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EFFECT OF THE STANDARD DEVIATION OF CLEARANCE DISTRIBUTIONS ON ASYMMETRIC TILTING-PAD JOURNAL BEARINGS

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Abstract. *The clearance and the pre-load of Tilting-Pad Journal Bearings (TPJBs) are significantly affected by dimensional uncertainty, thus possibly leading the rotor-bearing system to an asymmetric configuration, i.e. the TPJB can present different clearances among its pads. Such asymmetry of the TPJB has consequences for the resultant dynamic coefficients of the rotor-bearing system. This work contributes to the further understanding of the dynamic behavior of TPJBs subjected to dimensional uncertainty. For that, it focuses on the question: What happens if the standard deviation of the clearance distribution changes? The results show that the dynamic coefficients still scatter around the mean value of the distribution, and this scattering of the dynamic coefficients is still limited by the values obtained from a symmetric TPJB with minimum and maximum clearances. Therefore, it is still possible to estimate the distribution of dynamic coefficients by fitting a Gamma distribution from three simulations runs of the symmetric case.*

Keywords: *lubricated bearings, uncertainty, stochastic analysis, thermohydrodynamics, dynamic coefficients.*

1. INTRODUCTION

Tilting-pad journal bearings (TPJBs) have been widely used in industry for the last 50 years to support heavy rotating machinery (e.g. centrifugal compressors, turbo-generators, and turbines) due to their inherent higher dynamic stability in comparison to other types of lubricated bearings. During this period of time, several studies in literature clarified the influence of different bearing parameters in the resultant dynamic characteristics of TPJBs. For example, Jones and Martin (1979) showed that bearing clearance (assembled clearance) is more important than machined clearance to the resultant dynamic coefficients. In fact, the variation of bearing clearance and pre-load due to manufacturing tolerances can strongly affect the bearing static and dynamic performances, specially at low loads (Fillon *et al.*, 2007; Dmochowski *et al.*, 2008). On the other hand, at high dynamic loads, pad compliance has a more significative influence on the dynamic behavior of the rotor-bearing system (Cha *et al.*, 2013).

Considering this strong effect of bearing clearances in the dynamic characteristics of TPJBs, researchers began to pay more attention to the actual clearances of such bearings after assembling. In fact, it has been recently noticed that, the machining tolerances of the pads can be of the same magnitude order of the oil film thickness in the bearing (Dang *et al.*, 2016). Consequently, the bearing clearance and the pre-load are significantly affected, thus possibly leading the system to an asymmetric configuration, i.e. the TPJB can present different clearances among its pads. This asymmetry of the TPJB has effect on the dynamic behavior of the rotor-bearing system. The consequences observed so far in literature are:

- the cross-coupling coefficients increase, depending on how asymmetric the system is (Gomez *et al.*, 2013);
- the direct coefficients are more affected when the geometry of the loaded pads changes, and the cross-coupling coefficients are more affected when the geometry of the unloaded pads changes (Quintini *et al.*, 2014);
- the standard deviation of the coefficients increases, but the mean values remain near the nominal values (Ruiz and Diaz, 2016);
- the resultant dynamic coefficients of asymmetric TPJBs with random clearances scatter around the values of the dynamic coefficients of the symmetric TPJB with nominal clearance (Silva and Nicoletti, 2019);
- such scattering of the dynamic coefficients is limited by the dynamic coefficients that symmetric TPJBs present with maximum clearance or minimum clearance considered in the random distribution (case of load-on-pad configuration) (Silva and Nicoletti, 2019).

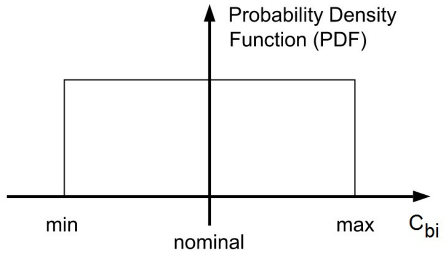

Figure 1. Probability density function of the adopted variation of bearing assembled clearances (Silva and Nicoletti, 2019).

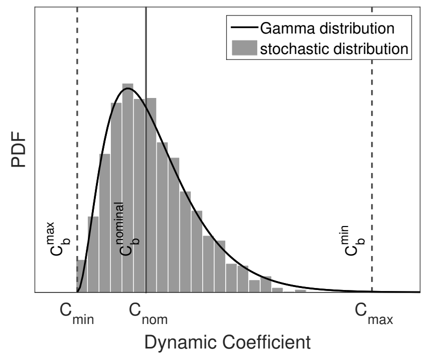

Figure 2. Probability density function of a Gamma distribution fitted to the stochastic results of dynamic coefficients (Silva and Nicoletti, 2019).

In Silva and Nicoletti (2019), the variability of the bearing clearances among the pads was represented by a uniform probability density function (PDF). This uniform PDF had an average value equal to the nominal clearance value ($C_{bi}^{nominal}$), and the extreme values (maximum and minimum possible clearances) were evenly distributed around the nominal clearance value ($C_{bi}^{nominal} - C_{bi}^{min} = C_{bi}^{max} - C_{bi}^{nominal}$), see Fig. 1. By performing a Monte Carlos simulation, 5,000 sets of clearances were randomly created and the resultant equivalent stiffness and damping of the TPJB were calculated for each set, by using a themo-hydrodynamic model of the bearing. As a result, such stochastic distribution of clearances resulted in a distribution of stiffness and damping of the bearing that can be represented by a Gamma distribution function (Fig. 2). The interesting fact is that, this Gamma distribution of the TJPB's dynamic coefficients can be calculated by knowing the dynamic coefficients of the bearing at the nominal, minimum, and maximum values of clearances of the bearing (Silva and Nicoletti, 2019). Therefore, instead of running thousands of simulations of the bearing, we only need to run three simulations to estimate the distribution of stiffness and damping of the bearing due to the clearance uncertainty represented by the uniform PDF shown in Fig. 1.

Despite these developments in the understanding of how TPJBs dynamically behave in face of dimensional uncertainty, a new question arose: what happens if the standard deviation of the clearance distribution changes? That means, what happens to the dynamic coefficients, and their distributions, if the dimensional variation around the nominal clearance value is doubled, tripled, or halved? What happens to the dynamic coefficients, and their distributions, if the nominal value is closer to the maximum value, or closer to the minimum value (uneven distribution around the nominal value)? Will the Gamma PDF still be a good representation of the coefficients' distribution? The objective of this work is to answer these questions and contribute to the further understanding of the dynamic behavior of TPJBs subjected to dimensional uncertainty.

2. MATHEMATICAL MODELING OF THE TPJB

The adopted mathematical model of the TPJB is the same as that used in Silva and Nicoletti (2019): a thermo-hydrodynamic (THD) model that neglects temperature variation across the film thickness. Hence, it comprises the numerical integration of the Reynolds and the Energy equations in each pad to evaluate the hydrodynamic pressure and temperature in the oil film between the pads and the rotor. After achieving static and thermal equilibria, we apply the perturbation method to calculate the dynamic coefficients of the system, by imposing variations of displacement and velocity to the pads and to the rotor positions (Allaire *et al.*, 1981). Such procedure results in 6×6 stiffness and damping matrices, because the TPJB has four pads (four angular degrees-of-freedom) and a rotor with vertical and horizontal degrees-of-freedom. To reduce theses matrices to the global degrees-of-freedom (horizontal and vertical displacements of the rotor), we adopt the condensation method described in Dimond *et al.* (2011). That results in global stiffness and damping matrices with direct coefficients in the main diagonal (k_{xx}, k_{yy}, d_{xx}, d_{yy}) and cross-coupling coefficient in the off-diagonal (k_{xy}, k_{yx}, d_{xy}, d_{yx}). The directions X and Y refer to the reference frame depicted in Fig. 3. The dynamic coefficients can be represented in adimensional form ($K_{ij} = C_p\, k_{ij}/W$ and $D_{ij} = C_p\, \Omega\, d_{ij}/W$) and they are presented as a function of the Sommerfeld number (So). Because of the temperature variation in the oil film, the Sommerfeld number is calculated considering the average oil viscosity allover the bearing. In the present work, the external load is applied to the shaft in the vertical direction (Y direction) towards pad 4 (Fig. 3). The parameter values used in the numerical simulation of the TPJB are listed in Table 1, and more details of the model are found in Silva and Nicoletti (2019).

In the mathematical model, the asymmetry of the TPJB is simulated by adopting different bearing clearances (C_{bi}) for each pad, based on the Monte Carlo method. Therefore, we adopt a uniform distribution of values for the bearing clearances (range between minimum and maximum possible values, Fig. 1) and the clearance values are randomly chosen from this distribution. Consequently, we can find sets of random clearances (C_{b1}, C_{b2}, C_{b3}, C_{b4}) and we can run the

Table 1. Bearing parameters used in the numerical simulations.

Parameter	Value	Unit	Parameter	Value	Unit
rotor radius	40.0	mm	oil viscosity @40^{o}C	0.032	N.s.m^{-2}
pad inner radius	40.14	mm	oil thermal conductivity	0.14	W.m^{-1}.K^{-1}
pad width	80.0	mm	oil specific heat	1,800	J.kg^{-1}.K^{-1}
pad angle	80.0	degree	oil density	840.0	kg.m^{-3}
radial clearance (nominal)	70.0	μm	oil supply temperature	40.0	^{o}C
preload (nominal)	0.5	—	oil supply pressure	1.8×10^5	N.m^{-2}
pivot position	pad center		rotating speed	4,200	rpm
pad configuration	load-on-pad		loading	250 to 6,000	N

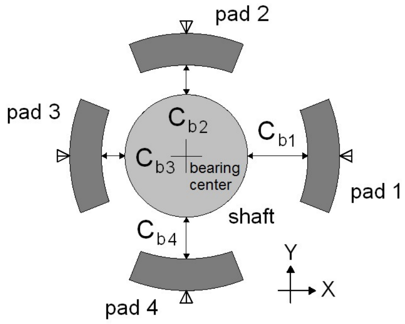

Figure 3. TPJB in load-on-pad configuration: asymmetric geometry due to variation of bearing assembled clearances in each pad (Silva and Nicoletti, 2019).

model and calculate the resultant dynamic coefficients of the TPJB using these clearances. We repeat this procedure 5,000 times (5,000 runs with different combinations of clearances in the TPJB).

From the results (distribution of dynamic coefficients at a given Sommerfeld number), we can fit a Probability Density Function (PDF) curve of a Gamma distribution. This PDF curve can be calculated from only three parameters: the minimum value of the dynamic coefficient C_{min} (obtained for a symmetric TPJB with maximum clearance), the maximum value of the dynamic coefficient C_{max} (obtained for a symmetric TPJB with minimum clearance), and the nominal value of the dynamic coefficient C_{nom} (obtained for a symmetric TPJB with nominal clearance). If we consider that (Silva and Nicoletti, 2019):

a) C_{min} is the starting value of the Gamma distribution;

b) all the distribution (e.g. 99.9%) is between C_{min} and C_{max};

c) the median of the distribution is C_{nom},

then we can identify the parameters k and θ of the PDF function by solving the system of equations:

$$\begin{cases} P(C_{max} - C_{min}) = 0.999 \\ P(C_{nom} - C_{min}) = 0.5 \end{cases} \tag{1}$$

where $P(.)$ is the cumulative distribution function of the Gamma distribution. The PDF curves presented in the following sections are obtained with this approach.

3. NUMERICAL RESULTS

The standard deviation of a uniform distribution is given by:

$$\sigma = \frac{b-a}{\sqrt{12}} \tag{2}$$

where a is the minimum possible value of the parameter in the distribution (e.g. C_{bi}^{min} in Fig. 1), and b is the maximum possible value of the parameter (e.g. C_{bi}^{max} in Fig. 1). In this work, we tested three different uniform distributions of the bearing clearances, with standard deviations of 5.77, 11.55, and 23.09, all of them with mean value of 70 μm (nominal bearing clearance). By using the THD mathematical model of the bearing and performing the stochastic analysis, we

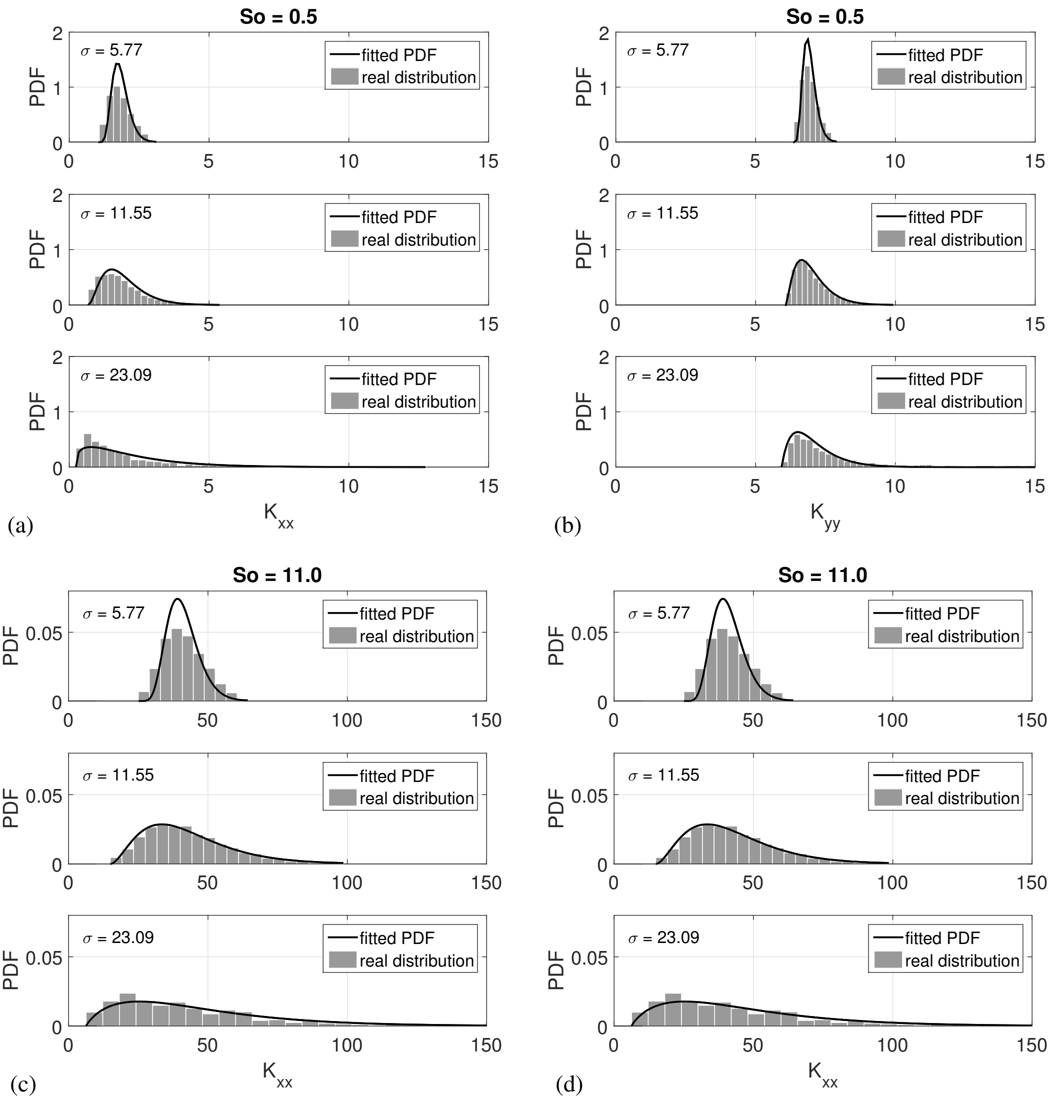

Figure 4. PDF of the adimensional stiffness coefficients of the TPJB for different distributions of bearing clearances. Comparison between the real distribution and the fitted PDF: (a) K_{xx} ($So = 0.5$), (b) K_{yy} ($So = 0.5$), (c) K_{xx} ($So = 11.0$), (d) K_{yy} ($So = 11.0$).

obtained the results shown in Fig. 4 and 5. We only present results for the direct coefficients because the values of the cross-coupling coefficients remained 2-3 orders of magnitude smaller than those of the direct coefficients.

In Fig. 4, by increasing the standard deviation of the distribution of bearing clearances, the resultant stiffness coefficients of the TPJB tend to scatter in wider ranges of values, as expected. However, irrespective of the standard deviation of the distribution, it is still possible to fit a PDF curve of a Gamma distribution to the resultant distribution of coefficients by the procedure described above. That is also true at different Sommerfeld numbers. The major differences occur at the results for a standard deviation of 5.77 (narrower distribution of bearing clearances), where the fitted Gamma PDFs tend to overestimate the probability at the center values (near the median) and underestimate at the extreme values of the distribution. These same observations can be drawn for the resultant distributions of damping coefficients (Fig. 5).

Hence, the Gamma function is still a good representation of the coefficients' distribution of the TPJB. In addition, we can still infer the coefficients' distribution of the TPJB by running only three simulations runs (symmetric bearing with nominal, maximum, and minimum bearing clearances).

3.1 Uneven Distribution Around the Nominal Value

In the analysis above, the nominal value of the bearing clearance coincided with the mean value of the distribution (see Fig. 1). Here, we investigate distributions whose mean values do not coincide with the nominal bearing clearance. We tested two different distributions: one with the nominal clearance closer to the minimum possible clearance of the TPJB, and another with the nominal clearance closer to the maximum possible clearance of the TPJB (Fig. 6). In both

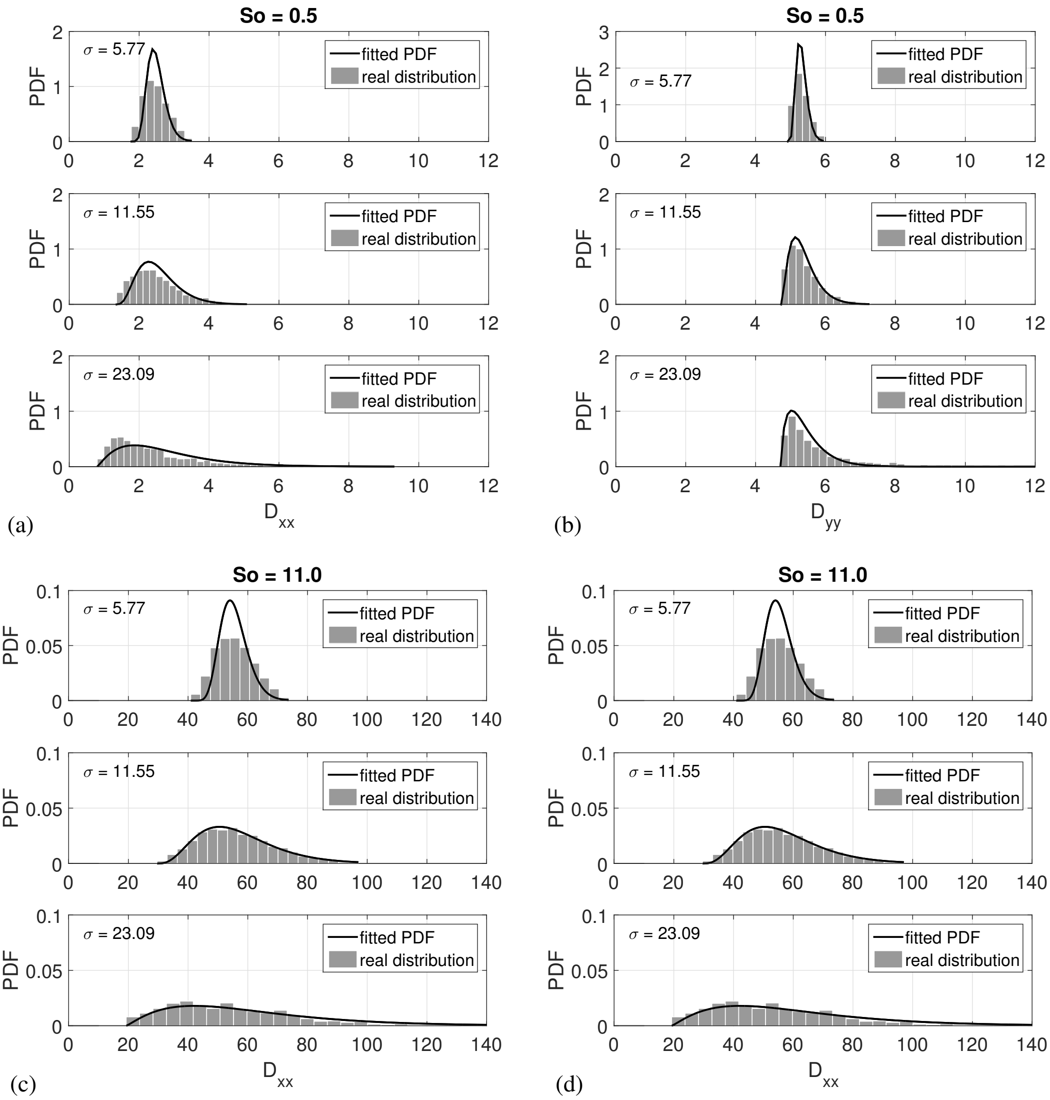

Figure 5. PDF of the adimensional damping coefficients of the TPJB for different distributions of bearing clearances. Comparison between the real distribution and the fitted PDF: (a) D_{xx} ($So = 0.5$), (b) D_{yy} ($So = 0.5$), (c) D_{xx} ($So = 11.0$), (d) D_{yy} ($So = 11.0$).

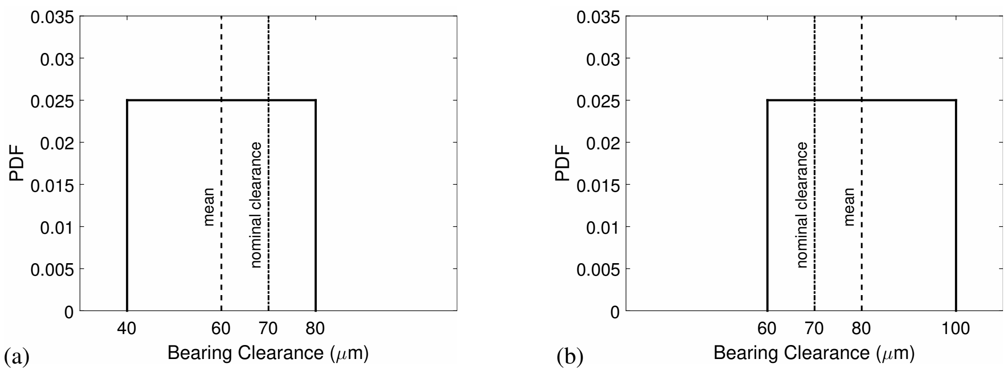

Figure 6. PDFs of the uniform distributions whose mean values do not coincide with the nominal bearing clearance: (a) mean value at 60 μm, (b) mean value at 80 μm.

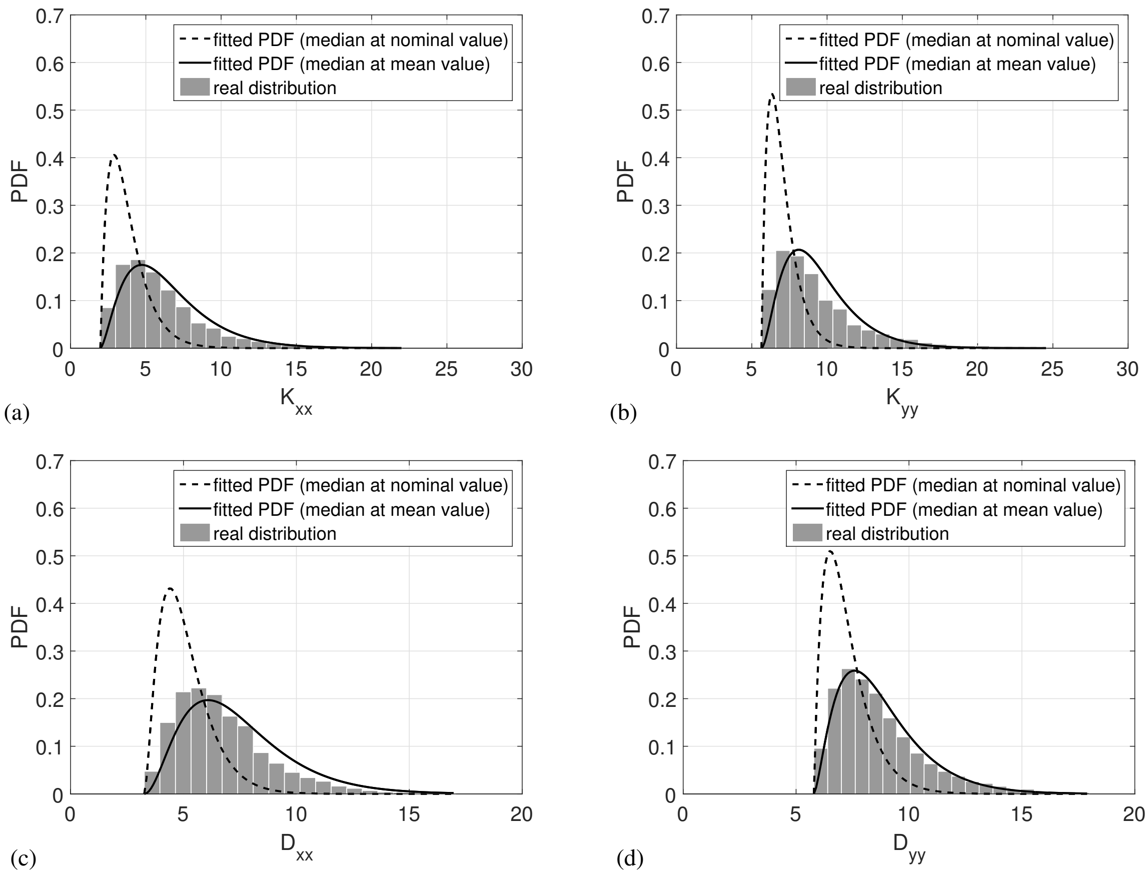

Figure 7. PDFs of the adimensional coefficients of the TPJB for the distribution of bearing clearances with mean value at 60 μm (So = 1.0). PDFs fitted with median at the nominal and at the mean values of the distribution: (a) K_{xx}, (b) K_{yy}, (c) D_{xx}, (d) D_{yy}.

distributions, the standard deviation remained the same (σ = 11.55). The results are shown in Figs. 7 and 8. Again, we only present results for the direct stiffness and damping coefficients because the values of the cross-coupling coefficients remained 2-3 orders of magnitude smaller than those of the direct coefficients.

In Fig. 7, we have the distributions of dynamic coefficients for the TPJB with nominal clearance value of 70 μm and possible variation of the clearances between 40 and 80 μm (mean value at 60 μm), for the Sommerfeld number of 1.0. As we can see, we cannot estimate a reasonable PDF curve of a Gamma distribution by using the methodology described in Silva and Nicoletti (2019) (three simulation runs of the symmetric bearing with maximum, nominal, and minimum clearances). However, when we use the mean value of the distribution instead of the nominal value, the estimated PDF curves of the distributions become more representative. We can make the same observation from Fig. 8, whose data refer to the distribution shown in Fig. 6(b) (nominal clearance value of 70 μm and possible variation of the clearances between 60 and 100 μm, with mean value at 80 μm).

Hence, it is clear from these results that the methodology described in Silva and Nicoletti (2019) to estimate the distribution of dynamic coefficients in TPJBs must be corrected. In fact, if we consider that:

a) C_{min} is the starting value of the Gamma distribution;

b) all the distribution (e.g. 99.9%) is between C_{min} and C_{max};

c) **the median of the distribution is $\mathbf{C_{mean}}$**,

where C_{mean} represents the value of the dynamic coefficient evaluated for a symmetric TPJB with clearance equal to the mean value of all possible clearances. Then, we can find the shape parameter k and the scale parameter θ of the PDF function by solving the system of equations:

$$\begin{cases} P(C_{max} - C_{min}) = 0.999 \\ P(\mathbf{C_{mean}} - C_{min}) = 0.5 \end{cases} \tag{3}$$

where $P(.)$ is the cumulative distribution function of the Gamma distribution.

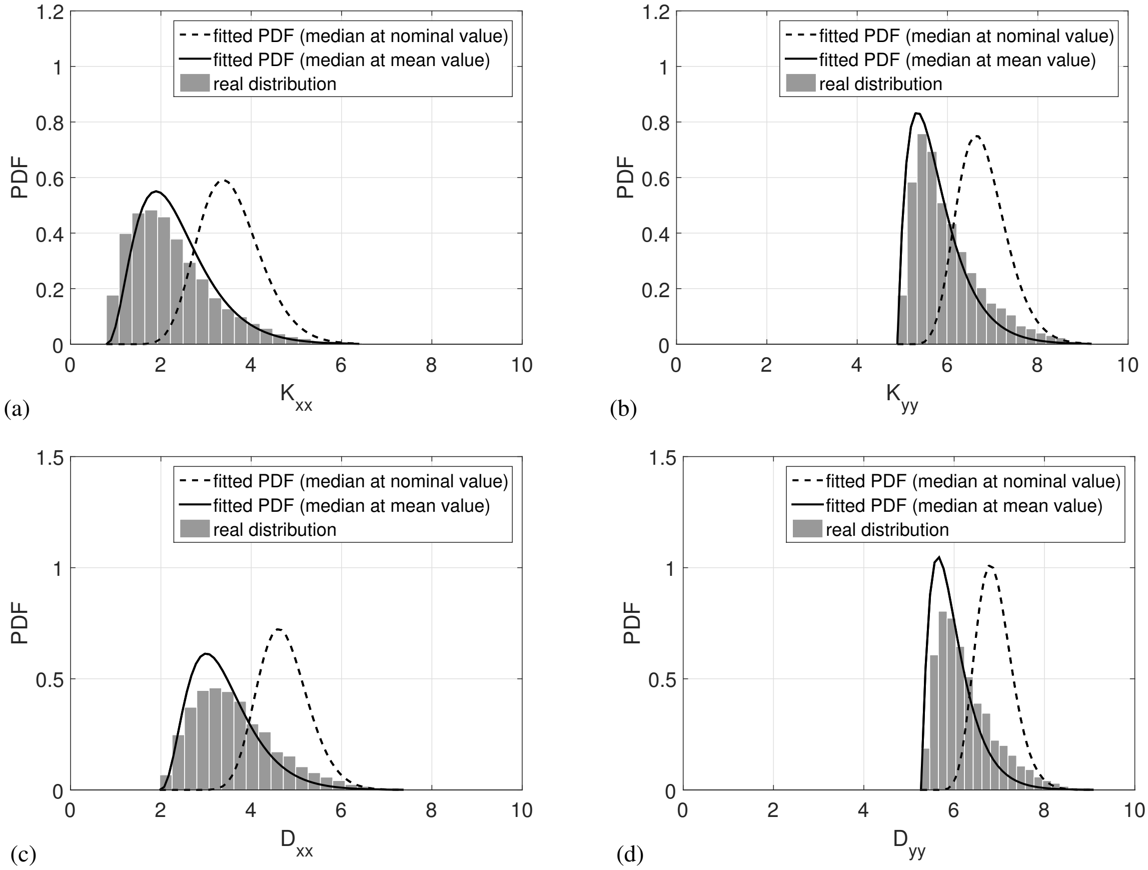

Figure 8. PDFs of the adimensional coefficients of the TPJB for the distribution of bearing clearances with mean value at 80 μm (So = 1.0). PDFs fitted with median at the nominal and at the mean values of the distribution: (a) K_{xx}, (b) K_{yy}, (c) D_{xx}, (d) D_{yy}.

The results presented in Silva and Nicoletti (2019) are still valid because the nominal clearance value of the analyzed TPJB was equal to the mean value of all possible clearances.

Hence, the distribution of dynamic coefficients can still be represented by a Gamma PDF in the case of uneven distribution of the bearing clearances around the nominal value. However, to predict this distribution by running only three simulations, we must use the mean value of all possible clearance instead of the nominal value, together with the minimum and the maximum possible clearances in the simulations of the symmetric TPJB.

4. CONCLUSION

This work contributes to the further understanding of the dynamic behavior of TPJBs subjected to dimensional uncertainty. For that, it focuses on the following two questions:

What happens if the standard deviation of the clearance distribution changes? Answer: the dynamic coefficients still scatter around the mean value of the distribution, however with different ranges of values depending on the standard deviation. The scattering of the dynamic coefficients is still limited by the values obtained from a symmetric TPJB with minimum and maximum clearances. Therefore, it is still possible to estimate the distribution of dynamic coefficients by fitting a Gamma distribution from three simulations runs of the symmetric case. The obtained Gamma PDFs are still a good representation of the coefficients' distribution.

What happens if the nominal value is not the mean value of the clearance distribution? Answer: again, the dynamic coefficient scatter around the mean value of the distribution, and this scattering is still limited by the values obtained from a symmetric TPJB with minimum and maximum clearances. It is still possible to estimate the distribution of dynamic coefficients by fitting a Gamma distribution from three simulations runs of the symmetric case. However, we must consider the cases of minimum, maximum, and mean clearance values to run the simulations. By doing so, the obtained Gamma PDFs become representative of the coefficients' distribution.

5. ACKNOWLEDGEMENTS

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