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# DYNAMICAL ANALYSIS OF BISTABLE ENERGY HARVESTER USING THE FINITE ELEMENT METHOD

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**Abstract.** *The conversion of vibrational energy from environment to electrical energy through piezoelectric devices has received increasing attention in recent years. The main challenges nowadays are to develop more efficient devices that operate over a wide frequency range, adapting to diverse sources of excitation available in the environment. In this work, a cantilever-type energy harvesting device is considered. An investigation of multimode and nonlinear effects on the system's performance is carried out. The nonlinearity included in the analysis is due to the nonlinear dependence between magnetic forces exerted by magnets and displacement of the beam's tip. The system dynamics is modeled using finite element method (FEM). Results show the influence of vibrations source in the dynamics and performance of the device for frequency range corresponding to first and second bending modes. Nonlinear effect is related to a rich dynamic, presenting periodic and non-periodic responses. All these effects can be combined in order to enhance the system capacity for energy harvesting purposes.*

**Keywords:** *Energy harvesting, finite element method, nonlinear harvester, magnetic interaction, ANSYS*

## 1. INTRODUCTION

The investigation of energy conversion from vibrations into electrical energy has become an important field of research in recent decades due to its great potential to replace/recharge batteries in wireless sensor and low-power-consuming electronic devices. Energy harvesting from ambient sources, especially from piezoelectric material, has gathered considerable focus due to its structural simplicity and high conversion capacity from mechanical vibrations. The most commonly used configurations for vibration-based energy harvesting using this material are known in the literature as piezoelectric beams. The literature presents a wide variety of works based on cantilever beam model, analyzing the generation of energy through piezoelectric linear devices (Erturk and Inman, 2008, 2009). Results show that the main limitation of linear oscillators is that they are suitable only for stationary excitation with bandwidth close to natural frequency, becoming less efficient when the ambient vibration energy is distributed over a wider frequency spectrum. This fact stimulated investigation of devices that amplify this limited bandwidth of linear harvesting systems (Zhu *et al.*, 2010).

An alternative to overcome the narrow operational bandwidth of linear devices is the consideration of multi-degrees-of-freedom systems, including harvester arrays (Lin *et al.*, 2013) and multimodal structures (Tadesse *et al.*, 2009). These devices have different response peaks and can be designed to obtain close resonant frequencies, thus increasing the operating range of the system. Another attempt to increase the performance of energy harvesting system is the exploration of nonlinearities through bistable devices. These nonlinear devices are investigated for different types of ambient excitation, such as harmonics (Erturk and Inman, 2011) and random (De Paula *et al.*, 2015) showing a considerable increase in energy generation.

Analytical and numerical solutions are available in the literature of piezoelectric energy harvesting devices based on simple structures. However, for complex systems, it is difficult to obtain models that accurately describe the behavior of these devices. In this regard, the finite element method (FEM) is an alternative for analyzing these energy harvesting systems (Zhu *et al.*, 2009; Upadrashta and Yang, 2015). Although many attempts have been made, the design of efficient energy harvesting devices capable of adapting to different ambient vibration sources remains a challenge in task.

This work presents an analysis of a vibration-based energy harvesting device consisting of a piezoelectric cantilever beam subjected to harmonic base excitation. A nonlinearity due to magnetic force, exerted by magnets in the beam's tip, is considered. The main goal is to investigate the system dynamics through a multimodal analysis over a wide frequency range. The nonlinear energy harvester device is modeled using FEM and numerical solutions are obtained using ANSYS software.

## 2. NUMERICAL MODEL

A typical configuration used to convert vibration to electrical energy is known as bimorph cantilever beam, shown schematically in Fig. 1. The system consists of a three-layer structure, two of piezoelectric material (bimorph) bonded to a substrate beam. The substrate is assumed aluminum and the piezoelectric material of PZT-5A. Electrical circuit is represented by a resistor connected, in series, to PZT's layers to generate power output. Also, ambient vibration is modeled as a base excitation. In order to make energy harvesting devices more efficient and with wider bandwidth, nonlinear effects are usually exploited. The use of magnetic forces is an interesting alternative that introduces a nonlinear dependence of magnetic force  $f_{My}$  and the beam's tip displacement  $w_L$ .

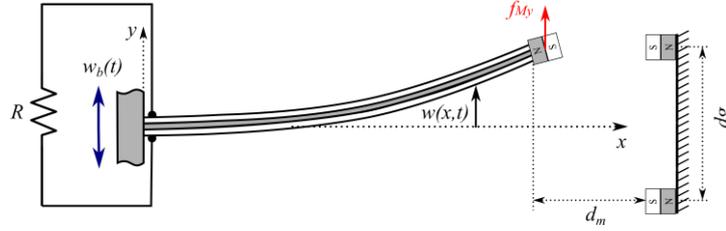


Figure 1. Schematic representation of nonlinear energy harvester.

The magnetic force is derived following similar procedure presented by (Kim and Seok 2015) and can be expressed as:

$$f_{My} = \sum_{k=1}^2 \frac{\mu_0}{4\pi} QQ' \left[ \frac{w_L - d_h}{(d_p^2 + (w_L + g_k)^2)^{3/2}} - \frac{w_L - d_h}{(d_m^2 + (w_L + g_k)^2)^{3/2}} \right] \quad (1)$$

where  $d_p = d_m + 2l_p$ ,  $g_k = (-1)^k d_h$  and  $w_L = w(L, t)$ ;  $2l_p$  is the length of each magnet and  $d_m$  is half of the distance between magnets; Also,  $\mu_0$  and  $Q$  are the vacuum permeability and total surface charge, respectively.

The energy harvesting system is modeled and analyzed using ANSYS 17. The substrate beam is modeled with 2D structural element PLANE82, which has 8 nodes and 2 DOFs per node, presenting translation motion in  $x$  and  $y$  directions. The PZT material is modeled using 2D 8 nodes element, PLANE223, having two translational DOFs and one additional DOF of voltage for each node. In addition, a resistor element (CIRC94) is connected to the PZT elements.

The equation of motion for a coupled-field analysis, in matrix form, is given by:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + \{f^i\} = \{F\} \quad (2)$$

where  $[M]$  and  $[C]$  are the mass and damping matrices, and  $\{F\}$  is the vector of applied element loads; the vector  $\{f^i\}$  is a nonlinear term related to the restitution force. Note that this term is no longer linearly proportional to the displacement vector  $\{u\}$  and a convergence procedure is conducted to calculate the stiffness matrix  $[K]$ , for each time step.

Table 1. Geometric and material proprieties of energy harvesting device.

Parameters	Substrate	Piezoceramic
Material	Aluminum	PZT-5A
Length and thickness (mm)	30×0.5	30×0.5
Mass density (kg/m <sup>3</sup> )	2700	7750
Young Modulus (GPa)	70	61
Poisson's Ratio	0.3	0.3
Piezoelectric constants (pC/N)	—	-171
Dielectric constants (nF/m)	—	15.05
Electrical resistance (Ohm)	1×10 <sup>6</sup>	
Damping ratio	$\alpha = 2.16$	$\beta = 5$

The dissipation mechanism is introduced, considering the Rayleigh method, by assuming the damping matrix proportional to the mass and stiffness matrices, defined as  $[C] = \alpha[M] + \beta[K]$ , which it is controlled by constants  $\alpha$  and  $\beta$ . Base excitation is assumed harmonic, and it is imposed by coupling the nodes located at  $x = 0$  in a component and applying displacement boundary condition. In addition, the voltage DOF on the upper and lower surfaces are coupled to provide uniform electrical potentials and thus emulate the electrodes of each piezoelectric layer. Geometric and material proprieties used in the harvester model are presented in Tab.1. The magnetic force is modeled using a nonlinear spring

element (COMBIN39) available in ANSYS as proposed by (Updrashta and Yang, 2015). The authors used COMBIN39 element to simulate the magnetic interaction in the finite element model by approximating the nonlinear force with piecewise linear segments. This force is applied at a node in the free-end-neutral-axis of the beam's tip. Parameter values used to calculate  $f_{My}$  are the following:  $d_m = 6$  mm,  $d_g = 14$  mm,  $l_m = b_m = 4$  mm,  $\mu_0 = 1.26 \times 10^{-6}$  A/m and magnetization of  $12 \times 10^{-6}$  N/A<sup>2</sup>.

### 3. RESULTS AND DISCUSSION

Since linear energy harvesters have better performance operating in a resonance condition, firstly, a harmonic analysis is performed for a frequency range from 0 to 1MHz. This can be obtained from the previously model by disregarding the magnetic force. The obtained amplitude response curve is presented in Fig. 2a. The first two bending natural frequencies are 85 and 535 Hz, and are quite away from each other. Besides, it is noticeable that the second bending mode has a wider bandwidth, but lower peak amplitude compared with the fundamental mode.

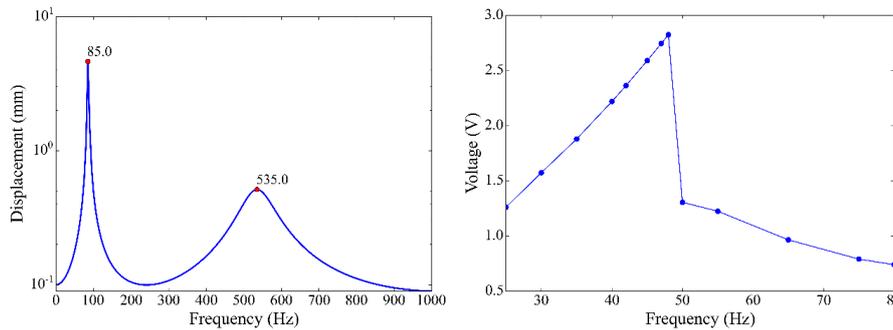


Figure 2. Amplitude response curve for linear (left) and nonlinear energy harvesters (right).

The nonlinear harvester dynamic is investigated by considering a frequency range of 25 to 100 Hz. Frequency spectrum for output voltage is built by evaluating the steady-state response peak for some points in this frequency range, as shown in Fig. 2b. The nonlinear harvester response is evaluated for a harmonic base excitation of 0.2 mm applied to the structure from rest condition. Note that magnetic force modifies the linear stiffness of the beam, reducing the frequency resonant condition. Besides, a dynamical jump is observed. As a result, the operating frequency is shifted to the left of fundamental frequency.

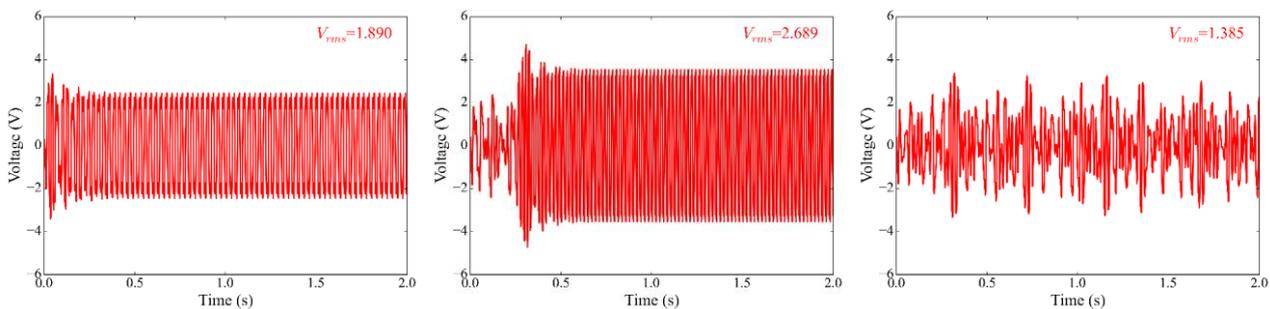


Figure 3. Time-history output voltage for base excitation of 0.2 mm at 35 Hz, 48 Hz and 50 Hz, respectively.

Time-history of the output voltage is presented in Fig. 3 considering three different operating frequencies: 35, 48 and 50 Hz. The system has periodic response with high amplitude oscillations for 35 and 48 Hz. In terms of output voltage, the system generates a rms voltage of 1.9 V for 35 Hz and 2.7 V for 48 Hz. For 50 Hz, the system presents a non-periodic response generating a rms voltage of approximately 1.4 V. Note that, despite of having similar voltage peaks, the periodic response at 35 Hz is preferable when compared with non-periodic response obtained at 50 Hz, which generated a rms voltage 26% greater.

Finally, the device performance is investigated in a region corresponding to second bending mode, around 535 Hz according to the linear analysis. Two operating frequencies are considered for nonlinear harvester: 300 Hz, which is away from resonance conditions; and 500 Hz which is expected to be close to resonant conditions. The resulting output voltage responses are shown in Fig. 4. In the first condition, a rms voltages of 1.5 V is generated by the nonlinear energy harvester, which is 80% less compared with the same results at 48 Hz. On the other hand, the second condition produces a rms voltage of 8.4 V, which is about 212% greater than the values of 48 Hz.

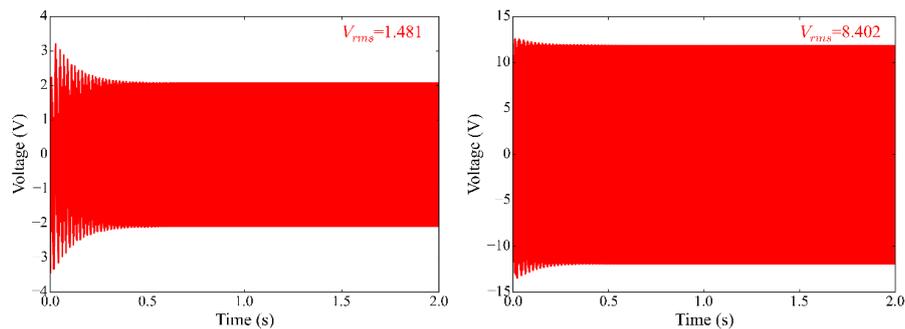


Figure 4: Time-history output voltage for base amplitude of 0.2 mm at 300 Hz and 500 Hz, respectively.

#### 4. CONCLUSIONS

This work deals with an analysis of a vibration-based piezoelectric energy harvesting device excited by harmonic base excitation. Nonlinear effects are due to magnetic interaction for broadening the harvested energy, being modeled using FEM. The influence of multimode and nonlinear effects on system performance is of special interest. Numerical simulations are carried out for different frequency range corresponding to first and second resonant conditions. FEM simulations are capable of capturing the rich dynamics characteristic of bistable systems presenting periodic and non-periodic responses, being related to low and high energy oscillation amplitudes. Although the first resonant condition is predominant for the system performance, results show that, the second resonant condition is also capable of generating enough energy to be used in low-power-consuming electronic devices. FEM shows to be an interesting tool to design complex, multimodal and nonlinear energy harvesting devices in order to enhance the system performance.

#### 5. ACKNOWLEDGEMENTS

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