



25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COB-2019-0648

SYNCHRONIZATION AND CHIMERA STATE IN A MECHANICAL PENDULUM-CHART SYSTEM

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Abstract. *Synchronization phenomenon appears in many natural systems including physical, chemical and biological processes. In brief, synchronization may be understood as a harmonization of two or more system behaviors following some common pattern. Synchronized patterns depend of the dynamical characteristics of the system and, among these possibilities, there is the chimera state that consists in a symmetry break in the behavior of identical oscillator network. This paper deals with synchronization analysis of a mechanical pendulum-chart system composed by an hierarchical network of three pendula coupled to each other through their own chart that receive continuous supply of energy via a base excitation. Equations of motions are established and numerical simulations carried out considering the Runge-Kutta fourth order method. In this work Some patterns are classified and investigated in order to understand the necessary conditions to each pattern and to describe the result of a hyperchaotic chimera state. Results indicate that there are not any direct relation between the periodicity and non-periodicity with the patterns formed.*

Keywords: *Synchronization, chimera state, mechanical system, mechanical oscillator*

1. INTRODUCTION

Synchronization is a combination of two or more system behaviors following some common pattern. This phenomenon was first discussed by Huygens (1666), which treated the synchrony of two pendulum clocks. This pioneer work observed the synchronization of the clocks in two different configurations: in-phase synchronization and anti-phase synchronization. Winfree (1967) formulated a model for a neural system through a large population of limit-cycle coupled oscillators. In this model, it was recreated the same frequency distribution of α -rhythm on a brain spectrograph, representing the most important brain frequency responsible for data transmission. A long period of synchronization pattern was discussed.

Kuramoto and Battogtokh, (2002) discovered an unexpected result on phase model: a partial synchronization in a symmetric ring of identical oscillators that represents a symmetry break. This partial synchronization behavior was named as chimera state sometime later, in reference of a mythological Greek creature. During the following decade, this behavior was observed in some numerical simulations. Tinsley et al. (2012) showed the first experimental evidence of chimera state in chemical oscillators based in a photosensitive reaction. Afterward, Martens et al. (2013) showed chimera state on mechanical oscillators using two charts coupled within fifteen metronomes above each chart. In this work, it was speculated that chimera state is the result of a competition between an in-phase synchronization and anti-phase synchronization since it was found in a border region that separates the behavior of each one of these patterns. Kapitaniak et al. (2015) presented chimera state in a ring of coupled metronomes, discussing a new variation of this state: an imperfect chimera state. Finally, Wojewoda et al. (2016) analyzed a ring three metronomes that represented the smallest system that could present a chimera state.

The synchronization can be observed in several situations including engineering applications. In this regard, it should be pointed out the increase the intensity of laser beams (Pikovky et al., 2003), civil structures as the millennium bridge, which oscillates sideways by the interaction among the pedestrians and the bridge (Strogatz et al., 2005). Power grid networks is another example where the design this to generate systems less susceptible to external perturbations (Dörfler et al., 2013).

This paper analyzes a mechanical system composed of pendulum-chart systems with base excitation. A hierarchical network is employed to couple the systems such as the one adopted in Martens et al. (2013). Different synchronization patterns are investigated, including chimera state.

2. MECHANICAL SYSTEM

Consider a mechanical system composed by pendulum-chart subsystems. Basically, the system has three charts, each one with a respective pendulum. These charts are connected to each other, to a reference wall and to a base, in a symmetric form as presented in Fig. 1. The base provides a harmonic excitation.

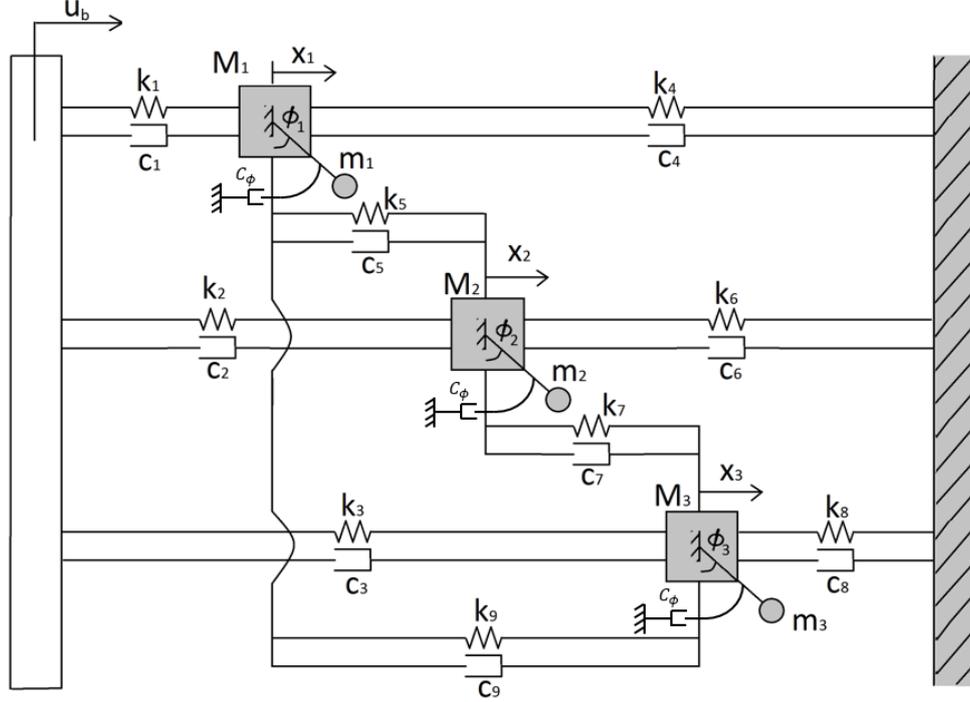


Figure 1. Representation of the mechanical model.

Mathematical model of this system is developed using Lagrange approach and the final equation is rescaled with respect to time $\tau = at$, where $\alpha = \sqrt{g/l}$. Dimensionless displacement is defined as $X_i = x_i/l$ ($i = 1, 2, 3$). Besides, dimensionless parameters are defined: $\beta_j = uk_j/\alpha ml$, $\eta_j = uC_j\omega/\alpha ml$, $\mu = M/m$, $\xi = C_\phi/\alpha ml^2$, $\omega = \Omega/\alpha$, $\rho_j = C_j/\alpha m$ and $\gamma_j = k_j/\alpha^2 m$, where $j=1, 2, \dots, 9$.

$$(\mu + 1 - \cos^2 \phi_1)\ddot{X}_1 - \xi \dot{\phi}_1 \cos \phi_1 - \sin \phi_1 \cos \phi_1 - \dot{\phi}_1^2 \sin \phi_1 + (\rho_1 + \rho_4 + \rho_5 + \rho_9)\dot{X}_1 - \rho_5 \dot{X}_2 - \rho_9 \dot{X}_3 + (\gamma_1 + \gamma_4 + \gamma_5 + \gamma_9)X_1 - \gamma_5 X_2 - \gamma_9 X_3 = \beta_1 \sin(\omega\tau) + \eta_1 \cos(\omega\tau)$$

$$(\mu + 1 - \cos^2 \phi_2)\ddot{X}_2 - \xi \dot{\phi}_2 \cos \phi_2 - \sin \phi_2 \cos \phi_2 - \dot{\phi}_2^2 \sin \phi_2 + (\rho_2 + \rho_5 + \rho_6 + \rho_7)\dot{X}_2 - \rho_5 \dot{X}_1 - \rho_7 \dot{X}_3 + (\gamma_2 + \gamma_5 + \gamma_6 + \gamma_7)X_2 - \gamma_5 X_1 - \gamma_7 X_3 = \beta_2 \sin(\omega\tau) + \eta_2 \cos(\omega\tau)$$

$$(\mu + 1 - \cos^2 \phi_3)\ddot{X}_3 - \xi \dot{\phi}_3 \cos \phi_3 - \sin \phi_3 \cos \phi_3 - \dot{\phi}_3^2 \sin \phi_3 + (\rho_3 + \rho_7 + \rho_8 + \rho_9)\dot{X}_3 - \rho_7 \dot{X}_2 - \rho_9 \dot{X}_1 + (\gamma_3 + \gamma_7 + \gamma_8 + \gamma_9)X_3 - \gamma_7 X_2 - \gamma_9 X_1 = \beta_3 \sin(\omega\tau) + \eta_3 \cos(\omega\tau)$$

$$\ddot{\phi}_1 + \xi \dot{\phi}_1 + \sin \phi_1 + \frac{\cos \phi_1}{\mu + 1 - \cos^2 \phi_1} [\xi \dot{\phi}_1 \cos \phi_1 + \sin \phi_1 \cos \phi_1 + \dot{\phi}_1^2 \sin \phi_1 - (\rho_1 + \rho_4 + \rho_5 + \rho_9)\dot{X}_1 + \rho_5 \dot{X}_2 + \rho_9 \dot{X}_3 - (\gamma_1 + \gamma_4 + \gamma_5 + \gamma_9)X_1 + \gamma_5 X_2 + \gamma_9 X_3 + \beta_1 \sin(\omega\tau) + \eta_1 \cos(\omega\tau)] = 0 \quad (1)$$

$$\ddot{\phi}_2 + \xi \dot{\phi}_2 + \sin \phi_2 + \frac{\cos \phi_2}{\mu + 1 - \cos^2 \phi_2} [\xi \dot{\phi}_2 \cos \phi_2 + \sin \phi_2 \cos \phi_2 + \dot{\phi}_2^2 \sin \phi_2 - (\rho_2 + \rho_5 + \rho_6 + \rho_7)\dot{X}_2 + \rho_5 \dot{X}_1 + \rho_7 \dot{X}_3 - (\gamma_2 + \gamma_5 + \gamma_6 + \gamma_7)X_2 + \gamma_5 X_1 + \gamma_7 X_3 + \beta_2 \sin(\omega\tau) + \eta_2 \cos(\omega\tau)] = 0$$

$$\ddot{\phi}_3 + \xi \dot{\phi}_3 + \sin \phi_3 + \frac{\cos \phi_3}{\mu + 1 - \cos^2 \phi_3} [\xi \dot{\phi}_3 \cos \phi_3 + \sin \phi_3 \cos \phi_3 + \dot{\phi}_3^2 \sin \phi_3 - (\rho_3 + \rho_7 + \rho_8 + \rho_9)\dot{X}_3 + \rho_7 \dot{X}_2 + \rho_9 \dot{X}_1 - (\gamma_3 + \gamma_7 + \gamma_8 + \gamma_9)X_3 + \gamma_7 X_2 + \gamma_9 X_1 + \beta_3 \sin(\omega\tau) + \eta_3 \cos(\omega\tau)] = 0$$

3. NUMERICAL SIMULATIONS

Numerical simulations are carried out considering the Runge-Kutta fourth-order method using the following parameters: $k_{4,6,8} = 0$ N/m, $k_{5,7,9} = 50$ N/m, $g = 9.81$ m/s², $l = 0.2$ m, $M = 1$ Kg, $m = 1$ Kg, $C_{1,2,3,5,7,9} = 0.56$ Ns/m, $C_{\phi_{1,2,3}} = 0.01$ Ns/m, $C_{4,6,8} = 0$ Ns/m. The parameters $k_{1,2,3}$, u , Ω and the initial condition of the charts are defined to each simulation. Initial conditions are null velocity and displacement for all pendula.

Figure 2 shows a synchronized chaotic response. That is, each pendulum has exactly the same behavior of the others even in a chaotic motion. Poincaré section is plotted together with state space showing the chaotic behavior that is verified by the calculation of Lyapunov exponents that presents one value greater than zero using the algorithm due to Wolf et al. (1985) presenting the value of 0.66.

Analyzing the effect of the initial conditions, it is observed that, for these specific parameters, just null initial condition are related to a synchronized state. The parametric analysis is carried out showing that the complete synchronized state is the major pattern behavior of this system. That is, a large domain on parameter state shows a synchronized state, but in the domain of Fig. 2, just null initial conditions present this state and, other initial conditions promotes a desynchronized behavior while still moves chaotic.

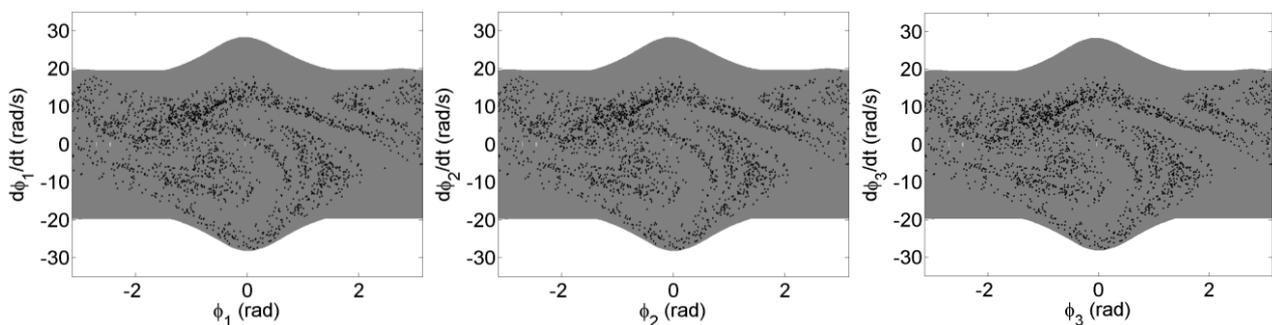


Figure 2. Synchronized and chaotic state. The initial conditions used was of zero displacement for all charts and $k_{1,2,3} = 19$ N/m, $u = 1$ m and $\Omega = 10$ rad/s.

Figure 3 shows a simulation with different system parameters. This response shows a frequency locked synchronization (Carlsen et al., 2012). This kind of pattern represents the characteristic of the oscillators for remaining in the same period. It means that, after the conclusion of one period, all the oscillators are in the same place on the state space, any of the oscillators can be used as reference for comparison. Note a period-2 oscillation through the Poincaré section plotted over the state space. It also should be pointed out that, pendulum 2 and 3 oscillate identically and the pendulum 1 is in a frequency locked with the others.

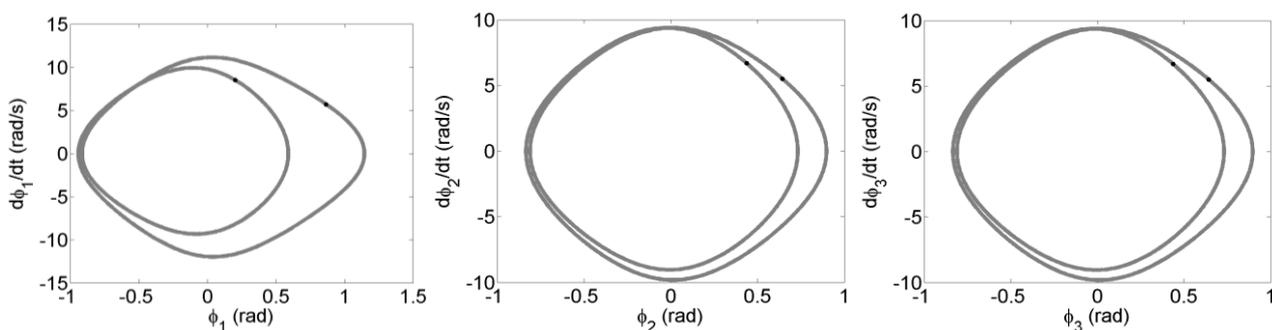


Figure 3. Frequency locked for initial condition of displacement of 1 m for chart 1 and zero to others, $u = 0.5$ m, $k_{1,2,3} = 13.5$ N/m and $\Omega = 9.9$ rad/s.

Figure 4 presents a hyperchaotic response associated with a chimera state, a behavior with partial synchronization where pendulum 1 and 3 behave identically until the second pendulum moves in a different way. This kind of partial synchronization is a break of symmetry in a system of identical oscillators symmetrically coupled. Hyperchaos is assured by the existence of two Lyapunov exponents greater than zero, presenting the following positive values: 0.89 and 0.40. Simulations show that chimera state appears for some initial conditions. Based on that, a desynchronized state and in-phase synchronized state is showed maintaining the parameters and changing the initial conditions. Any clear relation is observed in order to understand a specific kind of condition that induces chimera state. It should be pointed out that partial synchrony behavior is not always chimera state. This mechanical system has identical springs coupling

the charts in a symmetric manner and the three pendulum-charts are identical too. Then, the pendulum-chart model within these characteristics has the necessary condition to call this partial synchrony as chimera state.

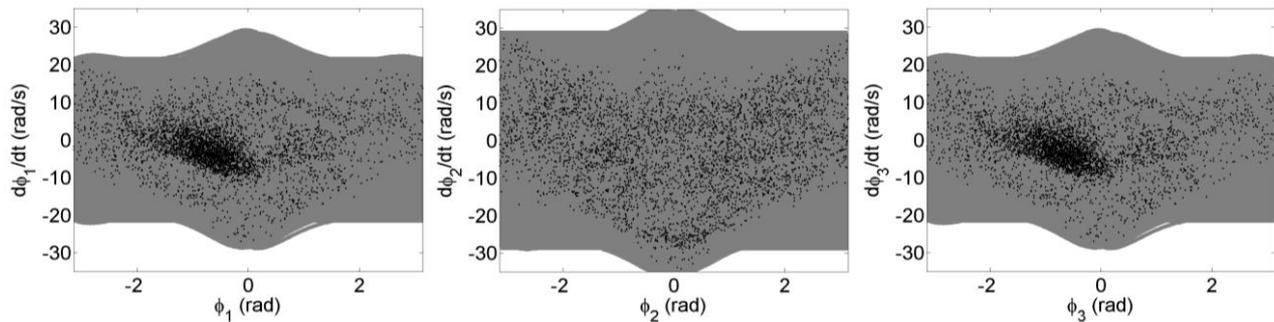


Figure 4. Hyperchaotic chimera state for initial condition null for chart 1 and 1 m of displacement to the others, $\Omega = 9$ rad/s, $u = 1$ m and $k_{1,2,3} = 20$ N/m.

4. CONCLUSIONS

A mechanical system is analyzed in synchronization perspective. Synchronized anti-phase response, long period synchronization and the desynchronized responses are observed. Synchronized, frequency locked and chimera state are presented. All these kind of patterns are identified, emphasizing the chimera state in a hyperchaotic motion. In general, it is possible to say that there is no relation between the periodicity of the system and the pattern of interaction of the oscillators. Besides, nonregular responses as chaos and hyperchaos do not affect the collective behavior of the oscillators. About the initial conditions, it is observed that null conditions are necessary to remain in a synchronized behavior in unstable domains. However, outside this region, the main system behavior is the synchronized pattern. The system has a strong dependency of the initial conditions in such a way that, not even hyperchaotic motion suppresses the chimera state.

5. ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the Brazilian Research Agencies CNPq, CAPES and FAPERJ. The Air Force Office of Scientific Research (AFOSR) is also acknowledged.

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