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STUDY OF EFFECTIVE SHEAR MODULUS ON FLEXIBLE COMPOSITES UNDER SIMPLE SHEAR

R.P. Faria
L.C.S. Nunes
Universidade Federal Fluminense –UFF Rua Passo da Pátria,156,Bloco E, Sala 210, Niterói, RJ CEP 24210-240, Brazil
renato_faria@id.uff.br
luizcsn@id.uff.br

Abstract. *The main purpose of this work is to evaluate the effective shear modulus of flexible composites, with fibers arranged perpendicularly to shear load, under simple shear state at linear elastic regime. This study focuses on evaluating the effect that different parameters related to fibers, such as: height, radius of gyration, volume fraction and cross-sectional area, have on effective shear modulus. This work uses numerical simulations to evaluate the effective shear modulus for different fibers patterns. Comparing the collected data, it is possible to identify the magnitude that each parameter has in the composite modulus. Also, if the same composite has its fibers arranged longitudinally to shear force, the effective shear modulus is similar as under pure shear state, mainly because fibers are not subjected to internal tensile forces. To illustrate that, some data for longitudinally arranged fibers composites are calculated, and they are compared with Reuss model. The obtained results shows the parameters such as height and cross-sectional shape of fibers have a huge impact on effective shear modulus of a composite and must be accounted in future studies which aims to model this material.*

Keywords: *Flexible composites, Effective shear modulus, Simple shear, Unidirectional composite*

1. INTRODUCTION

Composite materials have been commonly used in many different areas, for example, reinforced concrete walls used in civil engineering (Dall`Asta et al., 2017), fiberglass composites used to manufacture freight car parts in mechanical engineering (Zaripov and Gavrilovs, 2017), and in prosthesis for disable individuals, which are well established in medical areas (Nurhanisah et al., 2017). Because of its wide range of applicability, it is important to investigate the mechanical behavior of composites in order to properly design and improve different technologies and tools. For this reason, during many years, the problem of determining mechanical properties, in composite materials, has played a considerable role in classical theory of elasticity. For example, one of the first and most remarkable studies related to this topic, was the mathematical model proposed by Reuss (1929), known as the inverse rule of mixtures, which is capable of estimating the mechanical properties of composites.

In the Reuss's model, some parameters were evaluated such as fiber and matrix moduli and volume fraction. Moreover, Halpin and Kardos (1976) developed a model capable of accounting the previous parameters and also the influence of different fibers cross-sectional areas on a composite modulus. Sideridis (1988) investigated the effect of the interface between fibers and matrices on the composite modulus.

Those studies mainly focus on composites in a state of pure shear, on other hand, when the subject is related to simple shear, fewer studies can be found in literature. According to Belik (1998), pure shear can be defined by the condition of Tr $(\sigma) = 0$ (where "σ" refers to Cauchy Green Tensor) and according to Jones and Treloar (1975) simple shear can be defined as pure shear added to rigid rotation movement. Thus, it can be seen some distinct boundary conditions between both cases, in order to properly model a shear state. For example, it was investigated by Horgan and Murphy (2011a), Horgan and Murphy (2011b) and Destrade et al. (2012), that homogeneous non-linear elastic materials and composites, under simple shear, have internal tensile forces due combination of shear deformation with materials fixed height. These tensile forces make the material stiffer and, as a result, modifies its shear modulus. In fact, those tensile forces have been investigated as the one of the factors related to the Poynting effect (Destrade et al., 2014).

Recently, some studies shows interest on evaluating composites under simple shear with unidirectional fibers and the effect of parameters such as fibers diameters, as seen in Moreira and Nunes (2016), and the orientation of fibers as investigated by Horgan and Murphy (2017). These studies illustrates there are considerably information to cover in simple shear subject, under flexible unidirectional composites. Since its behavior varies dramatically due to boundary conditions comparing to pure shear, classic models such as Reuss are limited to determine composite properties under simple shear. This occurs for perpendicularly arranged fibers, because of generated internal tensile forces.

Therefore, the purpose of this work is to evaluate different composite parameters and its influence on the effective shear modulus, in a unidirectional flexible composite under simple shear. Fibers were arranged perpendicularly to the applied shear load to investigate the composite modulus at this condition. In order to estimate this effect, numerical simulations were performed in a composite with simple shear boundary conditions, by varying height, radius of gyration, cross-sectional area and volume fraction of fibers. As for the numerical approach, this study implements this method motivated by studies such in Destrade et al. (2013), Destrade et al. (2015), Giner et al. (2015) and Raju (2018).

In addition, for comparative purposes, the effective shear modulus of composites under simple shear with fibers longitudinally arranged, were also obtained. For this case, tensile forces have no influence in effective shear modulus. Therefore, Reuss model can accurately estimate the composite modulus. This data can be useful to validate the simulations since they are expected to be in good agreement with Reuss model and also, clarify the differences between modulus for a longitudinal and perpendicular fiber arrangement.

2. NUMERICAL METHODS

Simulations were performed using Ansys[TM] software and the applied geometries represent a biphasic solid reinforced with unidirectional fibers. The composite has its fibers arranged perpendicularly to an applied load (*F*), causing a displacement (*d*). Also, a fixed support boundary condition is defined in its base in order to prevent any type of displacement, as shown in Fig.1.

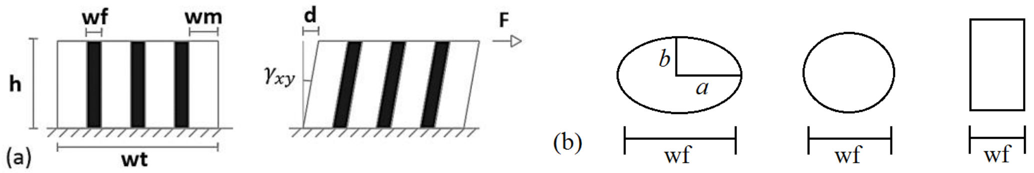

Figure 1. (a) Representation of composite, its dimensions and boundary conditions (b) cross sectional areas of fibers: elliptical, circular and rectangular

Each of the fibers, indicated by the black color, have the same width denoted by w_f. The same applies for each of the matrices, indicated by the white color, theirs width are denoted by w_m. The total composite width, total composite height, and angular deformation are denoted respectively as w_t, h, γ_{xy}. The values of w_t (70mm) and the thickness of the composite "z" (3.4mm) are the only parameters kept fixed in all analyzes. For ellipses, dimension "*a*", denotes the horizontal radius and "*b*" the vertical radius, for circular and rectangular cross sectional shaped fibers "w_f" refers respectively to diameter and width.

In terms of mechanical properties, fibers and matrices were assigned with a shear modulus of 147.5 MPa and 0.39 MPa and Poisson coefficient of 0.4 and 0.499 respectively. Perfect bonding condition was defined between fibers and matrices, preventing any sliding or separation between both phases.

As for the mesh, the element type used was solid 186 (hexagonal 20 nodes). In order to obtain convergence, mesh was refined until data variation became lower than 3% from its previous value. In most of the cases, the number of elements suitable for this scenario varies in a range between 50000 and 150000.

Simulations were performed for composites with rectangular, circular and elliptical cross-sectional area fibers with different height values and volumetric fraction. The effective shear modulus for each situation was calculated and the obtained data was used to evaluate the influence of these parameters.

In order to verify the applied boundary conditions and its influence on simple shear two types of arrangement of fibers were use, perpendicularly and longitudinally. The same shear conditions were applied for both types of composites. For longitudinally arrangement, collected data should agree with Reuss model, which can be expressed as

$$G_{eff} = (G_f \cdot G_m) / (\phi_m \cdot G_f + \phi_f \cdot G_m) \quad (1)$$

Where ϕ denotes the volume fraction of the material, G_{eff} the effective shear modulus, G the shear modulus and the subscripts *f* and *m* denote fiber and matrix, respectively. Due to both materials have the same height values, the volume fractions can be obtained by

$$\phi_f = A_f / A \quad (2)$$

$$\phi_m = 1 - \phi_f = A_m / A \quad (3)$$

Where A denotes total cross sectional area of the composite material.

3. RESULTS AND DISCUSSION

For fibers with rectangular, circular and elliptical cross-sectional areas, different volume fractions were obtained by fixing the matrix width value (w_m =1mm), and varying the width of fibers (w_f). By changing its height (h) and maintaining the volume fraction of fibers (ϕ_f), it is possible to identify the influence of h on the effective shear modulus (G_{eff}). The results are shown in Figs. 2(a), 2(b) and 2(c) and it is noticeable that, when the height increases, the effective shear modulus decreases.

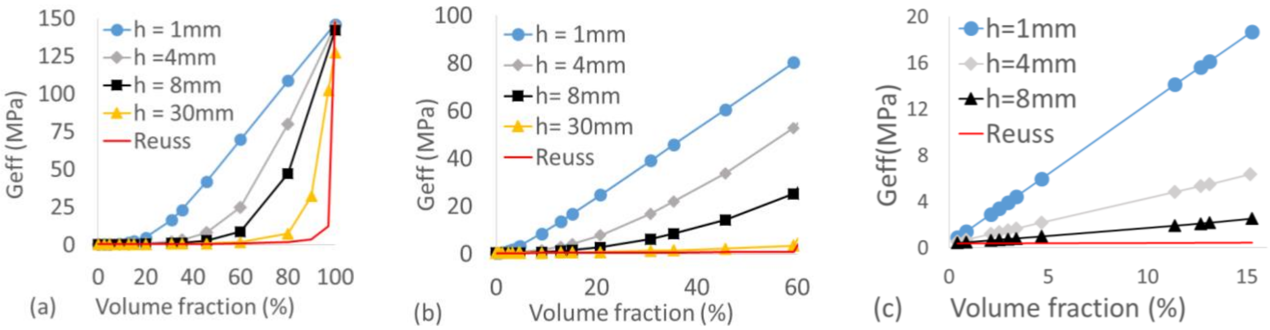

Figure 2. Values of effective shear modulus due to the influence of volume fraction and height of fibers with (a) rectangular, (b) circular and (c) elliptical cross-sectional areas.

To verify the influence of radius of gyration, fibers height was maintained constant and different volume fractions for a fixed w_f were obtained. For fibers with elliptical cross sectional area, the horizontal radius (a) and vertical radius (b) had its values kept constant, and by increasing the number of fibers, different volume fractions were obtained. The results are shown in Figs. 3(a), 3(b) and 3(c), respectively. It can be identified as higher the radius of gyration becomes, the effective shear modulus increases.

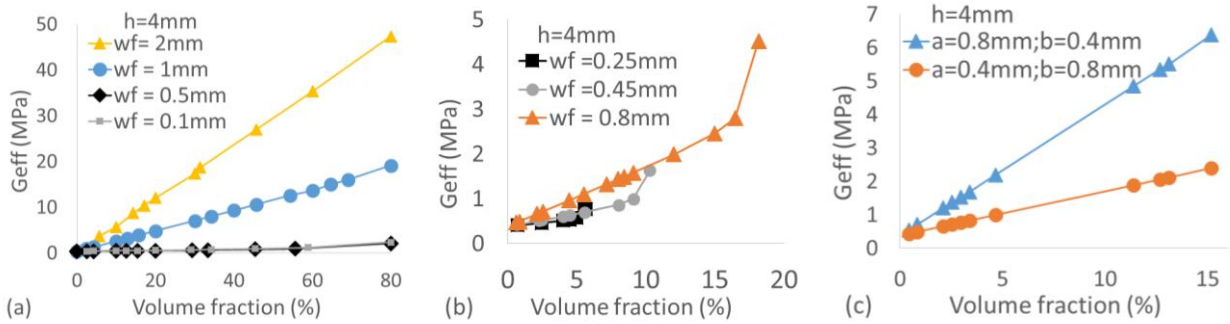

Figure 3. Values of effective shear modulus due to the influence of volume fraction and width of fibers/radius of gyration for (a) rectangular, (b) circular and (c) elliptical.

Since elliptical cross sectional fibers may change in both directions (horizontal radius "a" and vertical radius "b") simulations varying each parameter were also performed. In the first case, dimension b was kept constant and radius a had the following values, in mm: 0.2, 0.4, 0.6, 0.8, 3.2, 6.4 and 17. In the second case, dimension a was kept constant and radius b had the following values in mm: 0.2, 0.4, 0.6, 1.2 and 1.6. In addition, in both cases, the values of w_m were fixed as 1mm, between both ends (see Fig.1), in each simulation. The results for each scenario and for different height values are shown in Figs. 4(a) 4(b) 4(c) and 4(d), respectively. The data shows, because of the radius of gyration of elliptical fibers depends strongly of de a value, the effective shear modulus tends to increase dramatically as well. Also, variations of a have more effect than b in composite effective shear modulus. Eventually, both curves have a similar point where volume fraction and effective shear modulus are the same. That value indicates there is common cross-sectional area between both evaluated scenarios, thus indicating at that point the cross sectional fibers have $a = b$. It is important to mention the number of fibers may change due to the fibers dimensions, which can change the volume fraction value where both of those curves match together. Also, by comparing each Figure, it is possible to understand that as higher the composite becomes more pronounced is the effect of increasing the value of a which is not likely to occur in the same scenario for increasing b. For both cases at elevated h values, since the bending moment effect occurs, it is expect the values to *Geff* tends to decrease which can be seen as well.

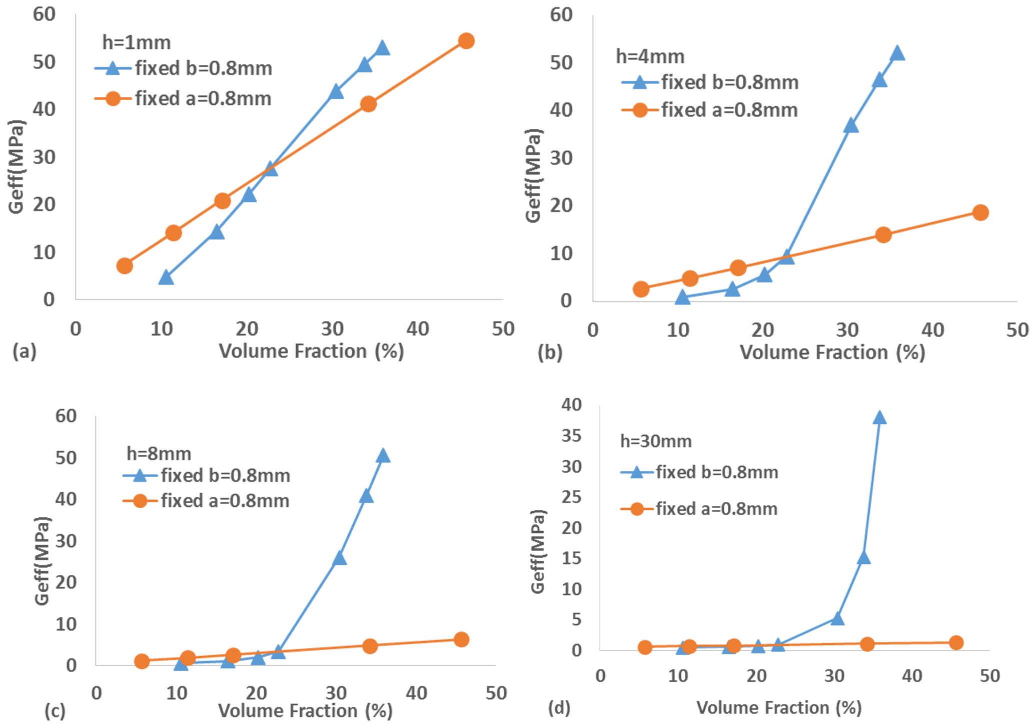

Figure 4. Values of effective shear modulus due to the influence of volume fraction and radius of gyration for elliptical fibers with composite heights equals to (a) 1mm, (b) 4mm, (c) 8mm and (d) 30mm.

Simulations with longitudinally arranged fibers were performed for comparison with Reuss model, For fibers with rectangle cross sectional shaped fibers, the data obtained are shown in Fig.5. It is noticeable by comparison of results from longitudinally and perpendicularly composite arranged fibers, that the longitudinally ones are well predicted by Reuss model. In the other hand, for perpendicular fibers, it is clear that due to the different boundary conditions, this problem cannot be modeled by Reuss model.

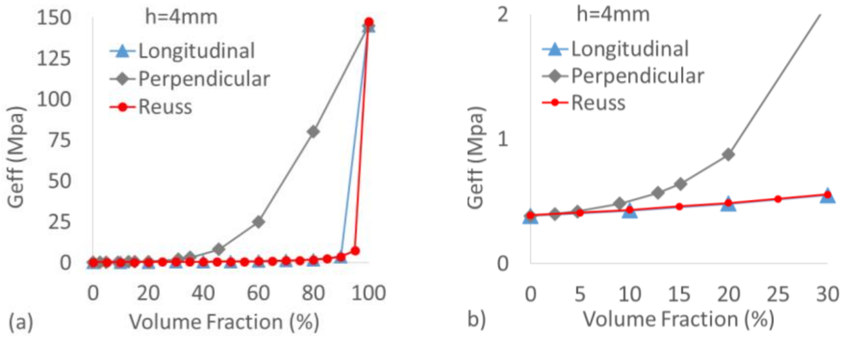

Figure 5. (a) Variation of G_{eff} values due to the perpendicular and longitudinal situations and (b) variation of G_{eff} for small volume fractions.

4. CONCLUSION

In this work, it is possible to identify that fibers perpendicularly arranged in a flexible composite results in a significant variation on effective shear modulus, due to internal tensile forces. Because of this, parameters related to fibers, such as height and radius of gyration, also have a huge impact on the estimation of this modulus. The data obtained in this work shows that by increasing the radius of gyration, the effective shear modulus becomes higher, making the composite stiffer. In the other hand, the opposite effect is noticeable when the height of fibers increases, making the effective shear modulus decreases, becoming closer to estimations in Reuss model. This occurs because internal tensile forces are less predominant in simple shear for high values of fibers height. In this situation, it is observed a predominant bending effect causing the composite to reduce its stiffness.

These results show, for models that aims to estimate mechanical properties of flexible composites, in simple shear, the shear modulus of fibers cannot be considered as its own value. Although, this approach is valid for pure shear, it is important to understand the same does not apply for simple shear. For the scenario of fibers orientated perpendicularly to applied shear load, internal tensile forces that do prevail in simple shear have a considerable impact. Besides that, the same does not occur in pure shear state or in a simple shear state with a composite with longitudinal fibers. Instead of this, it is necessary a proper model for the shear modulus of fibers that is capable of account the effects presented in this work. Also, the present analysis is useful for future studies related to composites under simple shear on its elastic regime.

5. ACKNOWLEDGEMENTS

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