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## COB-2019-0412 OPERATIONAL MODAL ANALYSIS APPLIED TO A NUMERICAL ROTOR MODEL

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**Abstract.** Operational Modal Analysis (OMA), used initially at civil structures, has also increasingly gained importance in the mechanical field. The purpose of this paper is to investigate the use of OMA algorithms in a numerical model of a rotating machine with hydrodynamic bearings. The rotor is modelled using finite element method and the bearings are approximated by a linearized approach, extracting the dynamic coefficients from an analytical model. Then, the rotor is excited with a white gaussian noise and its time response is used as input for the OMA technique, estimating the modal parameters from a time domain method, Stochastic Subspace Identification, and a frequency domain method, Enhanced Frequency Domain Decomposition, using software ARTEMIS<sup>®</sup>. The model data allowed to study the influence in the modal factors of different parameters of the simulations. Frequency domain methods showed a straightforward peak picking technique of resonance regions at the singular value decomposition of the power spectral densities of the signal, but the parameters obtained from time domain methods presented higher accuracy for closed spaced modes and better estimation for damping ratios and natural frequencies.

**Keywords:** operational modal analysis, rotating system, numerical modelling

### 1. INTRODUCTION

The increase in demand of productive sectors and the need for high-scale, low-cost production has pushed the boundaries of the industry, which is increasingly in need of refined, robust and reliable equipment. In this way, the interest in the study and development of models and techniques of parameters identification that can be used for an analysis of the performance and integrity of systems have grown fast. In parallel, techniques of dynamic structural analysis of machines have become relevant for formulation of more refined models used to study and predict the behavior of mechanical systems.

In this context, it is difficult to find machines or mechanical systems that do not have any rotating components, such as shafts, gears, bearings, wheels, among others. Gas turbines, pumping units, reducers, compressors, motors (combustion and electric) and generators are some examples (Lorenzo, 2017). These machines are subjected to conditions that are often complex and non-linear during their operation cycle and require an extra attention in their design and activity so that losses through breaks and downtimes for maintenance are the smallest possible. Therefore, to analyze the dynamic parameters of these machines in operation is primordial to have a control of their structural health, as well as becomes fundamental for the elaboration of more refined numerical models, which consider effects caused by the adversity of operation condition.

The dynamic behavior can be analyzed, for example, by measuring the vibration of these machines. Consequently, techniques for identifying parameters of the structure vibration based on these measurements emerge as a fundamental tool and of great interest in engineering and structural dynamics. These techniques of vibration analysis are known as modal analysis.

Modal analysis is frequently performed to obtain the modal parameters (natural frequencies, damping factors and modes of vibration) of the object of study and how the vibration of a given structure occurs. These vibrations can be measured in the physical domain by instruments that get the response of the structure, such as accelerations, velocities

and displacements and contain data of various modes of vibration. The final vibration is inherent of the structure, composed of infinite modes. The estimation of the modal parameters of these vibrations can be performed in two major ways (Brincker and Ventura, 2015): experimental modal analysis, known as EMA, and operational modal analysis, known as OMA.

In EMA, the analysis is input-output type and the excitations are usually inserted into the structure by instrumented shakers and hammers. These instruments are used because they allow excitations to be measured. The most common test setup used in this type of analysis is to suspend the structure, so that it is exposed only to the inputs forced by the instruments described above. To measure the response, accelerometers are placed in positions where the responses of the modes of interest can be measured. In addition, the excitation point should also be evaluated to be a convenient point for the measurement of modes (Brincker and Ventura, 2015). A comparison between the excitation signal and the response results in a set of frequency response functions (FRF) that normally shows peaks at resonant frequencies and hence modal models can be extracted. In this type of test a controlled environment is required, usually performed in the laboratory, which makes it difficult to test large machines and structures, as well as ignore the environment in which the equipment is inserted.

Therefore, it is exactly taking into account the operating conditions that the method known as Operational Modal Analysis (OMA), of main interest in this paper, arises. Initially focused in civil structures, the study of system behavior from the operating condition itself in some cases turns out to be more representative than in laboratory tests. The excitations to which the equipment is subjected in operation may have several distinct sources and often do not have a defined behavior, making it difficult (or even impossible) to be measured and reproduced. In this way, the operational modal analysis is a method known as output-only, since the modal parameters are estimated only from the dynamic response of the structure, without necessarily knowing the excitation the system is submitted to. Using the OMA, the modal parameters reflect the operating behavior of the structure, with all components assembled (Lorenzo, 2017). For this reason, OMA has been widely applied in the aerospace, automotive and civil engineering, and has also gained some notoriety in the oil and gas industry.

In this context, the purpose of this paper is to investigate the use of OMA and its algorithms in rotating machines employing a numerical model with hydrodynamic bearings, evaluating the modal parameters for different identification methods and comparing them with the values of the numerical model.

## 1.1 Operational Modal Analysis

For a correct implementation of Operational Modal Analysis in rotating machines, some assumptions must be satisfied in order to make a correct identification of the structure. For example, the main requirement is to ensure that the excitations have approximately gaussian white noise characteristics, with energy distributed over a wide frequency range that covers the frequency range of interest for the structure modal characteristics (Brincker and Ventura, 2015). However, how to do that, if presumably the input forces are not measured?

Based on the Central Limit Theorem, even when the individual load contributions for the system might not be Gaussian, the summation of these independent and identically distributed random variables will tend to be Gaussian (Brincker and Ventura, 2015). Furthermore, the random forces must produce a clear multiple-input force for the system, in order to distinguish closed spaced modes, see Brincker et al, 2003.

Since the process is based on a random excitation, the analysis of random data involves different considerations from the deterministic data (Bendat and Piersol, 2010). The ground of the analysis starts estimating the correlations and consequently autocorrelation functions for the response signal. Additionally, the random data used in the identification techniques in OMA must be ergodic, i.e., the time-averaged mean value and autocorrelation functions do not vary in time.

When applied to rotating machinery, the use of OMA techniques still needs to consider the possible presence of harmonic excitations, i.e, combination of deterministic signals, inherent from operational conditions and therefore conflicting with white noise consideration of excitation forces. Such disturbances are consequence of unbalanced mass, misalignment and moving components, for example, and the effect of having such components depends on the nature of its occurrence, such as frequency, level and number of excitations. In some cases, they can be mistaken or even bias the estimation of structural modes. Hence, special attention is required to identify and remove the harmonic influence from structural modes in the modal parameter extraction process. Some methods for harmonic identification are better described in Agneni et al, 2012, and are mainly based on the Kurtosis, the fourth central moment of the probability density function. Also, the removal of the harmonic components is made as preprocessing tool and the main algorithms are based in an orthogonal projection, see for instance Gres et al, 2018.

Also, when dealing with rotating machines, it must be considered the possibility of existing close spaced modes. As shall be seen, this consideration may lead to some implications, forcing the use of a specific method to overcome this issue.

In the present work, the identification of modal parameters are based on two main techniques: Enhanced Frequency Domain Decomposition (EFDD), a non-parametric frequency domain method, usually used as a first step in the identification process, see for instance Brincker et al., 2000, and on Stochastic Subspace Identification methods, a

parametric time domain, first proposed by Overschee and Moor, 1996, with its application for Operational Modal Analysis better described in Møller et al., 2005.

The EFDD is an improvement over the Frequency Domain Decomposition: a method based on a peak-picking technique using a Singular Value Decomposition (SVD) of each of the Spectral Density matrices of the system, a matrix which its dimension is correlated with the input measurement channels, but not necessarily the same number. The EFDD improves the method by adding the identification of a Single Degree of Freedom (SDOF) Spectra Bell over the identified modes shapes and performing a correlation analysis based on the Modal Assurance Criterion (MAC) between the reference vector and the singular vectors corresponding to a certain frequency. Additionally, the selected singular vectors are weighted averaged and, in this way, it is possible to get better estimation of modal parameters. This method also allows the estimation transforming the SDOF Spectra bell into time domain, obtaining a correlation function that can provide an estimation of the damping ratio of the system, a parameter that cannot be extracted in the basic FDD technique.

The SSI is an identification method that fits a parametric model directly into the raw time series data of the system. The methods are based upon a stochastic state space model described by Eq (1):

$$\begin{cases} x_{t+1} = [A]x_t + w_t \\ y_t = [C]x_t + v_t \end{cases} \quad (1)$$

Where  $x_t$  is the state vector at time  $t$ ,  $[A]$  is the system matrix (state matrix),  $y_t$  is the response vector at time  $t$ , and  $[C]$  is the observation matrix. The response is generated by two stochastic processes  $w_t$  and  $v_t$  called the process noise and the measurement noise respectively. Modal models are estimated for the different state space dimensions up to a selected maximum state space dimension. The setting of maximum state space dimension depends upon the number of modes, which is searched for, the excitation, the number of sinusoidal components in the response signals and the number of noise modes needed to fit (predict) the measured response signals (Herlufsen et al, 2005).

## 2. NUMERICAL MODEL

The numerical model is based on the implementation of the Finite Element Method for an offset rotor supported on hydrodynamic bearings (Fig. 1). For the shaft model, it is used the Euler Bernoulli beam model with four degrees of freedom per node, two translational and two angular (bending). It is assumed a rigid support structure in order to isolate the effects by the bearings. The resulting equation of motion is given by Eq (2):

$$[M]\{\ddot{x}\} + ([C] + \Omega[G])\{\dot{x}\} + [K]\{x\} = \{F\} \quad (2)$$

Being  $[M]$ ,  $[C]$ ,  $[K]$  and  $[G]$  the global mass, damping, stiffness and gyroscopic matrices, respectively (damping and stiffness matrices include the influence of hydrodynamic bearings).  $\Omega$  represents the rotating speed of the rotor and  $\{x\}$  is the vector that represents the degrees of freedom from the system. The external force  $\{F\}$  is exciting the system due to the white Gaussian noise applied at the disk position, according to Fig. 1. It is possible to modify its power to analyze the effects on the identification process.  $[C]$ , which represents the shaft equivalent structural damping plus the hydrodynamic bearing damping, has the structural damping modeled as proportional to the stiffness matrix ( $\beta [K]$ ), with  $\beta$  adjusted to  $1.5 \times 10^{-5}$ . Overall, the model has 5 elements and 6 nodes, totalizing 24 degrees of freedom.

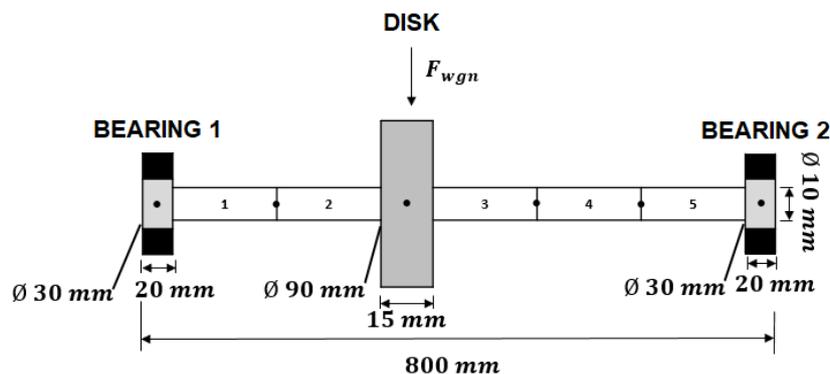


Figure 1. Finite element model for the rotating system.

The steel shaft has an elastic modulus of  $2.1 \times 10^{11}$  N/m<sup>2</sup>, 7800 kg/m<sup>3</sup> density and poisson coefficient 0.3. The hydrodynamics bearings are approximated by a linearized model based on the assumptions made by Machado et al., 2018.

The purpose of the numerical model is to evaluate the temporal responses at different rotating speeds, obtain the Campbell diagram and analyze the modal parameters of the system at a given rotating frequency, set as  $\Omega=50$  Hz. The simulation is used to provide time responses based on the random excitation from the white noise and, for a preliminary analysis, it will not be considered the existence of harmonic components on the signal. Though, the interface of numerical model and identification algorithms need to fulfill some assumptions: it is necessary to recreate the discrete data characteristic of data acquisition systems. Since the time responses are obtained from a variable time step numerical integrator, it is used an interpolation method to recreate a sampling that meet the requirement of an equally spaced time series.

With the model well established, it is possible to compare the numerical modal parameters with the ones given by the identification processes of the Operational Modal Analysis and to investigate the precision on the parameter estimation. Besides, it is explored how to work with some variables of the identification process, since it is easier to test different conditions, like varying the power of the excitation, the time interval of data and it is not necessary to deal with noisy data and other difficulties inherent to experimental data acquisition in this first study.

### 3. RESULTS AND DISCUSSION

Before getting into the Operational Modal Analysis of the model, it is important to have some useful information about the system. In practice, this is not the situation found in the OMA testing, but as mentioned, this is an investigation about how the methods will react to the model and what precision can be expected for each one. Therefore, analyzing the rotor Campbell diagram, Fig. 2, it is chosen the first three modes in the angular frequency of 50 Hz as the modes of interest on the OMA tests.

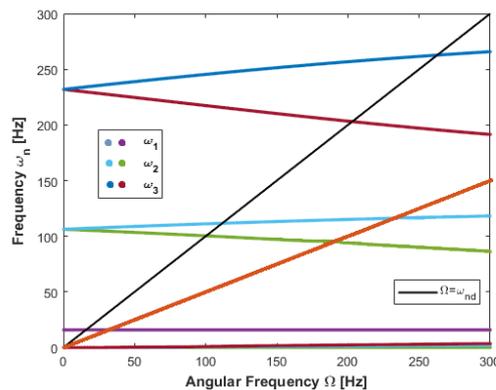


Figure 2. Campbell Diagram for the rotor model.

As so, the time response for each degree of freedom is taken as OMA input. Fig 3 is the time response of the disk in the vertical direction. The first step is to analyze the EFDD of the system, as Fig. 4 shows identification results for this frequency domain method, with the singular value decomposition of the spectral densities matrices for the rotor. This decomposition corresponds to a SDOF identification of the system, with the peaks being the resonances for the model. In this case, since the system is free from harmonic forces, the peak picking identification can be straightforward.

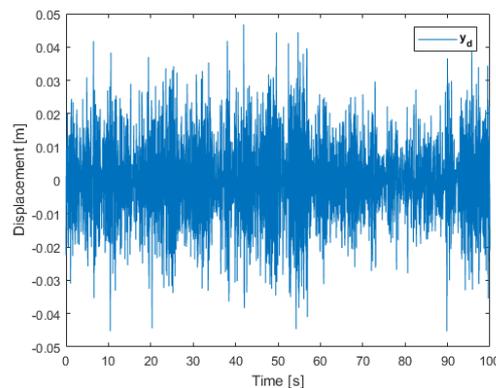


Figure 3. Rotor disk vertical displacement time series.

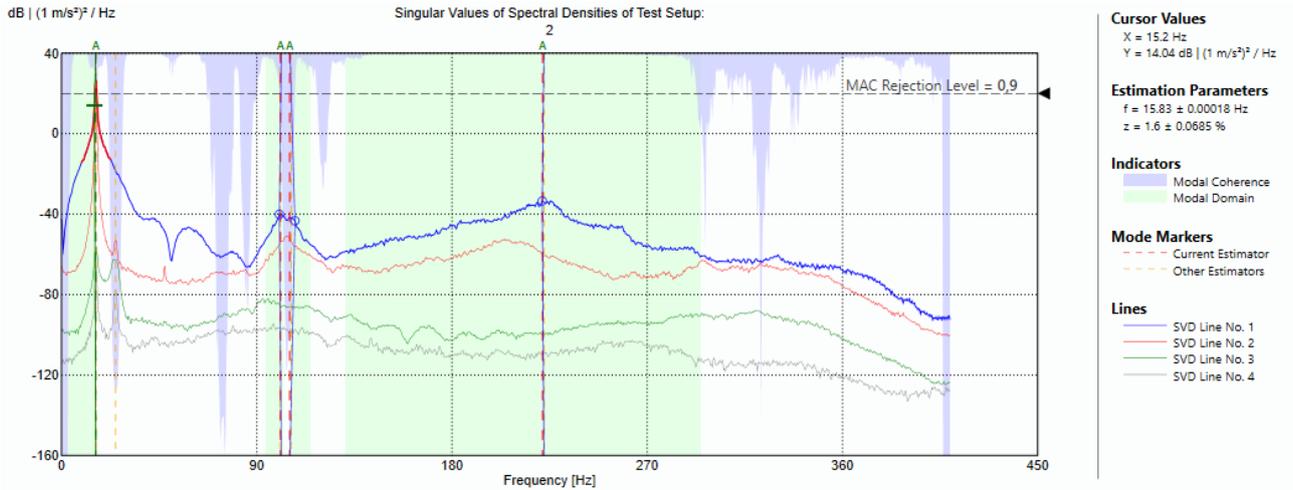


Figure 4. EFDD Singular Values of Spectral Densities.

Still in the case shown at Fig. 4, the Modal Assurance Criterion showed that the modes estimated, one for first mode, two for the second and one for the third mode, had a low correlation between each one (less than 0,2), verifying their orthogonality. In addition, the method was unable to identify the first mode forward whirl. This is a consequence of a closely spaced mode, since the numerical model showed a first mode with natural frequencies of 15.82 Hz for backward whirl and 15.86 Hz for forward whirl. A validation of the parameters can be made, considering the spectra bell for each mode identified in Fig. 4. As an example, it is chosen the first mode, with corresponding spectra bell selected in Fig. 4 as the red line in the SVD line number 1. The correlation function for this mode can be seen in Fig. 5.

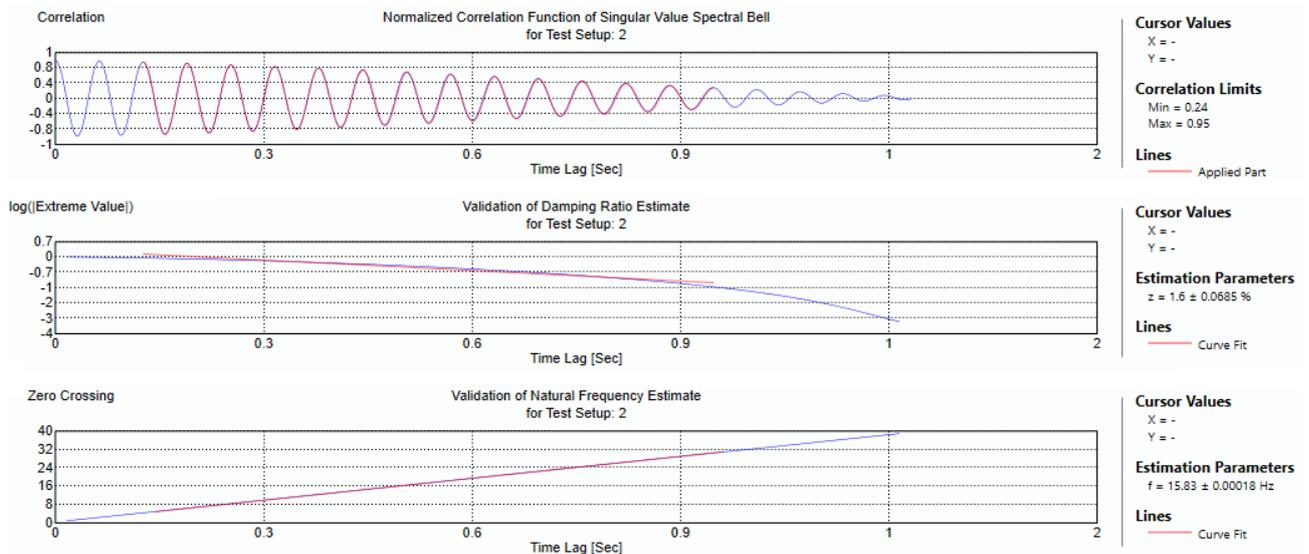


Figure 5. EFDD validation procedure.

The curve fit in this case shows a good estimation of the modal parameter, as seen on Zhang and Tamura, 2003. For the assumptions made for the model, the correlation function needs to be as close as possible of a SDOF system for the mode analyzed, with damping estimation being the slope of the middle curve and the natural frequency estimation the number of zero crossing. However, for the modes 2 and 3 the algorithm was less precise for estimating damping coefficients, even when the input white noise signal was changed. The correlation function for these modes were unable to reproduce a good pattern of SDOF system, because the spectra bell was miss fit.

Figure 6 is an attempt to get a better estimate of the first mode, switching the modal identification to a time domain process: the SSI Unweighted Principal Components (SSI-UPC), an unbiased parametric estimator that uses the raw time data for analysis. It is possible to change the dimension of the system and evaluate the consequences of this State Space dimension on the precision of the parameters. Also, it is used a software tool to eliminate the computational and spurious modes inherent of an SSI estimation. This tool works based on specified number of eigenvalues for matrix  $[A]$

on Eq. (1). This number is not necessarily equal to the number of modes to be identified, but rather needs to be large enough so the algorithm can reproduce the same behavior of the system.

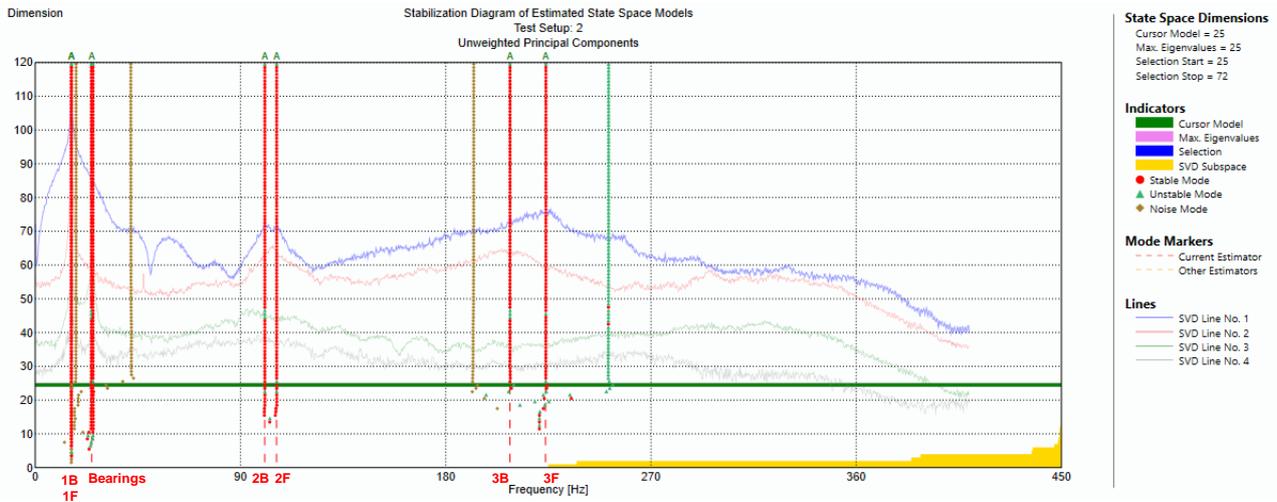


Figure 6. SSI-UPC stabilization diagram.

It is shown that both backward and forward whirl of all modes were identified. Also, the mode identified marked as Bearings, is a rigid body mode equal to  $\Omega/2$ . The SSI is expected to have better estimation of damping values and identifying closed spaced modes. Table 1 and Fig. 7 features a comparison between the results of each method for the identification of natural frequency and Table 2 present the values identified for damping coefficients.

Table 1. Natural frequency estimation results for OMA analysis of the rotor

Mode		Numerical [Hz]	EFDD [Hz]	SSI-UPC [Hz]
1	Backward	15,82	15,83	15,83
	Forward	15,86	-	15,85
2	Backward	103,25	101,70	100,87
	Forward	108,63	105,43	106,05
3	Backward	222,20	-	208,90
	Forward	239,50	221,81	223,80

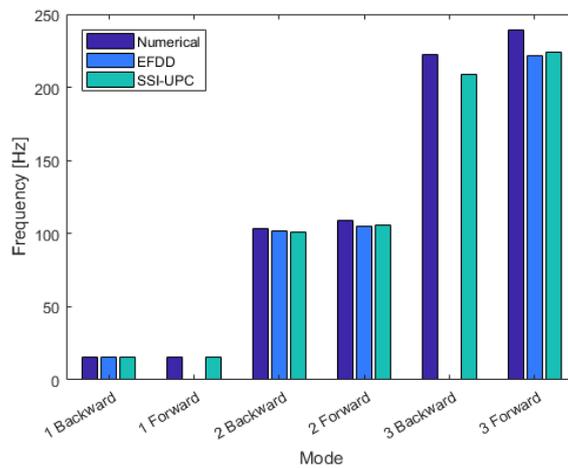


Figure 7. Natural frequency estimation comparison.

Table 2. Damping estimation results for OMA analysis of the rotor

Mode		Numerical [%]	EFDD [%]	SSI-UPC [%]
1	Backward	0,23	0,81	0,16
	Forward	0,56	-	0,71
2	Backward	3,47	2,18	1,48
	Forward	3,42	1,578	1,59
3	Backward	7,41	-	4,77
	Forward	7,40	4,19	2,66

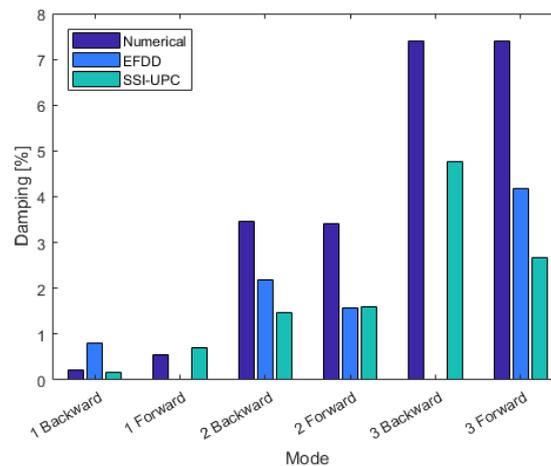


Figure 8. Modal damping estimation comparison.

In Fig. 7 it is clear that the only method that was capable of identifying all backward and forward whirl modes was the SSI-UPC, as expected. Analyzing Tab. 2 and Fig. 8, the methods showed an imprecise estimation of the damping. This might be due to two issues: the rotor modelling and the additional bearing damping on the system.

The rotor modelling may introduce some uncertainties to the estimation since it was established as an ordinary differential equation of first order, using the widely adopted state space modelling. The gaussian white noise input was then integrated as a continuous function, as this was found to be the best way to include it in the numerical integrator. Although, since this external force is rather a stochastic variable, models based on stochastic differential equations for the problem might have a better suit and therefore will be tested as improvement for the modelling.

Initially, when dealing with the stiffness and damping of the shaft, the matrices were considered to be proportional. However, inserting the hydrodynamic bearing into the system implies adding its stiffness and damping to the respective node into the global matrices and therefore making the global damping not proportional to the global stiffness. This could generate a potential difficulty to the OMA identification process and will be verified in further experimental analysis.

#### 4. CONCLUSIONS

It is shown that the Operational Modal Analysis is a considerable important tool when analyzing the dynamical behavior of machines, even when the input forces are not truly gaussian or have influence of deterministic forces, such as harmonics.

The SSI based identification technique is shown to be an efficient method for identifying closed spaced modes, like the ones found in the first mode of the rotor. Comparing the results obtained in the frequency domain FDD method, SSI was also capable of a better estimation for the natural frequencies when compared with the numerical model, indicating that this type of technique should be a better estimator for operational Modal Analysis of rotating machines.

Further investigation will be considered for a rotor with harmonics, a question of great interest in rotating machines. Analyzing a real machine can also bring great information for the research, dealing with acquisition and sampling that can overcome the numerical issues and approach to a more realistic operation measurement.

## 5. ACKNOWLEDGEMENTS

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