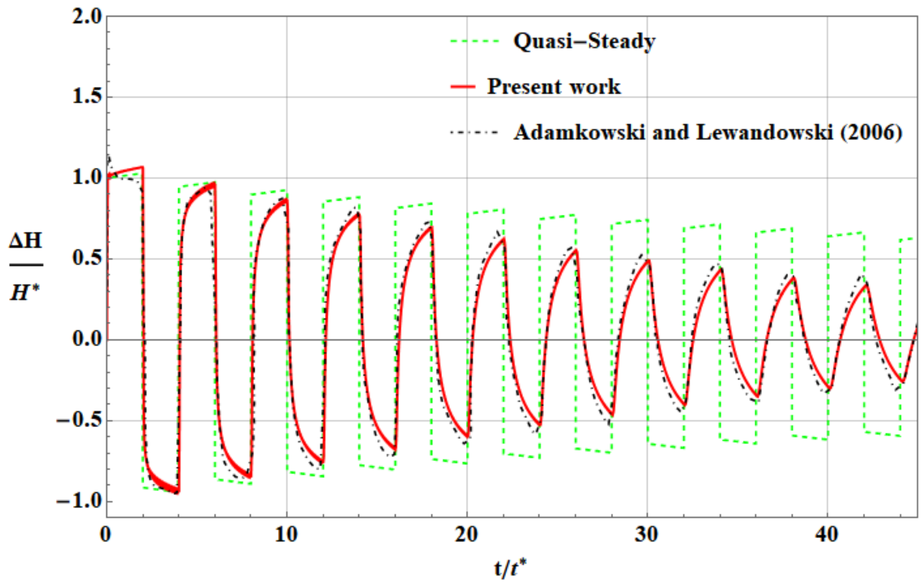

Figure 3. Normalized head time history at the valve of the pipe in the Adamkowski and Lewandowski (2006) experiment. Comparison among the present model, the quasi-steady one in Gonzaga Filho (2017). In which $H^*$is the Joukowsky head and $t^*$is at/L.

Figure 4 presents the normalized rate of energy dissipation time history at the mid-length of the pipe predicted by the proposed and quasi-steady models. One can notice that the dissipation predicted by the thermomechanical model developed in this work is always positive, obeying strictly the second law of thermodynamics (SLT).

Although the quasi-steady model does not violate the SLT, its results may be interpreted as unrealistic since the pressure decay is a consequence of the loss of energy of the fluid flow. As may be seen by comparing Fig. 5 and Fig.6, the dissipation and the mean velocity are not in phase. In fluid transients, reverse flow appearances make the velocity to have high gradients next to the wall meanwhile the stream has small average velocities (Brunone et al., 2004). Therefore, as the quasi-steady only can see the mean velocity, the overall dissipation computed by the quasi-steady model is underestimated.

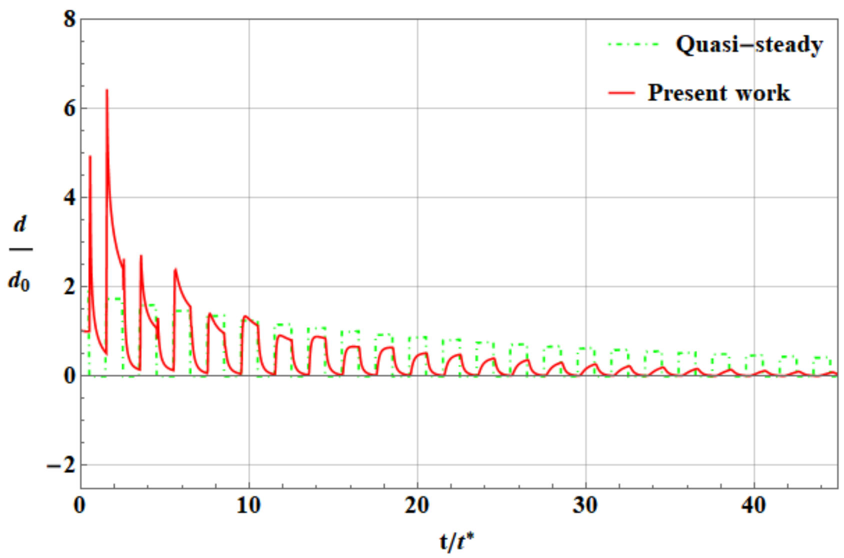

Figure 4. Normalized rate of energy dissipation at the mid-length of the pipe in the Adamkowski and Lewandowski experiment. Comparison among the present model, quasi-steady models computed in Gonzaga Filho (2017). In which d_0 is the steady-state rate of energy dissipation and $t^*$is at/L.

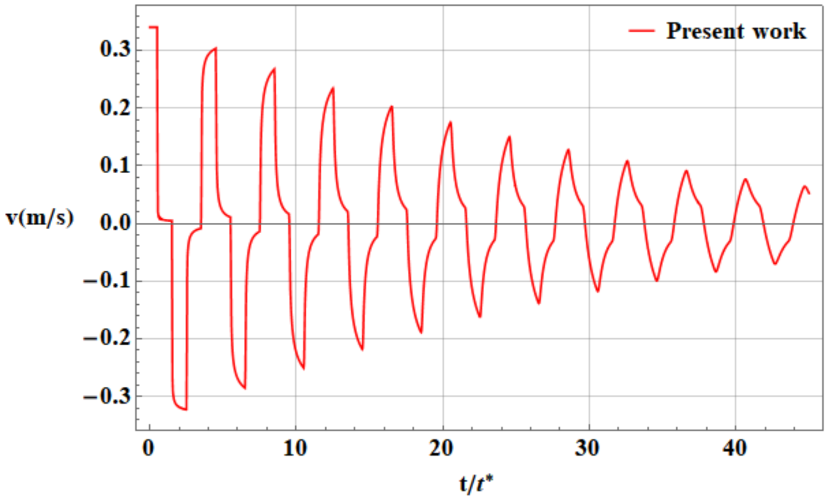

Figure 5. Average velocity at the mid-length of the pipe in the Adamkowski and Lewandowski experiment.

4. CONCLUSION

This study presented a new model for frequency-dependent friction in turbulent flows framed on the continuum mixture theory. The fluid is interpreted as a set of virtual constituents that allows viewing in detail the transient flow momentum transfer and further flow energy dissipation. The proposed unsteady friction model has shown a great potential for being a useful tool for hydraulic applications since it predicted with accuracy pressure fields of fluid transients. In addition, the presented model is assured to be thermodynamically consistent, what may be not true for other models found in the literature.

5. REFERENCES

Adamkowski, A.; Lewandowski, M., 2006. “Experimental Examination of Unsteady Friction Models for Transient Pipe Flow Simulation”. *Journal of Fluids Engineering*, Vol. 128, pp. 1351–1363.

Andrade, D.M., Rachid, F. B. F., 2005. “A Model for Unsteady Turbulent Friction in Pipe Flows”. In *Proceedings of the 17th Brazilian Congress of Themal Sciences and Engineering-ENCIT* (2018). Águas de Lindóia, Brazil.

Brunone, B., Ferrante, M., and Cacciamani, M. “Decay of pressure and energy dissipation in the laminar transient flow.” *J. Fluids Eng.*, Vol. 126, pp. 928–934, 2004.

Gonzaga Filho, J. S. Uma avaliação de modelos para descrever o atrito em escoamento unidimensional transiente. Universidade federal Fluminense, 2018.

Brunone, B., Golia, U. M., and Greco, M. “Effects of Two-Dimensionality on Pipe Transients Modeling”. *J. Hydraul. Eng.*, vol. 121, pp. 906 − 912, 1995.

Drew, D. A. and Passman, S. L., 1998. *Theory of multicomponent fluids.* Springer-Verlag, New York.

Cunha, M. and Rachid, F. B. F., 2005. “A Model for Frequency-Dependent Friction Onedimensional Unsteady Fluid Flows.”. In *Proceedings of the 18th International Congress of Mechanical Engineering-COBEN* (2005). Ouro Preto, Brazil.

Guidaoui, M. S., Zhao, M., McIlnnis, D. A., Axworthy, D.H., 2005. “A review of water hammer theory and Practice”. *Applied Mechanics Reviews*, Vol. 58, pp. 49-76.

Laufer, J. *The structure of turbulence in fully developed pipe flow*. NACA, 1954

Martins, N. M. C., Brunone, B., Menicone, S., Ramos, H. M., and Covas D. I. C., 2017. “CFD and 1D Approaches for the Unsteady Friction Analysis of Low Reynolds Number Turbulent Flows”. *Journal of Hydraulic Engineering*, vol. 143, 2017.

Vardy, A. E. and Brown, J. M. B., 2003. “Transient Turbulent Friction in Smooth Pipe Flows”. *Journal of Sound and Vibration*, Vol. 259, pp. 1011–1036.

Vardy, A. E.; Brown, J. M. B., 2007. “Approximation of Turbulent Wall Shear Stresses in Highly Transient Pipe Flows”. *Journal of Hydraulic Engineering*, 133, pp. 1219–1228.

Zielke, W. Frequency-Dependent Friction in Transient Pipe Flow. *Journal of Basic Engineering*, Vol. 91, pp. 109−115, 1968.

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