Numerical Simulation of Natural Convection Heat Transfer in Circular Fins

Sandi Itamar Schafer de Souza, sandi@ufrnet.br
Kleiber Lima de Bessa, klbessa@ct.ufrn.br
Universidade Federal do Rio Grande do Norte – UFRN Campus Universitário, S/N – Lagoa Nova - Natal, RN – Brazil - CEP 59072-970

Abstract. *This paper aims to increase knowledge about the devices used to enhance heat transfer processes, called Fins. For the engineering design of the fins, the classic literature suggests that the convective heat transfer coefficient must be constant, and to determine the effectiveness, this coefficient must be of the surface whitout fins. The insertion of fins on tube changes the geometry and causes a change of the convective heat transfer coefficient. So the objective of this study was to analyse and quantify the change of convective coefficients and its interference in the heat transfer process. For this, it was employed the computational fluid dynamics (CFD), and it was elected for this purpose a geometry previously studied using experimental techniques. Six cases were studied, varying the boundary conditions, wall temperature and temperature of the surrounding fluid. The studied geometry was a circular fin with non-uniform cross section. The tube without fins was also simulated, in order to determine the convective coefficient. In addition, the design using the analytical model for the fin efficiency and heat transfer rate was developed. The results obtained from the numerical simulation indicate a strong change of the convective coefficient and agree with the experimental results. This work shows the velocity and temperature fields, the convective coefficient for situations with and without fins. Comparisons among numerical results and those obtained using the analytical and experimental techniques are also shown.*

Keywords*: Heat Transfer, Natural Convection, Finned Tube, Numerical Simulation.*

1. INTRODUCTION

Heat transfer processes are present in our daily lives and are of great importance. From an engineering point of view, there are situations where the aim is to enhance the heat transfer process, and a form of intensification is the use of fins. These devices are an extension of the exchange surface of the solid body and are employed when the heat transfer occurs by diffusion within a solid and by convection or radiation in the solid border. The use of fins allows the reduction in size of the heat exchanger by increasing the heat transfer rate per unit area.

The classical literature of heat transfer presents the equations for the design of the fins to numerous configurations, including circular fins, which belong to the class of those having nonuniform sectional area. Some important simplifications are used in the design. One is the employment of a constant convective coefficient over the fin surface, but it is known that the flow of the fluid and the presence of the base surface create a complex flow around the solid. There is no correlation for the Nusselt number for all geometrical possibilities. It is also known that the convective coefficient depends on the geometry, and so it may be necessary to adopt other simplifying assumptions, diverting the solution of physical reality. In determining the effectiveness, according to the literature, it is necessary to know the convective coefficient of the tube without the presence of fin. The heat transfer coefficient on the tube wall is considered to be constant, but it is known that it varies radially and axially. These considerations may also cause Solution deviations from physical reality.

Kayansayan e Karabacak (1992), developed experimental studies using finned tubes with a heat source power of 2 kW , conditions were maintained for Rayleigh numbers in the range of $1x10^5$ to $5x10^7$. Four diameters fins were studied and different spacing, making a total of 16 settings. They investigated the behavior of Nusselt number as a function of Rayleigh, and the spacing effect on the convective coefficient. identified the Rayleight critical of laminar-turbulent transition. The results show that for low spacing values, the presence of the fins affects the convective coefficient. Hahne and Zhus (1994) have made experimental studies aiming to identify the effect of height on the heat transfer process. Three fins diameters were studied with heating rates ranging from 10 W to 60 W. The authors investigated the thermal gradient in the wall fin by thermography. The results for the Nusselt number are close to those obtained by correlation. Yildiz and Yüncü (2004) studied experimentally the behavior of the heat transfer process in finned tubes. Eighteen configurations were tested altering the diameter and spacing between the fins. The authors compared the results for the Nusselt number with those obtained by Churchill and Chu (1975) and Morgan (1975), for external laminar flow over a cylinder. Chen et Hsu (2007) investigated the thermal exchange, using the technique of inverse problem associated with experimental results. The authors present the results of the convective heat transfer coefficient and the efficiency as a function of spacing between fins. The thermal power used in the experiment was 200 W, fins 2 mm thick, 99 mm outer diameter, 27 mm internal diameter. The fin material was steel AISI 304. Yaghoubi and Mahdavi (2013) developed a numerical and experimental work aiming to investigate the natural convection tubes with aluminum fins used to cool the surrounding air. The physical dimensions of the fin were held constant, being: outside diameter 56 mm, thickness 0.4 mm and 2 mm spacing. The temperature of the environment and the tube wall was also kept constant. The tests were conducted for 3 temperatures of the base and three ambient temperatures, generating 9 settings. The results of the velocity field and the convective heat transfer coefficient as a function of ambient temperature and the wall temperature are presented. Kumar et al. (2016) developed a three-dimensional numerical study on the heat transfer in the finned tubes. The authors evaluated the effect of spacing, diameter and fin-ambient temperature difference on the heat exchange and buoyancy forces. The authors found a 8 mm value as the optimal spacing between fins.

2. PHYSIC MODEL AN BOUNDARY CONDITIONS

The Fig. 1 presents the physic model employed, which consists of two domains, the aluminum solid fin with thickness t of 0.4 mm and the fluid domain. The internal duct has a diameter d of 25.4 mm, and the fin diameter D_f is equal to 56 mm. The dimensions were adopted to match with those of the experimental study from Yaghoubi e Mahdavi (2013). The test section has a length H of $7D_f$, $L = 2D_f$, and the spacing S was set to 2 mm.

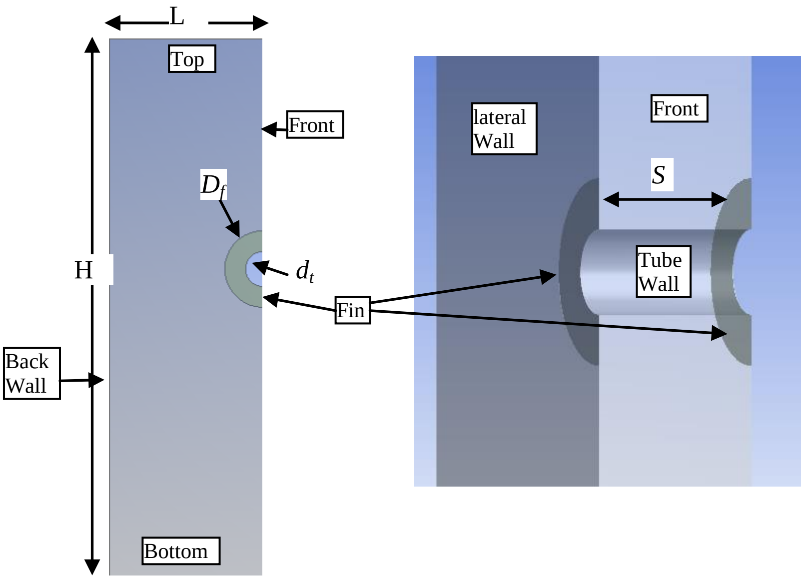

Figure 1. Physic model of finned tube.

In the fluid domain, air as an ideal gas was adopted as the working fluid and the effects of compressibility were neglected. In the solid domain, the material adopted, was the aluminum. The adopted boundary conditions were: Lateral walls and front were considered symmetry. To the back wall were applied adiabatic and no slip conditions. On the top and bottom it was applied the open boundary condition with static pressure equal to 1 atm, and the fluid temperature (T_∞) equal to 22 ºC, 27 ºC and 33 ºC. In the tube wall and the base of the fin were adopted the temperatures (T_w) of 8 °C, 12 °C and 15 °C.

3. GOVERNING EQUATIONS

It was used the Rayleigh number Eq. (1) to carry out preliminary reviews of the flow regime, inside the test cell, where it were found values in order of 2.5×10^5, featuring laminar flow.

$$Ra = \frac{g\beta(T_\infty - T_w)D_f^3}{\nu\alpha} \tag{1}$$

Where the g is the acceleration due the gravity, D_f, β, υ and α denote the fin's diameter, volumetric thermal expansion coefficient, kinematic viscosity and the thermal diffusivity of air at mean temperature. To model the laminar flow, the differential equations of momentum (ENS), energy and mass conservation, can be written in indicial notation as Eq. (2), Eq. (3) and Eq. (4). The ANSYS-CFX 13 was used to solve the governing equations and the Full Buoyancy Model was employed. The convergence criteria adopted was a Root Mean Square (RMS) of 1.0×10^{-5}, for all velocities and for pressure.

$$\frac{\partial}{\partial x_j}\left(\rho U_i U_j\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] + S_M \quad (2)$$

$$\frac{\partial}{\partial x_j}\left(\rho U_j T\right) = \frac{\partial}{\partial x_j}\left(\frac{k}{c_p}\frac{\partial T}{\partial x_j}\right) \quad (3)$$

$$\frac{\partial}{\partial x_j}\left(\rho U_j\right) = 0 \quad (4)$$

In the above equation, ρ is the density, x is the coordinate, U is the vector velocity, p represents the pressure, k and S_M represent the diffusive coefficient and the sum of the body forces, respectively; T is the temperature and c_p represents the specific heat at constant pressure.

4. RESULTS

The right side of Figure 2 shows the temperature field, whereas in the left side, the velocity streamlines, in a plane placed between the fins, are shown.

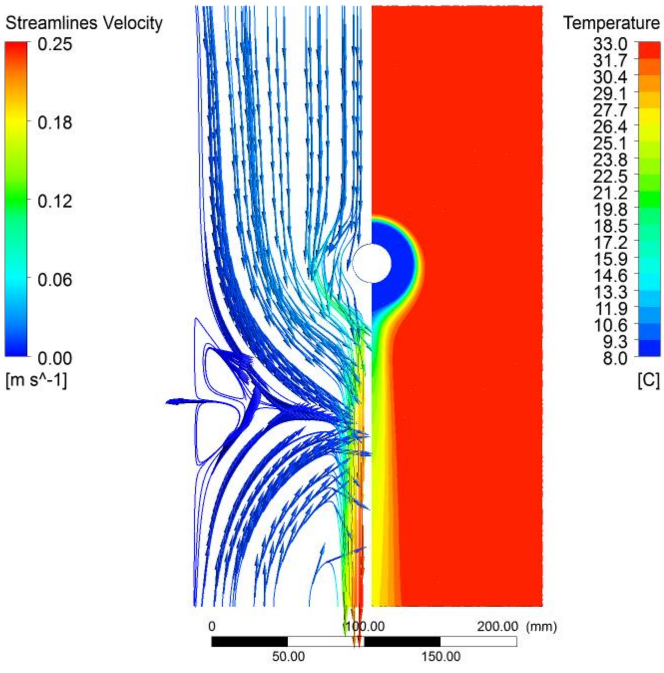

Figure 2. In the left the velocity streamlines and in the right the temperature field for the case T_∞ = 33 ºC an T_w = 8 ºC.

In the preceding figure, it is possible to perceive the streamlines in the downward direction, where the fluid is at 33°C at the inlet and outlet of the test cell, and is cooled by the walls of fin and the tube., which are kept at 8° C. The air which is between the fins has its temperature very close to the temperature of the wall. Thus the air in this region has a higher density compared with the surrounding fluid. This difference originates the buoyancy forces that promote air circulation. The field of density and downward buoyancy force, Eq. (5), are shown in Fig. 3, in a plane placed between the fins. When buoyancy is included, a source term is added to the momentum equations, Eq. (2), based on the difference between the fluid density and a reference density (ρ_{ref}).

$$S_M = g\left(\rho - \rho_{ref}\right) \quad (5)$$

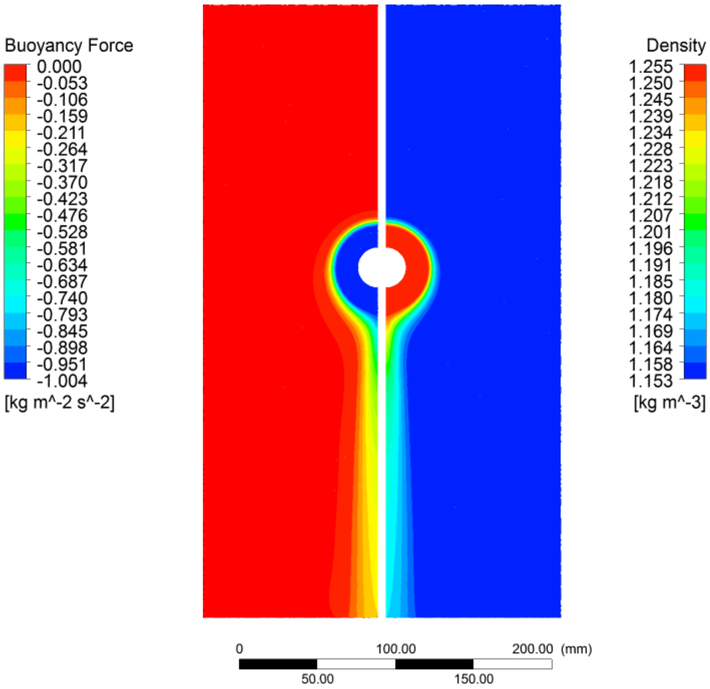

Figure 3. In the left the downward buoyancy force and the right the Density,
to the case T_∞ = 33 ºC an T_w = 8 ºC.

The temperature field over the wall fin is shown in Fig. 4. At the base of the fin, the temperature is constant at 8 °C, and due to convective heat exchange with the heated surrounding air, the temperature of the fin is not constant. The temperature varies radially (*r*) and angularly (*θ*), moving away from the simplified situation, where the heat conduction is unidirectional.

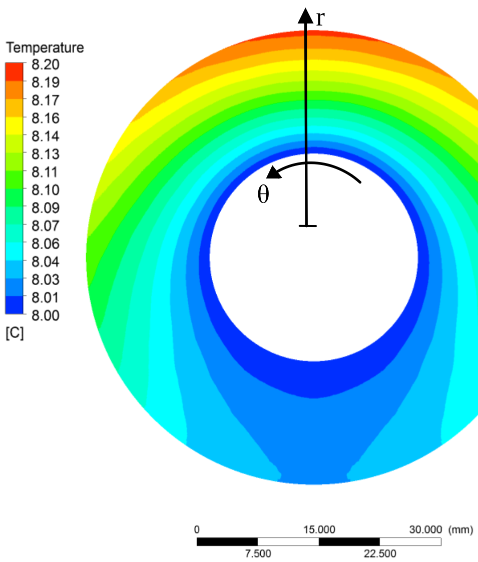

Figure 4. The temperature over the fin wall, to the case T_∞ = 33 ºC and T_w = 8 ºC.

The heat transfer from the fin to the air is larger in outer contours, as shown in Fig. 5. In the figure it can be seen the velocity field in this region. Due to the short distance between the fins, the velocities have little influence in the heat transfer process. At the tip of the fin, the heat transfer process is more intense because in this location are the largest velocities, and air temperature.

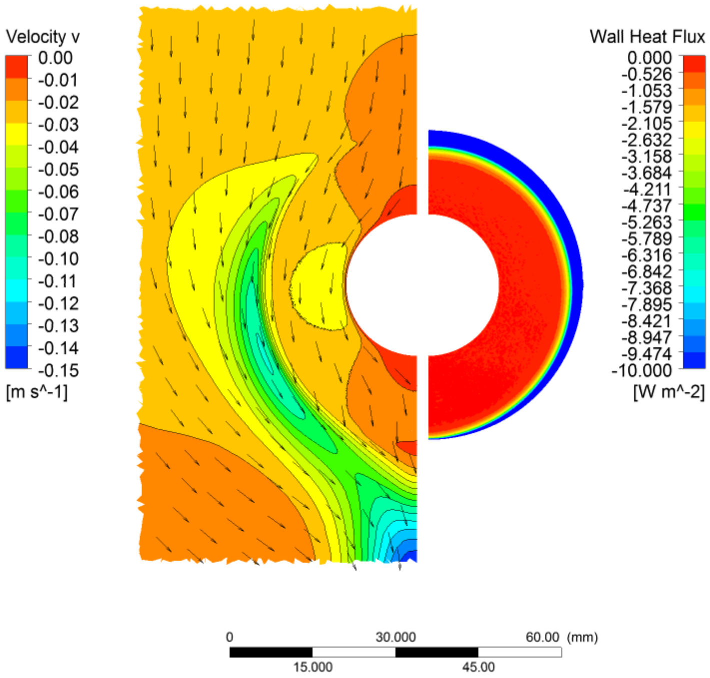

Figure 5. On the left side velocity *v* and the on the right the wall heat flux over wall fin, for the case T_∞ = 33 ºC an T_w = 8 ºC.

The results for the other boundary conditions of the simulation are show in the Tab. 1. The value of body force decreases with temperature difference (T_∞-T_w), agreeing with the physical principles of the problem. The vertical velocity *v* should follow the same behavior, but in two situations there is a slightly increase in speed. This behavior is due to changing patterns of recirculation of fluid on the device outlet, as seen in Fig. 2, and not interferes in the heat transfer process.

Table 1. Minimal Vertical velocities and Minimal Body force to all cases studied.

	T_∞=22°C		T_∞=27°C		T_∞=33°C	
	V_{min} (m/s)	S_{Mmin} (N)	V_{min} (m/s)	S_{Mmin} (N)	V_{min} (m/s)	S_{Mmin} (N)
T_w=8°C	-0.19	-0.585	-0.216	-0.78	-0.25	-1.0
T_w=12°C	-0.18	-0.412	-0.22	-0.607	-0.259	-0.83
T_w=15°C	-0.136	-0.285	-0.195	-0.481	-0.238	-0.707

To determine the effectiveness of the fin it is necessary to know the convective heat transfer without the presence of the fin. The Heat transfer was simulated on a tube without fins with an outer diameter of 25.4 mm and length of 100 mm. The dimensions of the outer box, *H* and *L* were the same as used for the simulation of the finned tube. The boundary conditions for the finned tube simulations were repeated in this simulation. Table 2 shows the Nusselt number (N_{u1}) obtained with numerical simulation and determined with Eq. 6 and Eq. 7, compared with two authors cited in the classical literature, Nu_2 obtained from the correlation of Churchil and Chu (1975), and Nu_3 from the correlation of Morgan (1975). Comparing the tabulated data it is noticed that the maximum relative difference is below 10%.

Table 2. Nusselt numbers to the tube without fin.

	T_∞=22°C			T_∞=27°C			T_∞=33°C		
	Nu_1	Nu_2	Nu_3	Nu_1	Nu_2	Nu_3	Nu_1	Nu_2	Nu_3
T_w=8°C	5.88	6.1	5.52	6.26	6.5	5.88	6.52	6.86	6.21
T_w=12°C	5.45	5.59	5.06	5.87	6.1	5.52	6.25	6.54	5.92
T_w=15°C	5.46	5.09	4.62	5.58	5.74	5.2	6.03	6.26	5.67

$$Nu = \frac{h D_f}{k_{air}} \quad (6)$$

$$h = \frac{q}{A(T_\infty - T_w)} \quad (7)$$

Where h is the convective heat transfer coefficient, A is the area of the tube, k_{air} the diffusive heat transfer coefficient of the air, considered constant, and q symbolizes the wall heat flux, obtained on the results of numerical simulation.

The results for the convective heat transfer coefficient h_1, of the fined tube are in Tab. 3, h_2 and h_3 are the experimental and numerical results obtained from Yaghoubi e Mahdavi (2013). The values to h_1, were obtained by Eq. 7, where the A is the sum of tube surface area with fin area. In Fig. 6, the Nusselt number for the finned tube was obtained by Eq. 6, where the characteristic dimension has become the spacing between the fins S. The values obtained are compared with the experimental Nusselt number presented by Yaghoubi e Mahdavi (2013) and with the values obtained with Eq. 8, Incropera (2008) and Bejan (1984), to the air flow between plates symmetrically heated and having the separation distance S, and vertical height D.

Table 3. Convective heat transfer coefficient for the finned tube.

	T_∞=22°C			T_∞=27°C			T_∞=33°C		
	h_1	h_2	h_3	h_1	h_2	h_3	h_1	h_2	h_3
T_w=8°C	0.68	0.71	0.7	0.76	0.74	0.73	0.85	0.81	0.8
T_w=12°C	0.59	0.69	0.63	0.67	0.68	0.67	0.78	0.74	0.72
T_w=15°C	0.52	0.61	0.58	0.61	0.65	0.63	0.72	0.69	0.66

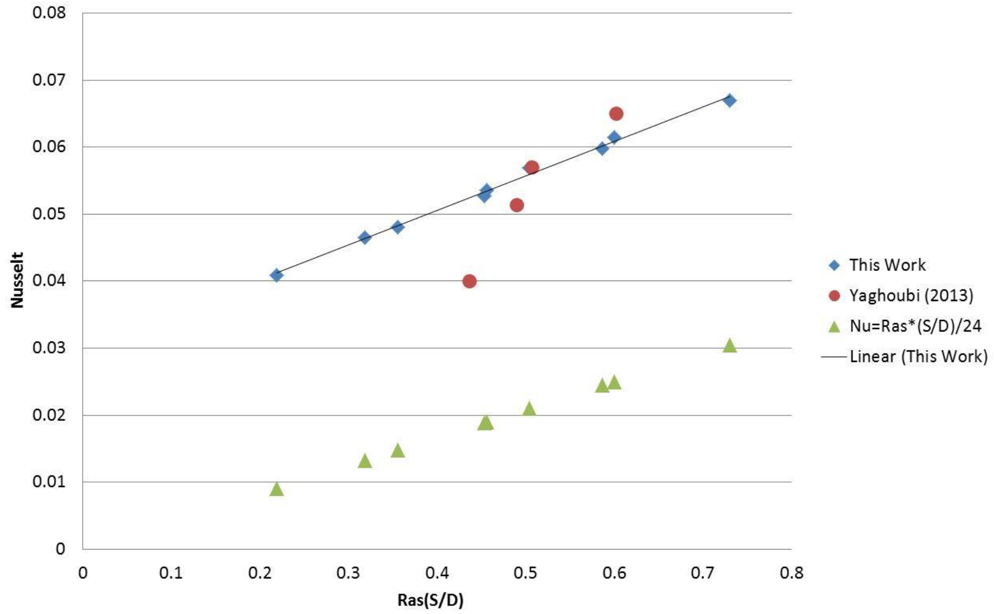

Figure 6. Comparison of the Nusselt number with the experimental solution from Yaghoubi (2013), and with the solution of the two vertical plates case.

$$Nu = \frac{Ra_S\left(\frac{S}{D}\right)}{24} \quad (8)$$

$$Ra_S = \frac{g\beta(T_\infty - T_w)S^3}{\nu\alpha} \quad (9)$$

Through the results shown, one realizes the good agreement of the simulation results with the experimental for both the convective coefficient and the Nusselt number. When the simulation results are compared to those generated with Eq. 8, it is clear that there is no agreement values. This is due to the differences in the geometric characteristics of the two channels. The agreement on the behavior of results suggests that the phenomena have an analogy. Also in Fig. 6, a trend line for the simulated results is exposed, generated by Eq. 10, emphasizing linear behavior of the solution to the number of Ra_s.

$$Nu = 0.515 Ra_S \left(\frac{S}{D}\right) + 0.03 \tag{10}$$

The effectiveness, Eq. 11, is defined as the ratio between heat transfer rate in the fin and the heat transfer rate that would exist in the tube without the fin. In Tab. 5, are the values determined by analytical solution and numerical simulation. Eq. 12 is the analytical solution of the governing equations for cross section of fins not uniform, with temperature specified at the base and tip adiabatic.

$$\varepsilon_f = \frac{q_f}{h A_{tr,b} (T_\infty - T_w)} \tag{11}$$

$$q_{fa} = \pi\, d\, t\, (T_\infty - T_w)\, m \left(\frac{K_1(mr_1) I_1(mr_{2c}) - I_1(mr_1) K_1(mr_{2c})}{K_1(mr_{2c}) I_0(mr_1) + I_1(mr_{2c}) K_0(mr_1)} \right) \tag{12}$$

$$m = \sqrt{\frac{2 h_1}{k t}} \tag{13}$$

Were q_f is the heat rate transferred in the fin, determined analytically, and by numerical simulation, $A_{tr,b}$ is the area of the cross section of the fin at its base. The convective heat transfer coefficient h, used to determine the effectiveness in Eq. 11, must be the one of the tube without fin, and h_1 is obtained from the simulation results, Tab. 3. The terms K_0, I_0, K_1, I_1, are modified Bessel functions of the first order, first and second kind, Incropera (2008), k is the diffusive heat transfer coefficient of the fin material. The effectiveness of the fin, determined with the results of analytical solution ε_{fa}, and by numerical simulation results, ε_{fn} are shown in Tab. 5. The relative difference is smaller than 5%, to every case.

Table 5. Effectiveness by analytical solution and numerical simulation.

	T_∞=22°C		T_∞=27°C		T_∞=33°C	
	ε_{fa}	ε_{fn}	ε_{fa}	ε_{fn}	ε_{fa}	ε_{fn}
T_w=8°C	14.02	14.49	14.72	15.35	15.80	16.45
T_w=12°C	13.12	13.67	13.84	14.37	15.13	15.69
T_w=15°C	12.46	13.0	13.26	13.83	14.46	15.06

5. CONCLUSIONS

The numerical simulation has proven to be an effective tool in the analysis of natural convection on a finned body. Comparisons with experimental and analytical results showed good agreement. It should be noted that the analytical solution used considers the boundary condition at the tip of the fin as adiabatic. The results show that for 2 mm spacing the convective process is of low intensity. When compared with the convection with the case of two vertical plates with similar dimensions it is noticed that the fin-tube geometry has better performance, shown in Fig. 6, because the tube base also contributes to the heat transfer. Despite of low intensity of the convection, the effectiveness shows that the use of fin for this case is advantageous due to the ε_f >2. With the dimensions of the spacing and the low velocity, which occur in this region, heat transfer occurs more intensively on the fin tips. The temperature of the fin is not constant, varying with r and θ, and the maximum fluctuations in temperature between the base and the tip fin are in the order of 0.2 °C. This is due to the material fin, which in this case was aluminum, having a diffusion coefficient of heat transfer in the order of 237 W/mK; and due to the low intensity of convective heat transfer, that occurs surrounding the fin. The fins design strongly depends on convective transfer coefficient, which in many situations is determined by correlations developed for similar geometries, which introduces uncertainty in the design, and may distance the solution of physical reality. The analytical technique has the exact solution to the problem, provided that the convective coefficient is correct, which in many cases does not occur. In these cases, the simulation proved its importance by providing values with good agreement with the experimental values.

6. REFERENCES

Kayansan, N. and Karabacak, R., 1992. “Natural Convection Heat Transfer Coefficients for a Horizontal Cylinder With Vertically Attached Circular Fins”. Heat Recovery Systems & CHP, Vol. 12, No. 6, pp. 457-468.

Hahne, E. and Zhus, D., 1994. “Natural Convection Heat Transfer on Fined Tubes in Air”. Int. J. Heat and Mass Transfer, Vol. 37, pp. 59-63.

Yildiz, Ş. and Yüncü, H., 2004. “An Experimental Investigation o Performance of Annular Fins on a Horizontal Cylinder in Free Convection Heat Transfer”, Heat and Mass Transfer, Vol. 40, pp. 239-251.

Churchill, S.W. and Chu, H.H.S., 1975. “Correlating Equations for Laminar and Turbulent Free Convection from Horizontal Cylinder”. Int. J. Heat and Mass Transfer, Vol. 18, pp. 1049-1053.

Morgan, V. T., 1975. “The Overall Convective Heat Transfer From Smooth Circular Cylinder”. Advances in Heat Transfer, Vol. 11, pp. 199-264.

Chen, H.T. and Hsu, W.L., 2007. “Estimation of Heat Transfer Coeficient on the Fin of Annular- Fined Tube Heat Exchangers in Natural Convection for Varios Fin Spacings”. Int. J. Heat and Mass Transfer, Vol. 50, pp. 1750-1761.

Yaghoubi, M. and Mahdavi, M., 2013. “An Investigation of Natural Convection Heat Transfer from a Horizontal Cooled Fined Tube”. Experimental Heat Transfer, Vol. 26, pp. 343-359.

Kumar, A., Joshi, B.J., Nayak, A.K., and Vjayan, P.K., 2016. “3D CFD simulations of Air Cooled Condenser-II: Natural Draft Around a Single Finned Tube Kep in a Small Chimney”. Int. J. Heat And Mass Transfer, Vol. 92, pp. 507-522.

Incropera, F.P., Dewitt, D.P., Bergman, T.L. and Lavine, A.S., 2008. Fundamentos de Trnsferêncai de Calor. LTC, Rio de Janeiro, RJ.

Bejan, A., 1984. Convective Heat Transfer. John Wiley & Sons, New York.

5. RESPONSIBILITY NOTICE

All authors are the only responsible for the printed material included in this paper.