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DIMENSIONLESS MODELS OF INTEGRAL SAUTER MEAN DIAMETER OF THE SPRAY OBTAINED FROM EFFERVESCENT ATOMIZERS

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Abstract. *The goal of this study was to develop dimensionless models to predict of the integral Sauter mean diameter (ID_{32}) associated with the atomization of light heating oil (LHO) in effervescent atomizers of outside-in gas injection configuration. The development of a new general dimensionless model was necessary to obtain an optimized dimensionless model. Five dimensionless models ($M1 - M5$) were established for the estimation of ID_{32} . For the establishment of each model ($M1 - M5$), 85 experimental results, obtained from 17 effervescent atomizers, were used. Finally, for the estimations of ID_{32} , the model $M3$ was selected, being its root mean square error (RMSE) equal to 1.973; thus considering excellent dimensionless model.*

Keywords: *Atomization, spray, light heating oil (LHO), dimensional analysis, model.*

1. INTRODUCTION

The effervescent atomization had its origins in a study made by Lefebvre and Wang (Lefebvre et al., 1988). Later on, significantly increased the interest in effervescent atomization in the coming years, which led to the development of studies that were crucial (Wang et al., 1989, Roesler and Lefebvre, 1989, Hanna and Zoughaib, 2017) to deepen the scientific knowledge about basic concepts related to the topic; such as types of atomizers according to gas injection (Otahal et al., 2007, Broniarz-Press et al., 2010, Jedelsky and Jicha, 2013), range of ideal operational conditions (Jedelsky et al., 2008), stability of sprays (Jedelsky and Jicha, 2006), geometric dimensioning of atomizers (Jedelsky et al., 2007, Jedelsky et al., 2009), besides other aspects (Sovani et al., 2001). Fundamentally, its main advantages are: use of small injection pressure of the liquid (Lefebvre et al., 1988, Jedelsky and Jicha, 2013), use of small gas-liquid ratio by mass (GLR) (Lefebvre et al., 1988), obtaining small Sauter mean diameter (SMD) for the droplets (Lefebvre et al., 1988, Jicha et al., 2002), and stable atomization for greater GLR values (Jedelsky and Jicha, 2006).

On the other hand, the dimensional analysis is a method that consists in relating a dependent variable with a set of independent variables, doing a nondimensionalization of the relation between them (Shames, 2002). It is a very useful method that is employed in fluid mechanics, thermodynamics, heat transfer, electricity, optics, chemistry, biology and various other research fields (Simon et al., 2017). The most outstanding advantage of this method is the study of less variables, which is possible by setting aside the direct study of the initial variables in order to work with dimensionless groups (the new variables). There is a great amount of specialized literature on dimensional analysis (Vaschy, 1892, Buckingham, 1914, Bridgman, 1922, Curtis et al., 1982, Gibbings, 2011, Sonin, 2011, Martins, 1981), however, there is no specific analytical method able to determine in a rigorous, simple and efficient way the exponent associated to each independent dimensionless group from the dimensionless model. In addition, there is also not a manner to establish dimensionless models that are not expressed by monomials.

The goal of this study was to establish dimensionless models to estimate the integral Sauter mean diameter obtained from the atomization of LHO in effervescent atomizers. In order to do so, it was necessary to develop a new general dimensionless model to be able to ascertain precisely the coefficients of the natural logarithms of the independent dimensionless groups.

2. MATERIAL AND METHOD

2.1 Independent variables in effervescent atomization

In this section, the independent variables that influence the spray obtained by the effervescent atomization with outside-in gas injection were identified, so then it was possible to determine the dimensional relation for the characteristic diameter of the spray droplets (ID_{32}). From the literature (Lefebvre et al., 1988, Roesler and Lefebvre, 1989, Jicha et al., 2002, Jedelsky et al., 2007, Sovani et al., 2001) about this type of atomizers, it was noted that the characteristic diameter of the spray droplets depend on three types of independent variables: geometric variables (number of aerator holes in the inner chamber n , diameter of each aeration hole d_a , length between first and last row of aerator holes Δl_m , diameter of internal mixing chamber d_c , length between the last row of aerator holes and the atomizer outlet port l_c , length of the atomizer outlet l_o , diameter of the atomizer outlet d_o), fluid dynamics variables (viscosity of the gas μ_g , viscosity of the liquid μ_l , surface tension of the liquid σ_l), and operational variables (mass flow rate of gas m_g , mass flow rate of the liquid m_l , and manometric pressure of the liquid P_l). In Eq. (1) it was established the dimensional relation of the characteristic parameters ID_{32} in function of the previously mentioned variables.

$$ID_{32} = f\left(n, d_a, \Delta l, d_c, l_c, l_o, d_o, \mu_g, \mu_l, \sigma_l, \dot{m}_g, \dot{m}_l, P_l\right) \quad (1)$$

being $\Delta l = \Delta l_m + c_{\Delta l}$, for some $c_{\Delta l} \in \mathbb{R}^+$. The term $c_{\Delta l}$ allows the independent dimensionless group (to be established latter) that has the variable Δl_m to be different from 0, even in case where is only one row of aerator holes. The importance of introducing the term $c_{\Delta l}$ becomes more evident once the dimensionless models have been established.

The theorem of Vaschy-Buckingham (Vaschy, 1892, Buckingham, 1914) was used to convert the dimensional relation presented in equation (1) into a dimensionless relation, so it would be possible to perform a dimensional analysis to determine the independent dimensionless groups. For further information on the theorem of Vaschy-Buckingham or the dimensional analysis, the corresponding literature (Simon, 2017, Bridgman, 1922, Curtis et al., 1982, Gibbings, 2011, Sonin, 2001, Martins, 1981) was consulted. In Eq. (2) the dimensionless functional relation of ID_{32}/d_o is presented.

$$\frac{ID_{32}}{d_o} = f\left(n, \frac{d_a}{d_o}, \frac{\Delta l}{d_o}, \frac{d_c}{d_o}, \frac{l_c}{d_o}, \frac{l_o}{d_o}, \frac{m_g}{\mu_g d_o}, \frac{m_l}{\mu_l d_o}, GLR, \frac{P_l d_o}{\sigma_l}\right) \quad (2)$$

2.2 General dimensionless model

From the literature review, it was observed that the current dimensional analysis does not enable the calculation of the exponent corresponding to each independent dimensionless group, for the expression of the classic general dimensionless model. As consequence of this considerable limitation, an analytical method was developed to determine the optimal coefficient, for the expression of the new general dimensionless model, related to the natural logarithm of each dimensionless independent group from the new proposed model. Thus, in addition to the goal of the present research, it was developed a contribution to the current dimensional analysis theory regarding the determination of the dimensionless groups. Due to the amplitude of the developed analytical method, it was not presented in its complete form in this work, for this reason, only the most necessary equations were presented in this manuscript.

The classical general dimensionless model is represented by the function $_{\text{Mod}}\pi_1$ in Eq. (3). The symbols $_{\text{Mod}}\pi_1$ and π_i designate the dependent dimensionless group and the independent dimensionless groups, respectively.

$$_{\text{Mod}}\pi_1 : \xi \in \mathbb{R}^{mn} \rightarrow _{\text{Mod}}\pi_1(\xi) =: _{\text{Mod}}\pi_1 := \sum_{j=1}^m a_j \left[\prod_{i=2}^n \left(\pi_i^{j e_i^*} \right) \right] \in \mathbb{R} \quad (3)$$

being:

$$\xi = \left(a_1, \dots, a_{j,1} e_2^*, \dots, j e_i^* \right) \quad a_j \neq 0, \forall i \in S_i \wedge \forall j \in S_j \quad (4)$$

One of the negative aspects of the classic dimensionless model is the excessive number of variables to be calculated, such as the coefficients of the model terms and the exponents of the independent dimensionless groups. The new general dimensionless model proposed in this work, only needs the calculation of the coefficients of the natural logarithms of each independent dimensionless group, thus significantly reducing the number of variables to be found. The new general dimensionless model is shown in Eq. (5).

$$\text{Mod } \boldsymbol{\pi}_1 : j \bar{e}_i \in \mathbb{R}^{m(n-1)} \rightarrow \text{Mod } \boldsymbol{\pi}_1 (j \bar{e}_i) =: \text{Mod } \boldsymbol{\pi}_1 := \sum_{j=1}^m \text{sgn}(j) \left[e^{i=2} \sum_{j=1}^n j e_i \ln(\boldsymbol{\pi}_i) \right] \in \mathbb{R} \quad (5)$$

being:

$$j \bar{e}_i = \left(1 e_2, \dots, 1 e_i, 2 e_2, \dots, 2 e_i, \dots, j e_i \right) \quad \forall i \in S_i \wedge \forall j \in S_j \quad (6)$$

In the proposed method for the new general dimensionless model, first it must be established a monomial dimensionless model and then add other terms (or monomials) if it is considered necessary. This monomial dimensionless model is presented in Eq. (7).

$$\text{Mod } \boldsymbol{\pi}_1 : 1 \bar{e}_i \in \mathbb{R}^{n-1} \rightarrow \text{Mod } \boldsymbol{\pi}_1 (1 \bar{e}_i) =: \text{Mod } \boldsymbol{\pi}_1 := e^{i=2} \sum_{i=1}^n 1 e_i \ln(\boldsymbol{\pi}_i) \quad (7)$$

2.3 Objective function and necessary condition of optimization for a monomial dimensionless model

In Eq. (8) it is presented the objective function established to be optimized and thus to determine the coefficients of the natural logarithms of the independent dimensionless groups presented in Eq. (7).

$$F_{\text{Obj},1} : j \bar{e}_i \in \mathbb{R}^{m(n-1)} \rightarrow F_{\text{Obj},1} (j \bar{e}_i) =: F_{\text{Obj},1} := \sum_{k=1}^p \left[\psi(\text{Expk } \boldsymbol{\pi}_1) - \psi(\text{Modk } \boldsymbol{\pi}_1) \right]^2 \in \mathbb{R} \quad (8)$$

The symbol ψ represents a bijective function (the identity function is mostly used in optimization), which is conveniently chosen in this study as the natural logarithm function, $\psi = \ln$. The necessary condition to optimize the dimensionless model from the Eq. (7) is shown in Eq. (9).

$$\sum_{k=1}^p \left[\ln(\text{Expk } \boldsymbol{\pi}_1) - \sum_{i=2}^n 1 e_i \ln(\boldsymbol{\pi}_i) \right] \ln(\boldsymbol{\pi}_i) = 0 \quad \forall i \in S_i \quad (9)$$

3. RESULTS AND DISCUSSION

In this section, five dimensionless models (M1 – M5) were presented in Tab. 1 for the ID_{32}/d_0 . In order to establish these models, 85 experimental results were used (from 17 effervescent atomizers of outside-in gas injection configuration) of the integral Sauter mean diameter ID_{32} obtained from the works of Jedelsky et al. (2007) and Jedelsky et al. (2009). Additionally, to compare the estimation accuracy of the dimensionless models, it was used as a parameter called the root mean square error (RMSE) (Devore and Berk, 2012) defined in Eq. (10) under the context of the present research.

$$\text{RMSE} = \sqrt{\frac{\sum_{k=1}^p (\text{Expk } \boldsymbol{\pi}_1 - \text{Modk } \boldsymbol{\pi}_1)^2}{p}} \quad (10)$$

LHO and air were used as atomization liquid and gas, respectively, in the two experimental studies (Jedelsky et al., 2007, Jedelsky et al., 2009) that were used to propose the models. It is important to mention that the quantity of data acquired was plentifully enough for the correct establishment of the dimensionless models proposed for each dimensionless characteristic parameter. For all these models, it was considered $c_{\Delta l} = 2.5 \text{ mm}$.

Table 1. Dimensionless models for ID_{32}/d_o .

Dimensionless group	M1	M2	M3	M4	M5
$_{Exp}\pi_1$	ID_{32}/d_o	ID_{32}/d_o	ID_{32}/d_o	ID_{32}/d_o	ID_{32}/d_o
π_2	n	n	n	n	n
π_3	d_a/d_o	d_a/d_o	d_a/d_o	d_a/d_o	d_a/d_o
π_4	$\Delta l/d_o$	$\Delta l/d_o$	$\Delta l/d_o$	$\Delta l/d_o$	d_c/d_o
π_5	d_c/d_o	d_c/d_o	d_c/d_o	d_c/d_o	l_c/d_o
π_6	l_c/d_o	l_c/d_o	l_c/d_o	l_c/d_o	GLR
π_7	l_o/d_o	l_o/d_o	l_o/d_o	GLR	$\ln(P_1 d_o/\sigma_1)$
π_8	$m_i/\mu_g d_o$	$m_i/\mu_l d_o$	GLR	$\ln(P_1 d_o/\sigma_1)$	—
π_9	$m_i/\mu_l d_o$	GLR	$\ln(P_1 d_o/\sigma_1)$	—	—
π_{10}	GLR	$\ln(P_1 d_o/\sigma_1)$	—	—	—
π_{11}	$\ln(P_1 d_o/\sigma_1)$	—	—	—	—

Before establishing the dimensionless models displayed on Tab. 1, other models were tested but they are not shown in this work. Among these other models, the same models presented in Table 1 were tested with the only difference of using the dimensionless group $P_1 d_o/\sigma_1$ instead of using $\ln(P_1 d_o/\sigma_1)$. From this substitution, it was noted that the dimensionless models with $P_1 d_o/\sigma_1$ returned greater RMSE in comparison with the ones with $\ln(P_1 d_o/\sigma_1)$. This happened because the length of the interval of $P_1 d_o/\sigma_1$ is very large as well as its contained values in the interval, thus producing much variation of the values of the dimensionless model. That is the reason why the natural algorithm was applied to $P_1 d_o/\sigma_1$, to reduce the influence of this dimensionless group in the values of the dimensionless model and, thus achieving, a smaller length of the interval of $\ln(P_1 d_o/\sigma_1)$ as well as its contained values in the interval. Finally, it was notable to observe that the term $\ln(P_1 d_o/\sigma_1)$ yields significant reductions in the variations of the results of the dimensionless model so that better estimates can be obtained.

In Tab. 2 the corresponding coefficients were presented for each natural logarithm of the independent dimensionless group belonging to a given dimensionless model for ID_{32}/d_o . The coefficients were obtained from the solution of systems of equations presented in Eq. (9), substituting the experimental results of the works of Jedelsky et al. (2007) and Jedelsky et al. (2009).

Table 2. Coefficients of the natural logarithms of dimensionless groups for estimation of ID_{32}/d_o .

Coefficient	M1	M2	M3	M4	M5
$1e_2$	-0.057	-0.057	-0.057	-0.095	-0.041
$1e_3$	-0.085	-0.085	-0.082	-0.134	-0.036
$1e_4$	0.014	0.014	0.015	0.038	-0.045
$1e_5$	-0.019	-0.019	-0.017	-0.033	-0.085
$1e_6$	-0.064	-0.064	-0.063	-0.108	-0.069
$1e_7$	-0.102	0.323	0.888	-0.069	-1.370
$1e_8$	-0.078	0.085	-0.074	-1.342	—
$1e_9$	0.163	-0.034	-0.944	—	—
$1e_{10}$	-0.034	-1.439	—	—	—
$1e_{11}$	-1.439	—	—	—	—
RMSE	1.935	1.927	1.973	2.900	3.035

From Tab. 2, for the estimation of ID_{32}/d_o , the models M4 and M5 are the ones that have the highest values of RMSE, and consequently considered not adequate for the estimation. Unlike models M4 and M5, the models M1 – M3 have RMSE values of less than 2. Specifically, for the models M1, M2 and M3 the values were 1.935, 1.927, and 1.973, respectively. Because the difference between the RMSE values for the models M1 – M3 is very small, the dimensionless model M3 was chosen to estimate ID_{32}/d_o , since it was considered not significant with respect to the

RMSE to consider one or two additional independent dimensionless groups in the dimensionless model. An important information is that the length of the interval of ID_{32} is 25.1.

4. CONCLUSIONS

The proposed dimensionless models are good for the estimation of the integral Sauter mean diameter. Evaluating the value of the RMSE and the number of terms of the dimensionless models M1 – M5, the model M3 was selected for the estimation of ID_{32}/d_o , due to its RMSE equal to 1.973. From the comparisons of the results calculated by the model M3 with the experimental data, satisfactory estimates were obtained by which the model M3, it was concluded that the estimations are satisfactory. Therefore, model M3 is useful to obtain predictions of the atomization of LHO in effervescent atomizers of outside-in gas injection configuration.

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