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A NEW TECHNIQUE TO SIMULATE THE PROCESS OF CONTACT FORMATION BETWEEN DEFORMABLE SOLIDS

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Abstract. *A new scheme to simulate the process of 2D contact formation between deformable solids is proposed. The strategy is developed based on the use of coupling finite elements (CFEs) recently developed by Bitencourt Jr. et al. (2015), and applied in this work in the context of small deformations. The technique is tested through a Hertzian contact model between cylinder and plate, and the results obtained show that the proposed approach is able to represent the interaction between solids.*

Keywords:*contact mechanics, finite element method, coupling finite element, formation of contact*

1. INTRODUCTION

Contact mechanics has deserved much interest in many fields of computational modeling in engineering and sciences.

The necessity to represent contact problems through finite element models arises as the geometries, constraints and loading conditions become more complex. Materials with nonlinear behavior also points to use finite elements models to predict accurate results in contact problems.

The first studies involving contact were conducted by Hertz (1881), who presented the analytical solution for frictionless contact between two elliptic elastic bodies.

At present, the techniques used to represent contact between bodies are widely used, for example, in vehicles and tires developments, and deep drawing process, respectively according to Klyavin *et al*. (2008), Moisescu and Frătilă (2011) and Othmen *et al*. (2017), and especially in tribological studies, as can be seen in Amor *et al*. (2016).

2. LITERATURE REVIEW

A broad overview about the state-of-the-art on the mathematical basics as well as related computational techniques of contact mechanics can be found in Wriggers (2006). According to Hartmann *et al*. (2010), the development of a computational contact strategy basically requires two ingredients:

- a suitable method for the contact constraint enforcement, and
- a technique to discretize the contact surfaces.

There are several techniques for the variational enforcement of the contact constraints, such as Penalty Method (Arnold, 1982) and Lagrangian Multiplier (Vulovic *et al*., 2007). These techniques are important because they are basis for other techniques, such as augmented Lagrange method (Simo and Laursen, 1992) and perturbed Lagrange method (Simo *et al*., 1985). There are other methods to enforce the contact constrains and an overview can be found in Wriggers (2006).

According to Weyler *et al*. (2012), the Lagrange Multipliers method introduces additional variables to the problem, the so-called "Lagrange multipliers", to ensure the exact fulfillment of the contact constraints, leading to accurate results. However an extra computational effort to determine these multipliers is required. On the other hand, the Penalty method avoids the need for additional variables by introducing an approximation of the constraint conditions, adding a term in the weak form of the equations that govern the problem, which penalizes the non-enforcement of the contact constraints, through a penalty parameter. Theoretically, when the penalty parameter tends to infinity, the contact constraints are met exactly and the result is equal to that obtained by the Lagrange Multipliers method. However, the resulting system of equations can become poorly conditioned, causing instabilities in the analysis. Therefore the choice of an appropriate penalty parameter becomes a compromise between the accuracy and stability of the analysis.

In addition, it is also necessary to apply techniques to discretize the contact domain.

The node-to-node approaches are usually applied in small deformation problems with matching meshes due its simplicity. According to Xu and Hjelmstad (2008), it is formulation can be also adapted to solve more complex problems, combined with an adaptive mesh technique.

Many of the contact algorithms developed in the past are based on a node-to-segment approach, developed by Hallquist *et al*. (1985). This approach is widely applied in problems involving large deformations. Since the non-smooth discretization of the contact, surfaces may leads to unreal oscillations of the contact forces in finite sliding problems, smoothing algorithms are necessaries.

In order to mitigate these issues of continuity, several smoothing techniques for contact domain have been proposed, including Hermit interpolation (Pietrzak and Curnier, 1999), Bezier interpolation (Lengiewicz *et al*., 2010) and B-spline interpolation (Padmanabhan and Laursen, 2001).

Other discretization strategy used is the segment-to-segment, based on a continuous treatment of the contact constraints.This strategy was first presented by Simo and Wriggers (1985) and extended for a large deformation case by Papadopoulos and Taylor (1992). This concept frequently is applied with mortar strategy, for coupling non-matching meshes as can be seen in Kallel *et al*. (2013).

Oliver *et al*. (2009) formulated a new approach to the contact constraints, called contact domain, which can be interpreted as a fictive intermediate region connecting the potential contact surfaces of the deformable bodies. The introduced contact domain is subdivided into a non-overlapping set of segments and is endowed with a displacement field, interpolated from the displacements at the contact surfaces.

3. OBJECTIVE

This work aims to extend the new technique to couple domains based on the use of "*coupling finite elements*" developed by Bitencourt *et al*. (2015), in order to represent the process of contact formation between deformable solids. The application context is frictionless 2D problems with non-matching meshes.

The new algorithm is formulated using a node-to-segment discretization contact domain concept and a Penalty method, enforcing displacement compatibility through a rigid coupling scheme between the non-matching meshes.

4. MECHANICAL MODEL

Figure 1 represents two body Ω^1 and Ω^2 before and during the process of contact formation. Each boundary body Γ is compound by Γ_u, Γ_f and Γ_c, that represents displacements, forces and contact boundary, respectively, such that:

$$\Gamma = \Gamma_u + \Gamma_f + \Gamma_c$$

This problem can be represented by the well-known weak form of the governing equations, adding the contact constrains term (last term) to frictionless problems, which leads to:

$$\int_{\Omega} \delta\boldsymbol{\varepsilon}^{\boldsymbol{T}}\boldsymbol{\sigma} d\Omega = \int_{\Gamma_f} \delta\boldsymbol{u}^T\overline{\boldsymbol{f}} d\Gamma + \int_{\Omega} \delta\boldsymbol{u}^T\boldsymbol{b} d\Omega + \int_{\Gamma_c} (\epsilon_N\overline{g_N}\delta\overline{g_N}) dA \qquad \forall\delta\boldsymbol{u} \in U_0;\ \ \epsilon_N > 0 \tag{1}$$

where $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ are the strain and stress fields, respectively, $\overline{\boldsymbol{f}}$ and $\boldsymbol{b}$ are the external and body loads, $\boldsymbol{u}$ is the displacement field, $\overline{g_N}$ is the normal gap function and ϵ_N the normal penalty parameter.

The nonlinear character is given by the undefined contact contour, a priori. Its determination implies an additional computational effort and demands specific algorithms.

The last term of the Eq.(1) arises to reinforce the contact non-penetration constraints, known as Kuhn-Tucker-Karush condition in optimization problems:

$$g_N \geq 0; \qquad t_N \leq 0; \qquad t_N . g_N = 0 \tag{2}$$

where t_N is the contact traction vector.

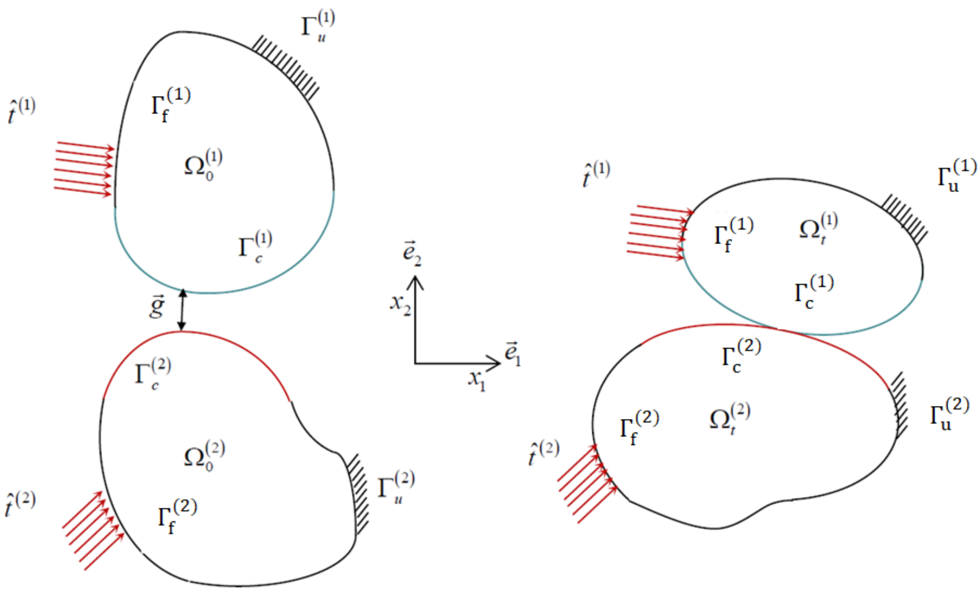

Figure 1. 2D contact between two deformable solids

5. COUPLING FINITE ELEMENT

The CFE, developed by Bitencourt *et al.* (2015) is a finite element composed by a standard isoparametric finite element and an additional node, called "coupling node" ($\mathrm{C_{Node}}$), situated at the material point $\mathbf{X_C}$. The coupling node can be anywhere in the element, but to represent contact problems, $\mathrm{C_{Node}}$ must be on its boundary. Figure 2 shows the coupling process for coupling two non-matching meshes.

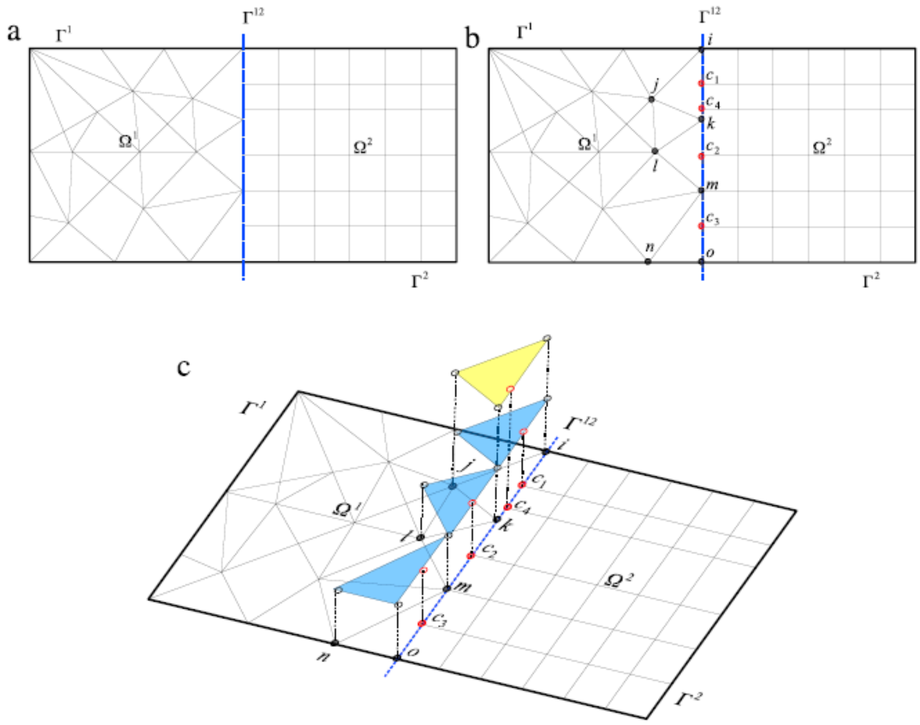

Figure 2. Coupling procedure for two non-matching finite element meshes: (a) 2D problem with two non-matching meshes; (b) process of identification of the nodes that will compose the CFEs, and (c) creation and insertion of the CFEs (adapted from Bitencourt *et al.*, 2015).

For problems composed by two adjacent bodies initially in contact, the original formulation states:

$$[\![\mathbf{U}]\!] = \mathbf{B}_e \mathbf{D}_e \tag{3}$$

where $[\![\mathbf{U}]\!]$ is the relative displacement, defined as the difference between the displacement of the $\mathrm{C_{Node}}$ and the displacement of the material point $\mathbf{X_C}$, $\mathbf{D_e}$ is the vector which stores the displacement components of the coupling finite element, and $\mathbf{B_e}$ is given by:

$$\mathbf{B}_e = [-\overline{\mathbf{N}}_1(X_c) \ -\overline{\mathbf{N}}_2(X_c) \dots -\overline{\mathbf{N}}_{nn}(X_c)I] \tag{4}$$

and $\overline{\mathbf{N}}_i = \overline{N}_i\mathbf{I}$, while **I** is the identity matrix of order 2, for 2D problems, and *nn* is the number of nodes of the isoparametric element.

Therefore, the internal force and tangent stiffness matrix can be written as:

$$\mathbf{F_{int}} = \mathbf{B_e^T C B_e\ D_e} \tag{5}$$
$$\mathbf{K}_e = \mathbf{B}_e^\mathbf{T}\mathbf{C}\mathbf{B}_e \tag{6}$$

where **C** is the matrix of contact penalty factors in global coordinates. For 2D frictionless problems:

$$\mathbf{C} = \mathbf{R}^t\mathbf{c}\mathbf{R} \tag{7}$$

in which, **R** is the rotation matrix, to convert from local to global coordinates, and ***c*** is a matrix that stores the penalty factors, in local coordinates. Note that ***c*** has only a normal penalty factor $\tilde{c}$.

The present work improves the original coupling finite element making it able to calculate the contact forces introducing the initial gap to the relative calculation, leading to:

$$[\![\mathbf{U}]\!] = \mathbf{B}_e\mathbf{D}_e + [\![\mathbf{U}]\!]_\mathbf{0} \tag{8}$$

where $[\![\mathbf{U}]\!]_\mathbf{0}$ is the initial gap, between bodies in the initial condition. To node-to-segment contact discretization, $[\![\mathbf{U}]\!]_\mathbf{0}$ stands to distance between master element edge (segment) and slave node. So, internal force and tangent stiffness matrix can be expressed by:

$$\mathbf{F}_{int} = \mathbf{B}_e^\mathbf{T}\mathbf{C}\mathbf{B}_e\ \mathbf{D}_e + \mathbf{B}_e^\mathbf{T}\mathbf{C}[\![\mathbf{U}]\!]_\mathbf{0} \tag{9}$$
$$\mathbf{K}_e = \mathbf{B}_e^\mathbf{T}\mathbf{C}\mathbf{B}_e \tag{10}$$

In other point of view, with this $\mathbf{F}_{int}$ formulation one can make the balance between total energy, to move from initial to final condition, and the energy to move from initial condition to contact. This balance will result only in the energy to deform elastically due to contact forces.

The proposed algorithm is able to check if the $\mathbf{F}_{int}$ is compressive or tensile.

If compressive force is verified $\tilde{c}$ stands to high value ($\tilde{c} \gg 0$), else $\tilde{c}$ stands to low value ($\tilde{c} \to 0$).

6. SIMULATION AND RESULTS

In order to validate the proposed method, the Hertzian contact between a 0,010 m radius cylinder and a plate, with 0,0001 m minimum gap was simulated. Both materials from cylinder and plate are steel ($\mathrm{E} = 210\mathrm{GPa}$ and $\nu = 0{,}3$), and the analysis is performed under plane stress state. The cylinder was moved downwards 0,0004 m in pre-determined constant steps. The penalty factor adopted was $\tilde{c} = 10^{10}$.

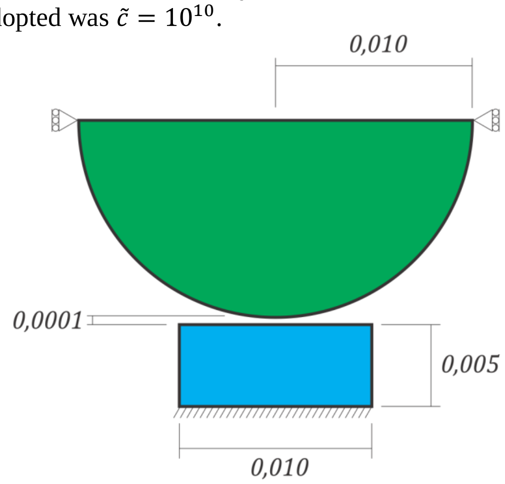

Figure 3. Configuration of contact model (dimensions in m).

Figure 3 shows the model configuration. As previously mentioned, it is a frictionless model.

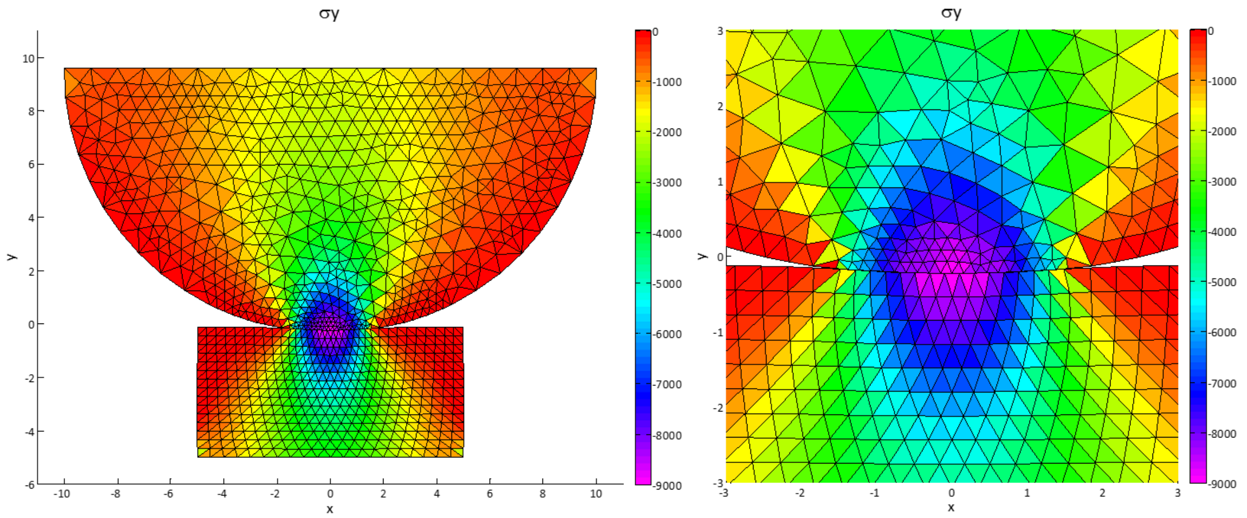

Figure 4. Result of the vertical stress σ_y (N/mm²). On the right, detail of the contact region.

Figure 4 shows the finite element mesh with vertical stress distribution σ_y, expressed in N/mm². Note that the discretization resulted in non-matching meshes at the contact edges, since this condition should be evaluated.

It is possible to see a good stress distribution compatible with the traditional Hertzian stress distribution.

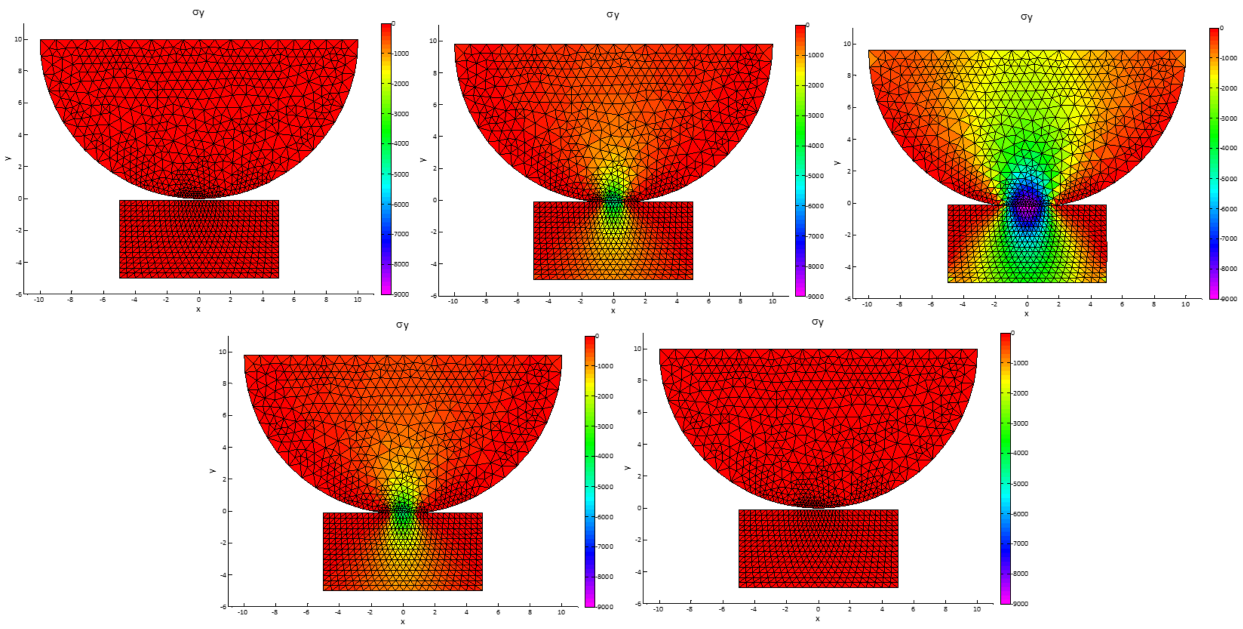

Figure 5. Result of the vertical stress σ_y for different load steps: 1; 5; 10; 15 and 20.

Figure 5 shows the results for different load steps: 1; 5; 10; 15 and 20 reaching the maximum value in step 10, and thereafter gradually attenuated until it returns to the original point in step 20.

It is important to note that the stress distribution remains consistent with the Hertzian stress in the intermediate steps during the contact formation. The same is observed during the steps of returning to the starting point. Therefore, the methodology has the capacity both to form the contact and to "lose" the contact, indicating that it is possible to employ this new method in problems with applications of more complex loads.

Figure 6 shows the analytical and numerical results in terms of pressure, to both master and slave domains.

The x-axis values represent the horizontal distance of nodes belonging to the contact edges, in relation to the cylinder center, while the y-axis values are the contact pressures on the respective nodes.

It can be observed that the results are in good agreement, for both domains, which demonstrates the capacity of the proposed method in providing a good representation of contact mechanism.

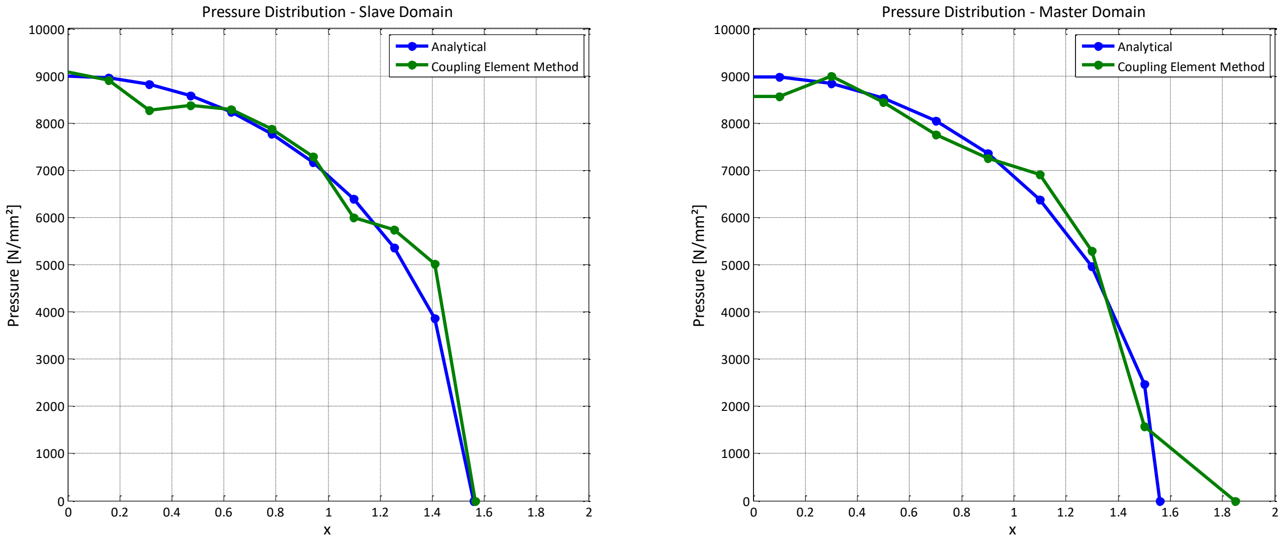

Figure 6. Comparison between contact pressures of the master and slave domain and their respective analytical results.

7. CONCLUSION

This paper proposes a new technique to represent problems with contact formation, in the context of small deformations without friction, using non-matching meshes, based on the use of coupling finite elements.

The fact that the stress profile resulting from the contact is compatible with the Hertzian contact stress and the agreement between the analytical and numerical results confirms the ability of the proposed method to represent contact formation problems.

In future works, the authors intend to extend the formulation to cover problems involving friction and 3D simulations.

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