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THEORETICAL STUDY OF HEAT TRANSFER INSIDE MICROTUBES WITH VISCOUS DISSIPATION VIA GITT

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Abstract. *This paper presents a solution obtained through of the Generalized Integral Transform Technique, to the problem of thermal entrance in forced convection in laminar flow inside microtubes of circular section. The model considers the effect of the slip flow, and viscous dissipation. The Generalized Integral Transform Technique is employed to reduce the original problem to a system of linear first-order differential equations, which is then solved utilizing available algorithms for the numerical solution of ordinary differential systems. Results are then presented for the fluid bulk temperature and local Nusselt number along the channel. Critical comparisons with previous results in the literature are also performed, in order to validate the present work, and to demonstrate the consistency of the final results.*

Keywords: *Convection , Microtubes, GITT*

1. INTRODUCTION

In the last two decades, there has been a growing number of studies on heat transfer in microtubes. This problem has received attention due to their potential for application in the refrigeration of electronic circuits, and equipment used in the industry. Recent advances in the field of micromechanics have enabled the fabrication of various micro-scale devices for use in scientific research and commercial products. However, it has been observed that the behavior of the fluid flow in such ducts deviates from the predictions based on the continuum hypothesis, which have been accepted in the macroscopic phenomena. For these reasons, the understanding of micro-scale transport phenomena has gained importance. Many studies have been conducted in order to analyse the behavior of the convective flow through microtubes. The goal of this article is to present the results of a hybrid, numerical-analytical solution for the temperature distribution in a fluid flowing through microtubes of circular cross-section, taking into account the velocity slip ande temperature jumps at the wall, and viscous dissipation. To this purpose , a more flexible approach was employed based on the ideas of the Generalized Integral Transform Technique (GITT). GITT is a well-established computational approach to the hybrid, numerical-analytical solution of different classes of heat conduction, convection, and fluid flow problems, Cotta (1993). The present solution is compared with the results available in the scientific literature. The present work can be inserted in the context of the problems of forced convection, is considered an extension of the studies carried out by Barron et al. (1996), who tried a solution to the problem through the separation of variables, however its solution became unstable from of the fifth term in the series, Mikhailov & Cotta (2005), Castellões (2004) and others, extending the application of GITT to the convection in microtubes.

2. ANALYSIS

We consider steady-state heat transfer in thermally developing, hydrodynamically developed forced laminar flow inside a microtube of circular section. The flow is assumed to be incompressible with slip at the wall, viscous dissipation and constant physical properties. Natural convection is negligible. The entrance temperature distribution is uniform. Including the effect of viscous dissipation in the model originally presented by Barron et al (1996). The temperature of the fluid is described in dimensionless form by the following problem:

$$Gz\frac{4Kn+\left(1-R^2\right)}{2\left(1+8Kn\right)}\frac{\partial\theta\left(R,Z\right)}{\partial Z}=\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta\left(R,Z\right)}{\partial R}\right)+Br\,16\frac{R^2}{\left(1+8\,Kn\right)^2}\quad,\ 0<R<1\ ,\ Z>0 \tag{1}$$

Contour conditions

$$\frac{\partial \theta(R,Z)}{\partial R}\Big|_{R=0}=0 \ , \ \theta(1,Z)=0 \ , \ Z>0 \tag{2,3}$$

Inlet condition

$$\theta(R,0)=1 \ , \ 0\leq R\leq 1 \tag{4}$$

The dimensionless velocity profile is given as (Barron *et al*, 1996):

$$u(R)=\frac{2u_m\left(1-R^2+4Kn\right)}{1+8Kn} \tag{5}$$

The dimensionless groups employed in the above equations are:

$$R=\frac{r}{r_1} \ , \ Z=\frac{z}{L} \ , \ \theta(R,Z)=\frac{T(r,z)-T_w}{T_0-T_w} \ , \ W(R)=\frac{Gzu(R)}{4u_m} \tag{6-9}$$

In the dimensionless form of the problem, once $\theta(R,Z)$ is determined, the average temperature $\theta_m(Z)$ and the local Nusselt number are found respectively from:

$$\theta_m(Z)=2\int_0^1 \frac{u(R)}{u_m}\theta(R,Z)\,R\,dR \ , \ Nu(Z)=-\frac{1}{\theta_m(Z)}\frac{\partial \theta(R,Z)}{\partial R}\Big|_{R=1} \tag{10,11}$$

3. METHOD OF SOLUTION

The approach used here is the Generalized Integral Transform Technique (GITT), (Cotta, 1993). The use of GITT allows for the selection of an appropriate auxiliary eigenvalue problem, without being restricted to the specific problem that would allow for a transformable solution. Here, an even simpler eigenvalue problem is considered. Following the formalism in the GITT an auxiliary eigenvalue problem is selected as:

$$\frac{d^2\widetilde{\Psi}_i(R)}{dR^2}+\mu_i^2\widetilde{\Psi}_i(R)=0 \ , \ \frac{d\widetilde{\Psi}_i(R)}{dR}\Big|_{R=0}=0 \ , \ \widetilde{\Psi}_i(1)=0 \tag{12,14}$$

Where $\widetilde{\Psi}_i(R)$ and μ_i are the normalized eigenfunction and eigenvalue, respectively. The auxiliary eigenvalue problem allows the definition of the following integral-transform pair:

$$\bar{\theta}_i(Z)=\int_0^1 \widetilde{\Psi}_i(R)\theta(R,Z)\,dR \ , \ \theta(R,Z)=\sum_{i=1}^{nt}\widetilde{\Psi}_i(R)\bar{\theta}_i(Z) \tag{15-16}$$

Equation (1) is now integral transformed through the operator $\int_0^1 \widetilde{\Psi}_i(R)\,dR$, to yield the transformed ordinary differential equations. Now we can rewrite the system of equations in matrix form as:

$$\frac{d\bar{\theta}(Z)}{dZ}=D\bar{\theta}(Z)+\frac{16\,Br}{(1+8\,Kn)^2}E \ , \tag{17}$$

Where $D=A^{-1}B$ and $E=A^{-1}C$, with $A_{i,j}=\int_0^1 R\,W(R)\,\widetilde{\Psi}_i(R)\widetilde{\Psi}_j(R)\,dR$ (18-20)

$$B_{i,j}=\int_0^1 \widetilde{\Psi}_i(R)\frac{d\widetilde{\Psi}_j(R)}{dR}dR-\mu_j^2\int_0^1 R\,\widetilde{\Psi}_i(R)\widetilde{\Psi}_j(R)\,dR \quad \text{and} \quad C_i=\int_0^1 R^3\,\widetilde{\Psi}_i(R)\,dR \tag{21,22}$$

We need now to transform the inlet condition by operating on equation (4) with $\int_0^1 \tilde{\Psi}_i(R)\,dR$ to obtain:

$$\bar{\theta}_i(0)=\int_0^1 \tilde{\Psi}_i(R)\theta(R,0)\,dR=\int_0^1 \tilde{\Psi}_i(R)1\,dR \tag{23}$$

Finally, the transformed system is defined by equations (17-23), and this linear initial value problem can be truncated and solved for transformed potentials. Various algorithms for the numerical integration of ordinary differential systems are available in scientific subroutines packages.

4. RESULTS AND DISCUSSIONS

Initially, convergence tests and comparisons are performed to verify the behavior of the present solution. The first table shows the convergence behavior of the local Nusselt number. The results were compared with the data of Barron et al. (1996), who calculated the limit value assumed by the Nusselt number as 3.657, in the case of $Kn=0$ and $Br=0$, which neglects the viscous dissipation effect. The second table shows the convergence behavior of the local Nusselt number. The results were compared with the data of Barron et al. (1996), who calculated the limit value assumed by the Nusselt number as 4.471, in the case of $Kn=0{,}12$ and $Br=0$. The third table shows the convergence behavior of the local Nusselt number. The results were compared with the data of Cetin et al (2008), who calculated the limit value assumed by the Nusselt number as 9.60, in the case of $Kn=0$ and $Br=0{,}01$, which considers the effect of viscous dissipation. The comparison shows a concordance between the solutions, which provides validation to the presented solution and the computational code used here.

Table 1. Convergence behavior of the local Nusselt number $Nu(Z)$, $Kn=0$, $Br=0$.

(Z/Gz)/nt	5	10	20	30	40	Barron *et al* (1996)
0,005	6,0604	6,0055	6,0020	6,0017	6,0016	-
0,010	4,9352	4,9170	4,9162	4,9161	4,9161	-
0,020	4,1819	4,1724	4,1724	4,1724	4,1724	-
0,040	3,7714	3,7689	3,7689	3,7689	3,7689	-
0,080	3,6628	3,6625	3,6625	3,6625	3,6625	-
0,100	3,6584	3,6581	3,6581	3,6581	3,6581	-
0,200	3,6571	3,6568	3,6568	3,6568	3,6568	-
0,500	3,6571	3,6571	3,6568	3,6568	3,6568	-
1,000	3,6571	3,6569	3,6568	3,6568	3,6568	-
2,000	3,6571	3,6569	3,6568	3,6568	3,6568	3,657

Table 2. Convergence behavior of the local Nusselt number $Nu(Z)$, $Kn=0{,}12$, $Br=0$.

(Z/Gz)/nt	5	10	20	30	40	Barron *et al* (1996)
0,005	7,7435	7,6688	7,6660	7,6657	7,6656	-
0,010	6,0991	6,0800	6,0795	6,0795	6,0795	-
0,020	5,0781	5,0699	5,0700	5,0700	5,0700	-
0,040	4,5804	4,5785	4,5785	4,5785	4,5785	-
0,080	4,4754	4,4750	4,4750	4,4750	4,4750	-
0,100	4,4724	4,4721	4,4720	4,4720	4,4720	-
0,200	4,4717	4,4714	4,4714	4,4714	4,4714	-
0,500	4,4717	4,4714	4,4714	4,4714	4,4439	-
1,000	4,4718	4,6279	2,8455	4,4737	4,4737	-
2,000	4,4717	4,4714	4,4712	4,4714	4,4714	4,471

Table 3. Convergence behavior of the local Nusselt number $Nu(Z)$, $Kn=0$, $Br=0{,}01$.

(Z/Gz)/nt	5	10	20	30	40	Cetin *et al* (2008)
0,005	6,1048	6,0500	6,0465	6,0462	6,0461	-
0,010	4,9910	4,9790	4,9721	4,9720	4,9720	-
0,020	4,2550	4,2457	4,2457	4,2457	4,2457	-
0,040	3,8767	3,8741	3,8741	3,8741	3,8741	-
0,080	3,8530	3,8526	3,8526	3,8526	3,8526	-
0,100	3,9106	3,9102	3,9103	3,9103	3,9103	-
0,200	4,6120	4,6113	4,6117	4,6117	4,6117	-
0,500	9,2329	9,2369	9,2377	9,2378	9,2378	-
1,000	9,5939	9,5989	9,5996	9,5997	9,5997	-
2,000	9,5942	9,5991	9,5999	9,6000	9,6000	9,60

The results obtained demonstrate the convergent behavior of the solution of the problem through GITT, despite the choice of a simple eigenvalue problem, the solution presented converges to the same values of the analytical solution presented by Barron et al. (1996). This indicates the validity of the present solution in the study of internal convection in microtubes. Figures 1 to 6 show the behavior of the mean temperature and the local Nusselt number along the length of the circular duct for a variation of the parameters Kn and Br . The graphs allow a better visualization of the behavior assumed by the solution along the length of the duct.

Table 4. Convergence behavior of the local Nusselt number $Nu(Z)$, $Kn=0$, $Br=0{,}01$.

(Z/Gz)/nt	5	10	20	30	40	Cetin *et al* (2008)
0,005	6,1048	6,0500	6,0465	6,0462	6,0461	-
0,010	4,9910	4,9790	4,9721	4,9720	4,9720	-
0,020	4,2550	4,2457	4,2457	4,2457	4,2457	-
0,040	3,8767	3,8741	3,8741	3,8741	3,8741	-
0,080	3,8530	3,8526	3,8526	3,8526	3,8526	-
0,100	3,9106	3,9102	3,9103	3,9103	3,9103	-
0,200	4,6120	4,6113	4,6117	4,6117	4,6117	-
0,500	9,2329	9,2369	9,2377	9,2378	9,2378	-
1,000	9,5939	9,5989	9,5996	9,5997	9,5997	-
2,000	9,5942	9,5991	9,5999	9,6000	9,6000	9,60

Table 5. Local Nusselt number behavior for a variation of Kn , $Br=0$, $nt=40$.

(Z/Gz)/Kn	0,02	0,04	0,06	0,08	0,10
0,005	6,4227	6,7665	7,0509	7,2894	7,4919
0,010	5,2070	5,4456	5,6444	5,8122	5,9556
0,020	4,3945	4,5775	4,7308	4,8610	4,9729
0,040	3,9669	4,1312	4,2695	4,3876	4,4896
0,080	3,8607	4,0253	4,1642	4,2829	4,3855
0,100	3,8567	4,0216	4,1607	4,2796	4,3824
0,200	3,8556	4,0207	4,1599	4,2789	4,3817
0,500	3,8556	4,0207	4,1599	4,2789	4,3817
1,000	3,8556	4,0207	4,1592	4,2688	4,3817
2,000	3,8556	4,0207	4,1599	4,2787	4,3817
Nu_∞	3,855	4,020	4,160	4,228	4,380

Tabela 6. Local Nusselt number behavior for a variation of Kn , $Br=0$, $nt=40$.

(Z/Gz)/Kn	0,01	0,03	0,05	0,07	0,09	0,11
0,005	6,2232	6,6030	6,9152	7,1753	7,3947	7,5820
0,010	5,0690	5,3320	5,5493	5,7317	5,8866	6,0197
0,020	4,2891	4,4902	4,6574	4,7984	4,9190	5,0231
0,040	3,8728	4,0527	4,2032	4,3310	4,4404	4,5355
0,080	3,7664	3,9466	4,0976	4,2258	4,3360	4,4317
0,100	3,7622	3,9428	4,0940	4,2224	4,3328	4,4287
0,200	3,7610	3,9418	4,0931	4,2216	4,3321	4,4280
0,500	3,7610	3,9418	4,0931	4,2216	4,3321	4,4280
1,000	3,7610	3,9418	4,0931	4,2216	4,3321	4,4280
2,000	3,7610	3,9418	4,0931	4,2216	4,3321	4,4280

Tables 6 and 7 shows the variations of the Nusselt number along the channel for a variation of Kn . This allows visualization of the values assumed by these quantities for different flow conditions. In all calculations, 40 terms were used in the series to provide more accurate data for these results. Table 5 compares the results obtained for the limit assumed by the fully developed Nusselt number, Nu_∞ , and the values presented by Barron et al (1996) for a variation of the parameter Kn .

Table 7. Local Nusselt number behavior for a variation of Kn , $Br=0{,}01$, $nt=40$.

(Z/Gz)/Kn	0,01	0,03	0,05	0,07	0,09	0,11
0,005	6,2612	6,6316	6,9374	7,1930	7,4092	7,5940
0,010	5,1172	5,3687	5,5783	5,7551	5,9059	6,0359
0,020	4,3527	4,5395	4,6967	4,8305	4,9456	5,0456
0,040	3,9651	4,1256	4,2621	4,3794	4,4812	4,5703
0,080	3,9363	4,0848	4,2122	4,3224	4,4187	4,5032
0,100	3,9896	4,1307	4,2520	4,3572	4,4492	4,5302
0,200	4,6649	4,7543	4,8267	4,8866	4,9370	4,9804
0,500	9,3971	9,6615	9,8714	10,0413	10,1821	10,2979
1,000	9,7410	9,9864	10,1817	10,3425	10,4771	10,5915
2,000	9,7443	9,9866	10,1818	10,3425	10,4772	10,5915

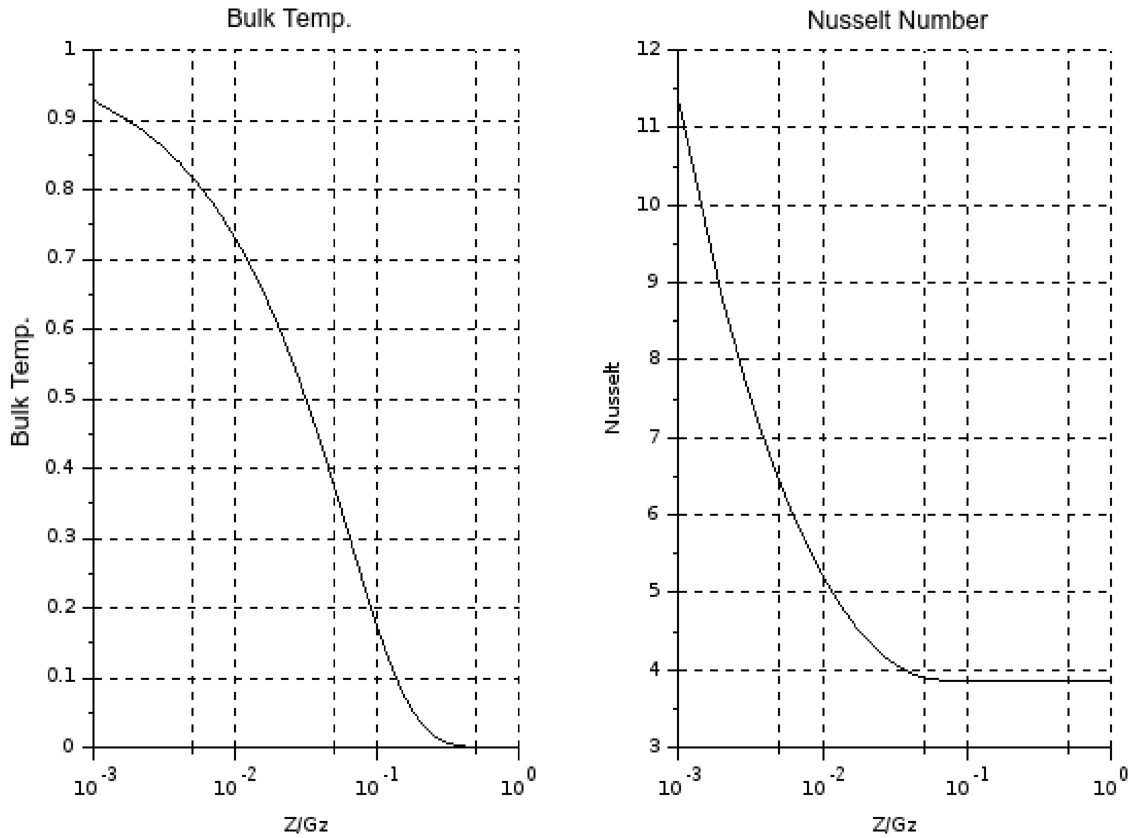

Figure 1. Bulk temperature and Nusselt number for $Kn = 0{,}02$, $Br = 0$, $nt = 40$.

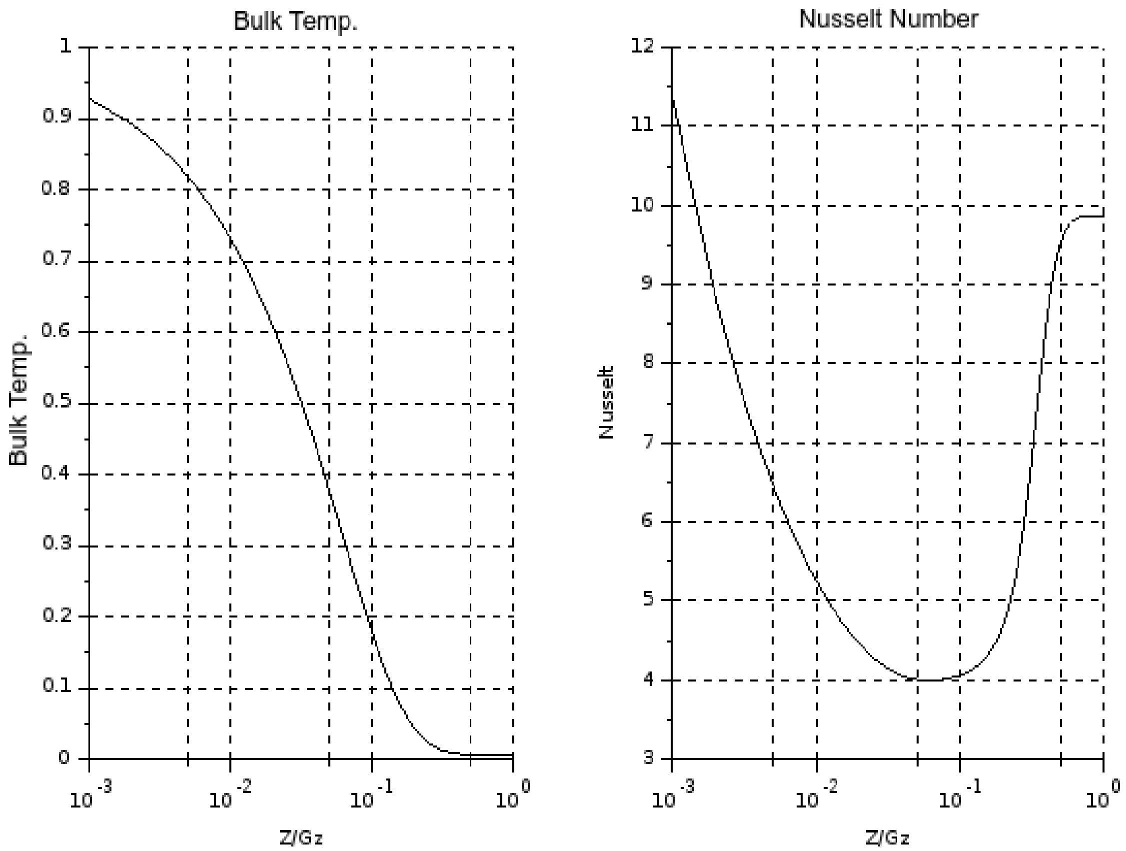

Figure 2. Bulk temperature and Nusselt number for $Kn = 0{,}02$, $Br = 0{,}01$, $nt = 40$.

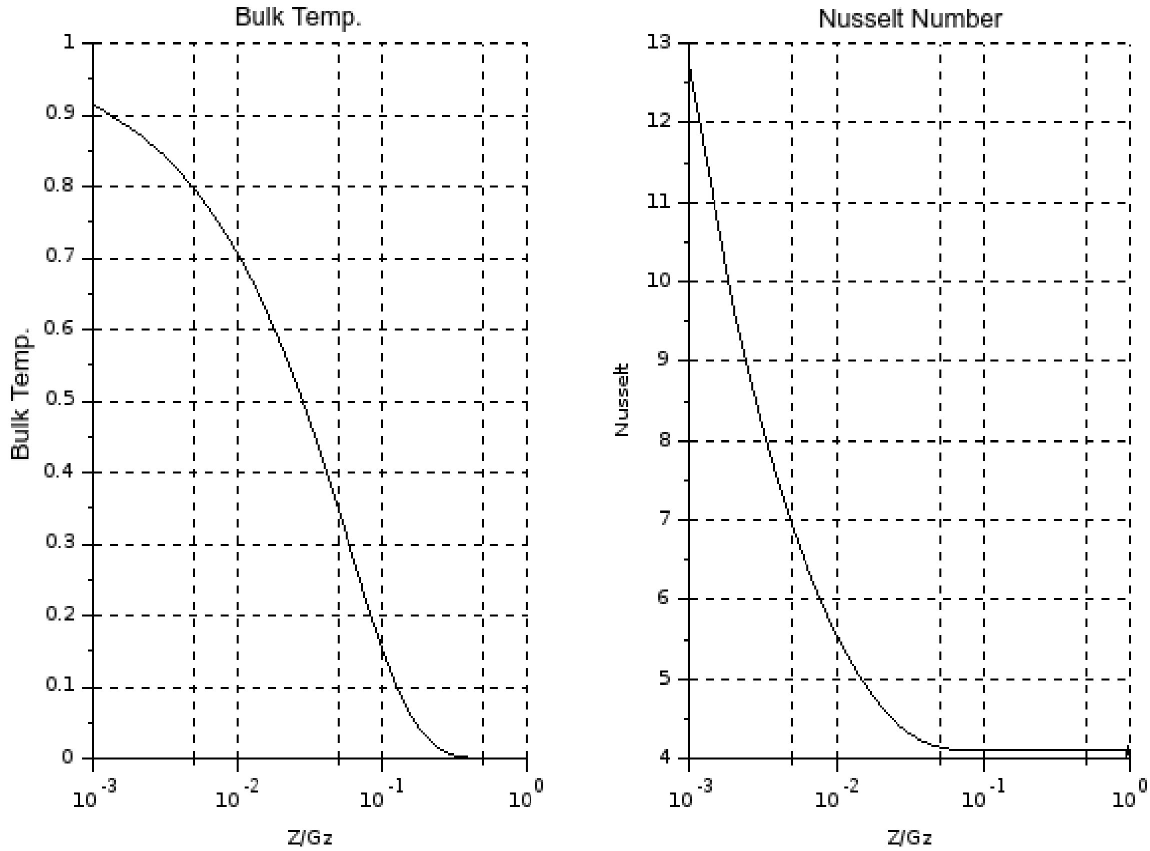

Figure 3. Bulk temperature and Nusselt number for $Kn = 0{,}05$, $Br = 0$, $nt = 40$.

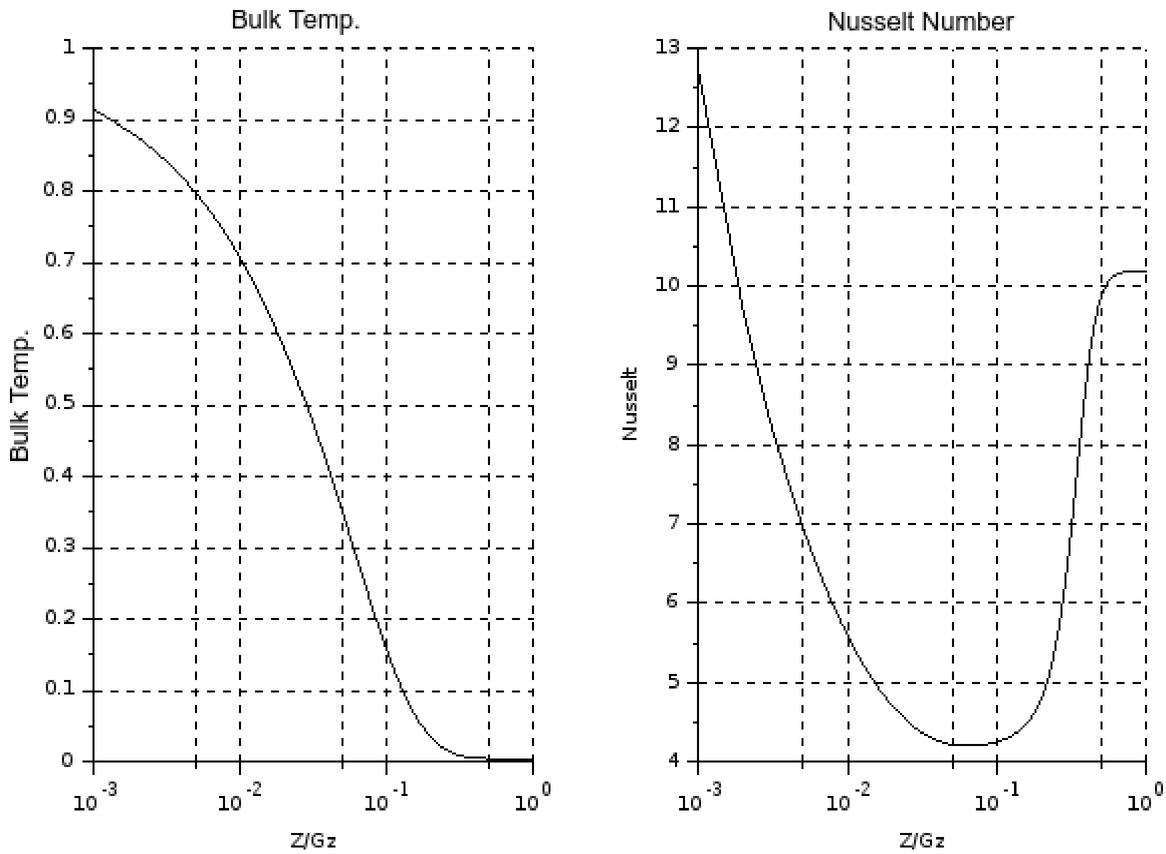

Figure 4. Bulk temperature and Nusselt number for $Kn = 0{,}05$, $Br = 0{,}01$, $nt = 40$.

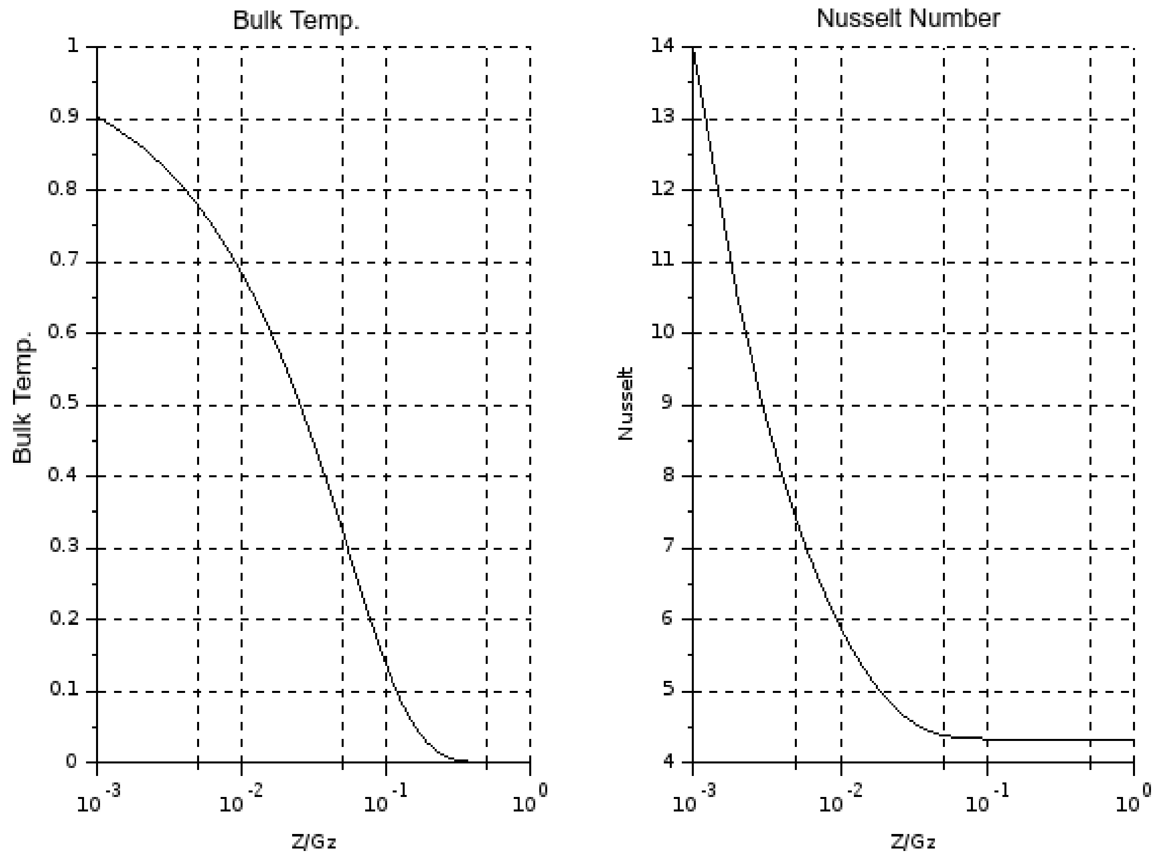

Figure 5. Bulk temperature and Nusselt number for $Kn=0{,}09$, $Br=0$, $nt=40$.

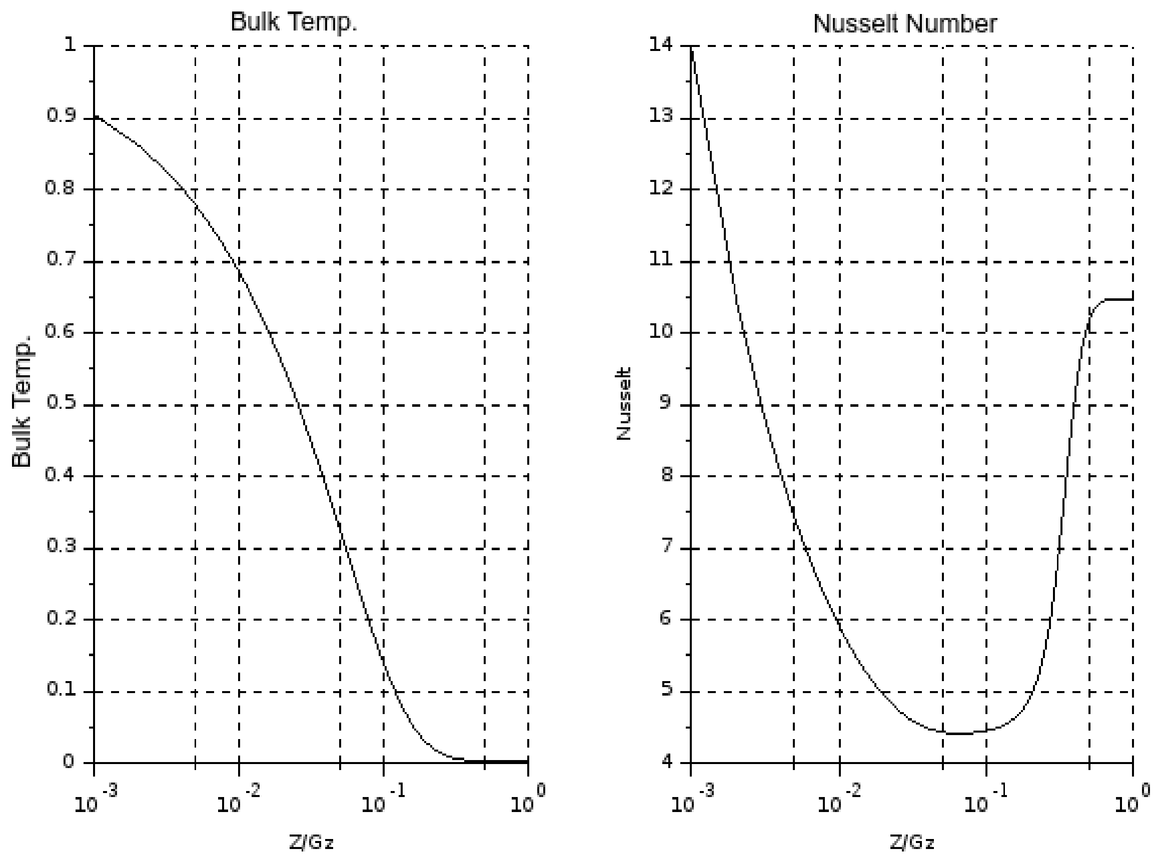

Figure 6. Bulk temperature and Nusselt number for $Kn=0{,}09$, $Br=0{,}01$, $nt=40$.

5. CONCLUSIONS

In the present work, a solution developed through GITT was presented for solving the problem of forced convection in the laminar flow inside microtubes of circular cross-section. Slip flow and the effect of viscous dissipation were considered. The results were obtained for the mean temperature and the Nusselt number. Comparisons were made with data previously published in the scientific literature, aiming to validate the numerical codes developed in this study and to demonstrate the consistency of the results presented in this work. This demonstrated the validity of the present solution as a viable alternative in the study of internal convection in microtubes.

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