

# NUMERICAL SIMULATION OF TWO-PHASE SLUG FLOW USING A HYBRID CODE BASED ON SLUG TRACKING AND SLUG CAPTURING METHODOLOGIES

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**Abstract:** Slug flow is the prevailing flow pattern in oil and gas production. In this kind of flow the phases are unevenly distributed into two different structures: an elongated gas bubble and a liquid slug. The intermittent behavior of those structures affects the oil and gas production, plays a decisive role on equipment design and may ultimately cause operational problems. Different methodologies to simulate slug flows are found in the literature. In this article a methodology that simultaneously uses a slug capturing and a slug tracking models will be presented. The slug capturing code developed at NUEM uses a simplified two-fluid model which solves for the gas domain using a finite-difference scheme whereas the liquid domain is modeled similarly to the shallow-water equations. This code can simulate the transition from stratified to slug flow and the evolution of the slug flow structures. A slug tracking approach led to the development of another computer code that uses the continuity and momentum equations applied to the unit cell – a structure composed by a liquid slug followed by an elongated gas bubble. This model simulates slug flows at relatively low computational effort. For the new hybrid model, the two-fluid model will be used to simulate the slug initiation whereas the slug tracking model will be used to simulate the slug propagation of a horizontal flow in a 40-m long, 0.026-m diameter pipe, and the results will be compared with those obtained from the slug capturing code.

**Keywords:** Slug Flow, Slug Capturing, Slug Tracking, Two-phase flow

## 1. INTRODUCTION

The slug flow regime is characterized by an alternating flow of gas pockets, which fill the cross section almost completely, and aerated liquid slugs (see Fig. 1). This flow pattern has been studied in the past 70 years by many researchers, since it has large application in the gas, nuclear and aerospace industry.

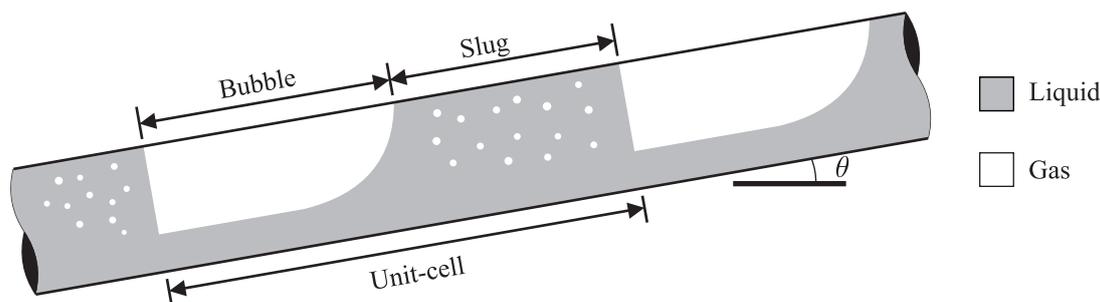


Figure 1: Physical model for slug flow pattern.

The first reported studies on the prediction of the flow characteristics assumed an intermittent passage of the structures, that is, the slug and elongated bubble. Experimental and analytical correlations were used and, due to their low computational effort, these methodologies are still in use. Wallis (1969) introduced the unit cell concept, considering each cell as being composed of one elongated bubble and one slug. The complete understanding of the dynamic behavior of one unit cell is extended to predict the overall flow behavior. Dukler and Hubbard (1975) studied the transition from stratified to slug flow, for horizontal and near horizontal pipes. Fernandes *et al.* (1983) also investigated this flow pattern, and proposed a methodology to predict the flow behavior in vertical pipes. Taitel and Barnea (1990a) and Taitel and Barnea (1990b) developed a model to predict the pressure gradient, profile and length of the elongated bubble.

Over the past two decades, due to the increasing computational capacity, transient models have been presented. Most of them use the two-fluid, drift-flux or slug tracking models. The two-fluid model was first introduced by Ishii (1975), and the conservation equations are separately applied to the liquid and gas phases in one-dimensional flows. In the drift-flux model, gas and liquid are not regarded as independent streams, and such model considers a slip velocity between

the phases. Both methodologies may use Eulerian or Lagrangian grids, and the convergence depends upon the mesh refinement. The first commercial package available was the software OLGA (Bendiksen *et al.*, 1991), based on the two-fluid model and still widely used by the oil and gas industry.

Renault (2007) presented a simplified two-fluid Lagrangian model, capable of simulating the transition from stratified to slug flow. The momentum equation for the gas is solved using a finite-difference scheme, whereas the momentum equation for the liquid, similar to the shallow water equations, is solved by means of an analytical solution for the Riemann problem. Based on this model, Conte (2014) and Conte *et al.* (2014) developed a code to simulate slug initiation, initiating from a stratified regime. Simulations for horizontal flows with a change of direction from inclined downward to horizontal flow were conducted, and the results were compared to experimental data, and a good agreement was found. The disadvantage of the model is the relatively high computational effort.

The slug tracking model uses the unit cell concept and a Lagrangian mesh. It was first presented by Taitel and Barnea (1993), using a simplified methodology for incompressible gas so as to predict the slug length. The following works investigated the effects in a hilly terrain pipeline (Zheng *et al.*, 1994) and the effects of gas compressibility (Taitel and Barnea, 1998). Franklin and Rosa (2004) presented a slug tracking model to simulate the slug propagation in horizontal sections, based on continuity equations for the unit cell, and momentum equation for the slug. Based on this methodology, Rodrigues (2009) developed a model to predict the flow characteristics in horizontal, inclined and vertical pipes.

The Rodrigues (2009) model applies the conservation equations to the slug flow structures to obtain two differential equations, where the main variables are the pressure in the elongated bubble and the liquid velocity in the slug. It is solved by a finite-difference scheme, and the other variables (film velocity, slug length and bubble length) are solved by means of auxiliary relationships. The boundaries are free to move and follow the unit cells. The model has a low computational effort, but needs the information about the unit cells that enter the pipe. Rosa *et al.* (2015) presented a slug tracking model including the advection term and the induced slug oscillations.

The purpose of this work is to present a hybrid methodology that is capable of using the two-fluid Lagrangian model to generate slug flow (Conte, 2014), and to follow the unit-cells using the slug tracking model developed by Rodrigues (2009). This new hybrid code is able to simultaneously use the two previous codes developed at NUEM.

## 2. MATHEMATICAL MODEL

This section presents the slug capturing model (Conte, 2014; Renault, 2007) and the slug tracking model (Rodrigues, 2009). The former will be used to simulate the slug initiation, whereas the latter, due to its low computational cost, will be used to simulate the propagation of the flow. Closure relationships will also be presented.

### 2.1 Slug capturing methodology

A simplified two fluid model uses the continuity and momentum equations for the gas and the liquid. The goal is to model the transition from stratified to slug flow (see Fig. 2). The liquid is incompressible and the gas is ideal. The temperature will be assumed constant with no mass change.

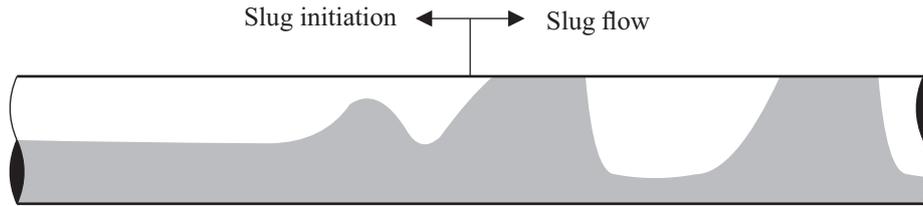


Figure 2: Transition from stratified to slug flow.

The following equations describe the one-dimensional flow:

$$\frac{\partial}{\partial t}(R_L) + \frac{\partial}{\partial x}(R_L U_L) = 0 \quad (1)$$

$$\frac{\partial}{\partial t}(\rho_G R_G) + \frac{\partial}{\partial x}(\rho_G R_G U_G) = 0 \quad (2)$$

$$\frac{\partial}{\partial t}(R_L U_L) + \frac{\partial}{\partial x} \left( R_L U_L^2 + \frac{1}{2} \kappa R_L^2 \right) = \frac{R_L}{\rho_L} F(R_L, U_L, U_G) \quad (3)$$

$$\frac{\partial}{\partial t}(\rho_G R_G U_G) + \frac{\partial}{\partial x}(\rho_G R_G U_G^2) = \left( -\frac{\tau_G S_G}{A} - \frac{\tau_I S_I}{A} - \rho_G g R_G \sin \theta - R_G \frac{\partial p}{\partial x} \right) \quad (4)$$

where  $F$  is the volumetric force:

$$F = -\frac{\tau_L S_L}{A_L} + \frac{\tau_G S_G}{A_G} + \tau_I S_I \left( \frac{1}{A_L} + \frac{1}{A_G} \right) - (\rho_L - \rho_G)g \sin \theta \quad (5)$$

and

$$\kappa = \frac{\rho_L - \rho_G}{\rho_L} g \cos \theta \frac{A}{\frac{dA_L}{dh_L}} - \frac{1}{R_G} (U_G - U_L)^2 \quad (6)$$

The symbol  $\rho$  denotes the density,  $t$  is the time,  $g$  is the gravity,  $R$  is the phase volume fraction,  $U$  is the velocity,  $\tau$  is the shear stress,  $p$  is the pressure,  $A$  is the cross-sectional area and  $S$  is the wetted perimeter. The subscript  $L$  denotes the liquid phase,  $G$  the gas phase and  $I$  the gas-liquid interface. Equations 1 and 2 represent the continuity, and Eqs. (3) and (4) represent the momentum.

## 2.2 Slug tracking methodology

The slug tracking model uses the continuity equation for the gas and the liquid in the elongated bubble and liquid slug, and the momentum equation for the liquid in the slug. For every step of time it is possible to obtain the average velocity of the liquid in the slug,  $U_{LS,i}$ , the pressure in the elongated bubble,  $p_{GB,i}$ , the velocity of the liquid,  $U_{LB,i}$ , the translational velocity of the bubble,  $U_{T,i}$ , and the positions of the boundaries,  $x_i$  and  $y_i$ , as indicated in Fig. 3.

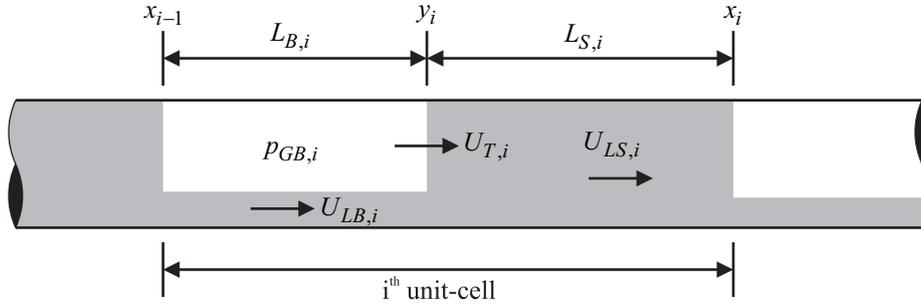


Figure 3: Unit-cell for the slug tracking model.

Using an integral analysis, two differential equations are obtained:

$$U_{LS,i-1} - U_{LS,i} = \frac{1 - R_{LS,i}}{R_{LS,i}} U_{D,i} - \frac{1 - R_{LS,i-1}}{R_{LS,i-1}} U_{D,i-1} + \frac{dp_{GB,i}}{dt} \left[ L_{B,i} \frac{(1 - R_{LB,i})}{p_{GB,i}} + L_{S,i} \frac{(1 - R_{LS,i})}{p_{GB,i}} + L_{S,i-1} \frac{(1 - R_{LS,i-1})}{p_{GB,i-1}} \right] \quad (7)$$

$$p_{GB,i} - p_{GB,i+1} = \rho_L R_{LS,i} L_{S,i} \frac{dU_{LS,i}}{dt} + 2C_{LS,i} \rho_L \frac{L_{S,i}}{D} U_{LS,i}^2 + 2C_{LB,i+1} \rho_L S_{LB,i+1} \frac{L_{B,i+1}}{D} U_{LB,i}^2 + (R_{LS,i} L_{S,i} + R_{LB,i+1} L_{B,i+1}) \rho_L g \sin \theta + \left[ \frac{L_{S,i}}{2} \left( \frac{dx_i}{dt} + \frac{dy_i}{dt} \right) - L_{S,i} U_{LS,i} \right] \rho_L \frac{dR_{LS,i}}{dt} \quad (8)$$

Equation 7 represents the mass balance (continuity), and Eq. (8) represents the momentum balance. The bubble nose has the following translational velocity:

$$\frac{dy_i}{dt} = U_{T,i} \quad (9)$$

Using a mass balance across the elongated bubble, it is possible to track the back border of the bubble down:

$$\frac{dx_{i-1}}{dt} = \frac{(R_{GB,i} - R_{GS,i}) \frac{dy_i}{dt} + \frac{L_{B,i} R_{GB,i}}{p_{GB,i}} \frac{dp_{GB,i}}{dt} + R_{GS,i} U_{GS,i} - R_{GS,i-1} U_{GS,i-1}}{R_{GB,i} - R_{GS,i-1}} + \frac{\left( \frac{L_{S,i} R_{GS,i}}{2 p_{GB,i}} + \frac{L_{S,i-1} R_{GS,i-1}}{2 p_{GB,i-1}} \right) \frac{dp_{GB,i}}{dt} - \frac{L_{S,i}}{2} \frac{dR_{LS,i}}{dt} - \frac{L_{S,i-1}}{2} \frac{dR_{LS,i-1}}{dt}}{R_{GB,i} - R_{GS,i-1}} \quad (10)$$

### 2.3 Closure relationships

The shear stress,  $\tau$ , and the shear stress coefficient,  $C_f$ , are calculated as:

$$\tau = \frac{1}{2} C_f \rho \|U\| U, \quad (11)$$

$$C_f = \max(16Re^{-1}, 0.079Re^{-0.25}), \quad (12)$$

where  $Re = \frac{\rho U D}{\mu}$  is the Reynolds number. For the two-fluid model, it is assumed that  $C_{f,I} = C_{f,G}$ . The slug frequency,  $f_S$ , is the number of slugs passing through a specific point along the pipeline per unit time. Considering the time that a unit cell takes to travel through a position is  $\Delta t_{UC}$ , the cell frequency is defined as:

$$f_{UC} = \frac{1}{\Delta t_{UC}} \quad (13)$$

The translational velocity of the bubble,  $U_T$ , is dependent on the mixture velocity of the slug,  $U_M$ , and on the drift velocity (Bendiksen, 1984). Also, the wake effect at the slug,  $\hbar$ , causes the acceleration of the bubble at the rear of that slug. The translational velocity is calculated as:

$$U_T = (C_0 U_M + U_D)(1 + \hbar) \quad (14)$$

The wake effect causes a pressure loss in the slug region, causing the bubble at the rear of that slug to accelerate. However, for long slugs, the flow may reorganize itself inside that slug. Some authors have studied the effect of the wake (Moïssis and Griffith, 1962; Grenier, 1997; Rodrigues, 2009) on the overall results. It is modeled as:

$$\hbar = a_W \exp\left(-b_W \frac{L_S}{D}\right) \quad (15)$$

The coefficients of the Eq. (15) will be  $a_W = 0.4$  and  $b_W = 1.0$ , as proposed by Rodrigues (2009) for water-air simulations.

## 3. NUMERICAL MODELING

In this section, the numerical solution procedures applied in the slug capturing and slug tracking models will be presented. Also, a methodology that simultaneously employ both models will be proposed.

### 3.1 Slug capturing solution

The Lagrangian solution uses a grid which is divided in cells (see Fig. 4), and each cell can be either a stratified section or a slug. An elongated bubble may be divided into several sections. The boundaries are free to move, but each section must have a maximum length,  $L_{max}$ , in order to maintain mesh refinement. It will be considered  $L_{max} = 2\Delta x$ , where  $\Delta x$  is the initial refinement upon the simulation startup.

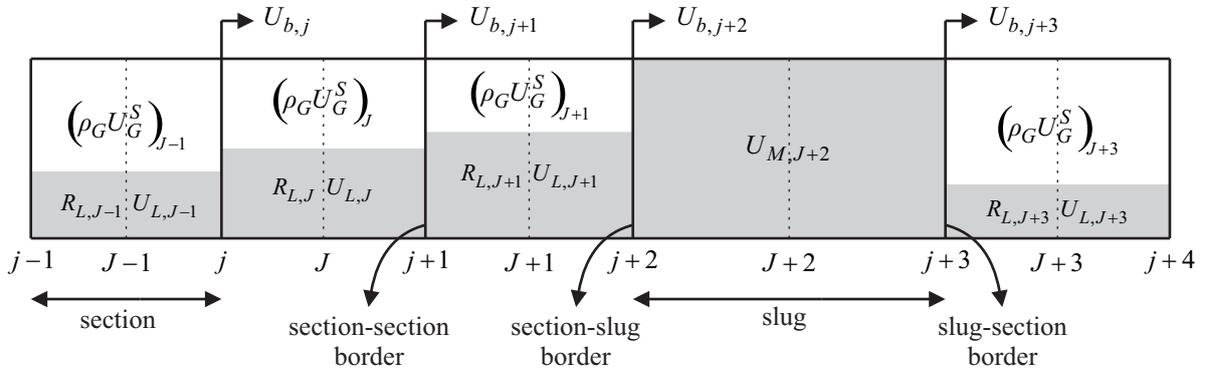


Figure 4: Computational grid for the slug capturing model.

A section  $J$  has the gas flux,  $(\rho_G U_G^S)_J$ , the liquid velocity,  $U_L$ , and the liquid hold-up,  $R_L$ . The cell  $J+2$  of a slug, for example, has the mixture velocity  $U_{M,J+2}$ . Each border  $j$  has the pressure,  $p_j$  and the border velocity,  $U_{b,j}$ . The slug is modeled as non-aerated. The gas equation is solved using a finite-difference scheme, and the liquid is solved using an analytical solution for the Riemann problem.

### 3.1.1 Gas solution

The Eq. (2) must be applied to every section. On doing that, the relationship between pressure and gas flux is obtained:

$$p_j^{n+1} = \chi_j^n \left[ (\rho_G U_G^S)_{J-1}^{n+1} - (\rho_G U_G^S)_J^{n+1} \right] + \psi_j^n \quad (16)$$

where:

$$\chi_j^n = \frac{\Delta t}{R_{G,j}^n L_j^n \left( \frac{d\rho_G}{dp} \right)_j} \quad (17)$$

$$\psi_j^n = p_j^n + \chi_j^n \left[ R_{G,J-1}^n U_{b,J-1}^n (\rho_{G,j}^n - \rho_{G,J-1}^n) + R_{G,J}^n U_{b,J}^n (\rho_{G,J}^n - \rho_{G,j}^n) \right] \quad (18)$$

Special care must be taken at the section-slug or slug-section borders, because there is no gas flux at the slug. Also, Eq. (4) is applied to every section, and a finite-difference scheme is used:

$$a_J^n (\rho_G U_G^S)_J^{n+1} = b_J^n (\rho_G U_G^S)_{J+1}^{n+1} + c_J^n (\rho_G U_G^S)_{J-1}^{n+1} + d_J^n \quad (19)$$

where:

$$a_J^n = \frac{1}{\Delta t} + b_J^n + c_J^n + \frac{1}{2} \left( \frac{S_G}{A_G} C_{f,G} \|U_G\| \right)_J^n + \frac{1}{2} \left( \frac{S_I}{A_G} C_{f,I} \|U_G - U_L\| \right)_J^n \quad (20)$$

$$b_J^n = -\frac{1}{L_J^n} \min(U_{G,j+1}^n - U_{b,j+1}^n, 0) + \frac{R_{G,J}^n}{L_J^n} \chi_{j+1}^n \quad (21)$$

$$c_J^n = \frac{1}{L_J^n} \max(U_{G,j}^n - U_{b,j}^n, 0) + \frac{R_{G,J}^n}{L_J^n} \chi_j^n \quad (22)$$

$$d_J^n = \left[ \begin{array}{l} \frac{1}{\Delta t} (\rho_G U_G^S)_J^n + \frac{R_{G,J}^n}{L_J^n} (\psi_j^n - \psi_{j+1}^n) \\ + \frac{1}{2} \left( \frac{S_I}{A} C_{f,I} \rho_G U_L \|U_G - U_L\| \right)_J^n - \rho_{G,J}^n g R_{G,J}^n \sin \theta \end{array} \right] \quad (23)$$

while the Eq. (3) is applied only for slugs, and the linearized system is obtained for the slug  $J+2$  (see Fig. 4):

$$a_{J+2}^n U_{M,J+2}^{n+1} = b_{J+2}^n (\rho_G U_G^S)_{J+3}^{n+1} + c_{J+2}^n (\rho_G U_G^S)_{J+1}^{n+1} + d_{J+2}^n \quad (24)$$

where:

$$a_{J+2}^n = \rho_L \frac{L_{J+2}^n}{\Delta t} + \rho_L \frac{R_{L,J+3}^n}{1 - R_{L,J+3}^n} (U_{M,J+2}^n - U_{L,J+1}^n) + \frac{C_{L,J+2}^n}{2D} L_{J+2}^n \|J_{J+2}^n + \rho_{G,j+3}^n \chi_{j+3}^n + \rho_{G,j+2}^n \chi_{j+2}^n \quad (25)$$

$$b_{J+2}^n = \chi_{j+3}^n \quad (26)$$

$$c_{J+2}^n = \chi_{j+2}^n \quad (27)$$

$$d_{J+3}^n = \left[ \begin{array}{l} \psi_{j+2}^n - \psi_{j+3}^n - g\rho_L \cos \theta (h_{L,R} - h_{L,L}) - g\rho_L L_{j+2}^n \sin \theta + \\ \rho_L \frac{L_{j+2}^n}{\Delta t} U_{M,J+2}^n + \rho_L \frac{R_{L,J+3}^n}{1-R_{L,J+3}^n} (U_{M,J+2}^n - U_{L,J+3}^n) U_{L,J+3}^n \end{array} \right] \quad (28)$$

Occasionally, for some time steps, the observed mass of gas in a bubble may be different from the mass at the time the bubble developed. Therefore, the pressure must be corrected in order to satisfy the gas mass conservation. The following correction must be applied to every section of the bubble:

$$corr = \frac{(m_G^0 - m_G^n)}{V_G^n} \left[ \frac{\partial \rho_G}{\partial p} \right]^{-1} \quad (29)$$

A system of equations is obtained to solve for the gas phase:  $a_j^n X_j^{n+1} = b_j^n X_{j+1}^{n+1} + c_j^n X_{j-1}^{n+1} + d_j^n$ , where  $X_j$  is either the gas flux  $(\rho_G U_G^S)_j$  or the mixture velocity at the slug,  $U_{M,j}$ . The TDMA (Tridiagonal Matrix Algorithm) method is used to solve the system.

### 3.1.2 Liquid solution

In order to solve the liquid equations, a solution for the Riemann equation is used, since the equations are similar to the shallow water equations. The first step consists in considering only the volumetric forces, hence the equation to be solved becomes:

$$\frac{\partial}{\partial t}(R_L U_L) = \frac{R_L}{\rho_L} F(U_L, R_L, U_G, R_G) \quad (30)$$

The intermediate solution for the liquid,  $U_L^{n+1/2}$ , is obtained by a finite-difference scheme for the Eq. (30). The next step is the solution for the Riemann problem, represented in Fig. 5.

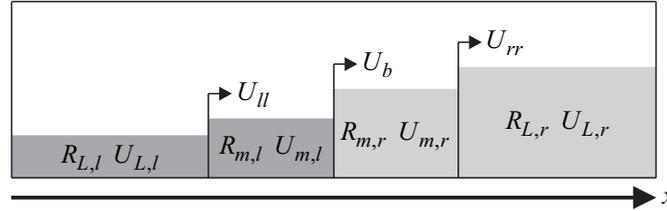


Figure 5: Solution of the Riemann problem .

Figure 6 represents the steps for the film solution.

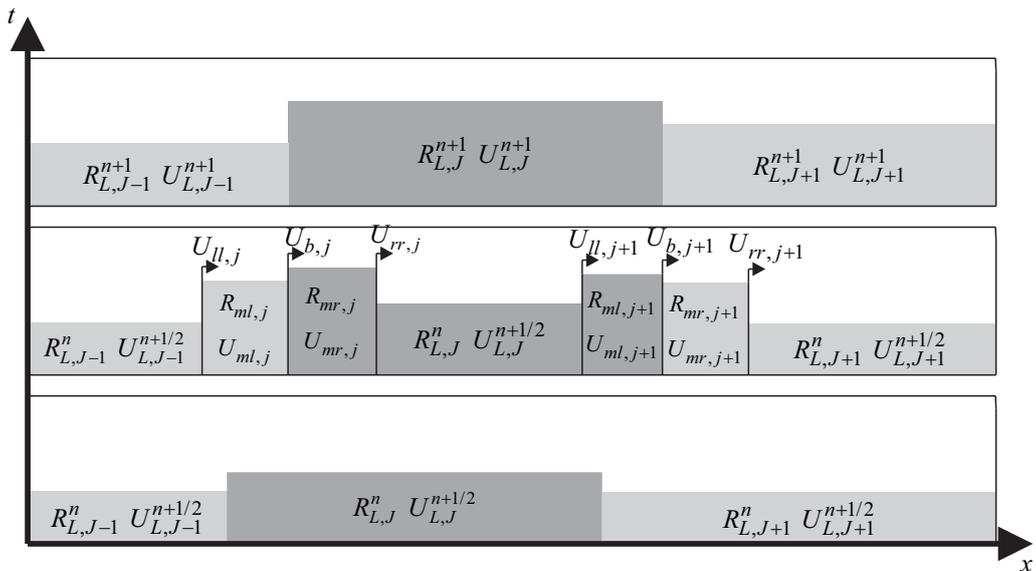


Figure 6: Liquid solution.

A detailed solution for the liquid phase is described in (Conte, 2014; Renault, 2007)

### 3.2 Slug tracking solution

In this section, the numerical solution of the slug-tracking model will be presented. Pressure and mixture velocity within the slug are obtained by solving Eqs. (7) and (8), while the position of the borders are computed in the next step using auxiliary relationships.

The Eqs. (7) and (8) are solved by means of a semi-implicit difference scheme. It is necessary to define  $H_{GB,i} = p_{GB,i}/\rho_L$ , for the sake of a better numerical stability. For non-aerated slugs and constant liquid hold-ups:

$$\begin{cases} a_i H_{GB,i}^{n+1} = -U_{LS,i}^{n+1} + U_{LS,i-1}^{n+1} + b_i \\ c_i U_{LS,i}^{n+1} = -H_{GB,i+1}^{n+1} + H_{GB,i}^{n+1} + d_i \end{cases} \quad (31)$$

where

$$a_i = \frac{2L_{B,i}^n R_{GB,i}}{H_{GB,i}^n \Delta t}, \quad (32)$$

$$b_i = U_{LS,i-1}^n - U_{LS,i}^n + \frac{2L_{B,i}^n R_{GB,i}}{\Delta t}, \quad (33)$$

$$c_i = \frac{2L_{S,i}^n}{\Delta t} + 4C_{LS,i}^m \frac{L_{S,i}^n}{D} U_{LS,i}^n, \quad (34)$$

$$d_i = H_{GB,i}^n - H_{GB,i+1}^n + \frac{2L_{S,i}^n U_{LS,i}^n}{\Delta t} - \frac{2}{\rho_L} (\Delta p_{S,i+1}^n + \Delta p_{G,i}^n). \quad (35)$$

The parameter  $\Delta p_{S,i+1}^n$  is the pressure loss within the film, and  $\Delta p_{G,i}^n$  is the gravitational effect over the pressure gradient. The system of equations is solved for  $H_{GB,i}$  and  $U_{LS,i}$  using the TDMA method.

### 3.3 Slug capturing - slug tracking coupling

The purpose of this work is to use an hybrid code to simulate slug flows (see Fig. 7). The slug capturing model presented in subsection 2.1 will be used to simulate the transition from stratified to slug flow, and the slug tracking model presented in subsection 2.2 will be used to simulate the propagation of the slugs. At the beginning, only the slug capturing model is used to simulate the flow, and after seven slugs are generated, the slug capturing and slug tracking models will work simultaneously.

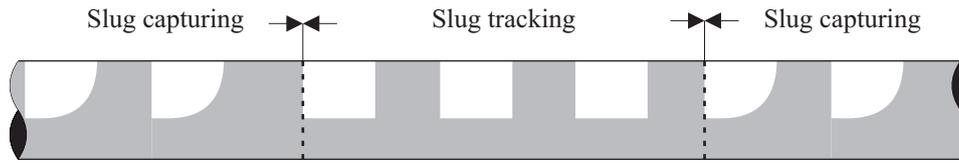


Figure 7: Hybrid methodology.

The models interface in such a way that the information that one model needs as a boundary condition will be provided by the other model. Figure 8 represents the coupling of the models.

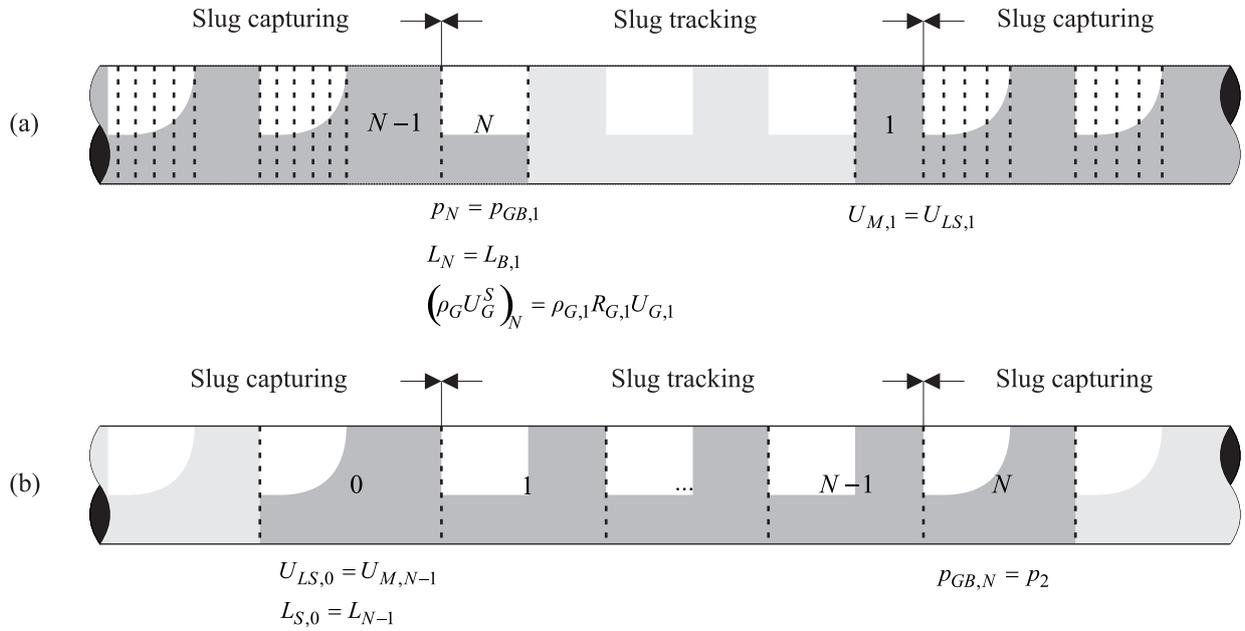


Figure 8: Coupling of the models.

The interface between the models will always be placed in a border between the front of a slug and the back of a bubble, meaning that the borders can move freely. The procedures are:

a. For the slug capturing model (see Fig. 8a):

- The pressure in the last cell will be the pressure in the first bubble of the slug tracking model,  $p_N = p_{GB,1}$ ;
- The length of the last cell will be the length of the first bubble of the slug tracking model,  $L_N = L_{B,1}$ ;
- The gas flux is also computed from the information supplied by the slug tracking model,  $(\rho_G U_G^S)_N = \rho_{G,1} R_{G,1} U_{G,1}$ .
- The mixture velocity at the first slug is taken from the last cell of the slug tracking model,  $U_{M,1} = U_{LS,N-1}$ .

b. For the slug tracking model (see Fig. 8b):

- The lengths of the bubble and of the slug, the pressure and the slug velocity in the cell are obtained from the last cell of the slug capturing model,  $p_{GB,0} = p_N$ ;
- The boundary condition at the exit (pressure) is taken from the second cell of the slug capturing model,  $p_{GB,N} = p_2$ .

As a initial condition, the flow is assumed as stratified. Using the slug capturing model, the transition to slug flow will be predicted. After seven unit cells enter the pipe, the slug tracking model will be used, as shown in Fig. 9.

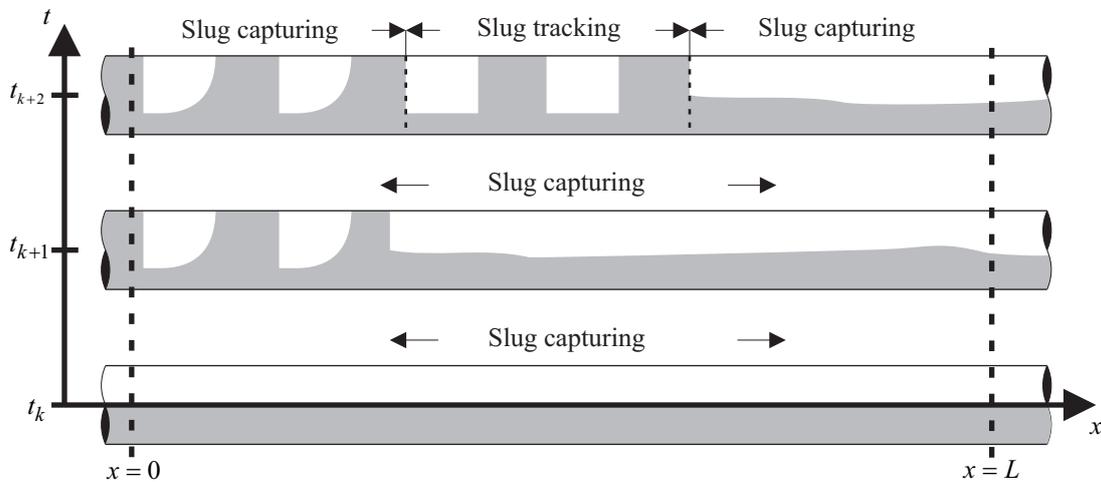


Figure 9: Stratified flow as a initial condition.

#### 4. RESULTS

Two simulations were carried out: one using the hybrid code and another using the slug capturing code. The results were compared. The velocity pair  $U_G^S = 0.3 [m/s]$  and  $U_L^S = 0.7 [m/s]$  was selected, in a horizontal pipe of length  $L = 40 [m]$  and diameter  $D = 0.026 [m]$ . The time step is  $\Delta t = 0.005 [s]$  and the initial mesh size is  $\Delta x = 0.01 [m]$ . As a initial condition, stratified flow with liquid hold-up  $R_L = 0.5$  was adopted. As a boundary condition, the pressure at the pipe outlet is  $p = 96 [kPa]$ . It was considered water-air flow at ambient temperature.

Figure 10 illustrates the average properties of the flow (bubble length, slug length, slug frequency and pressure) along the pipe.

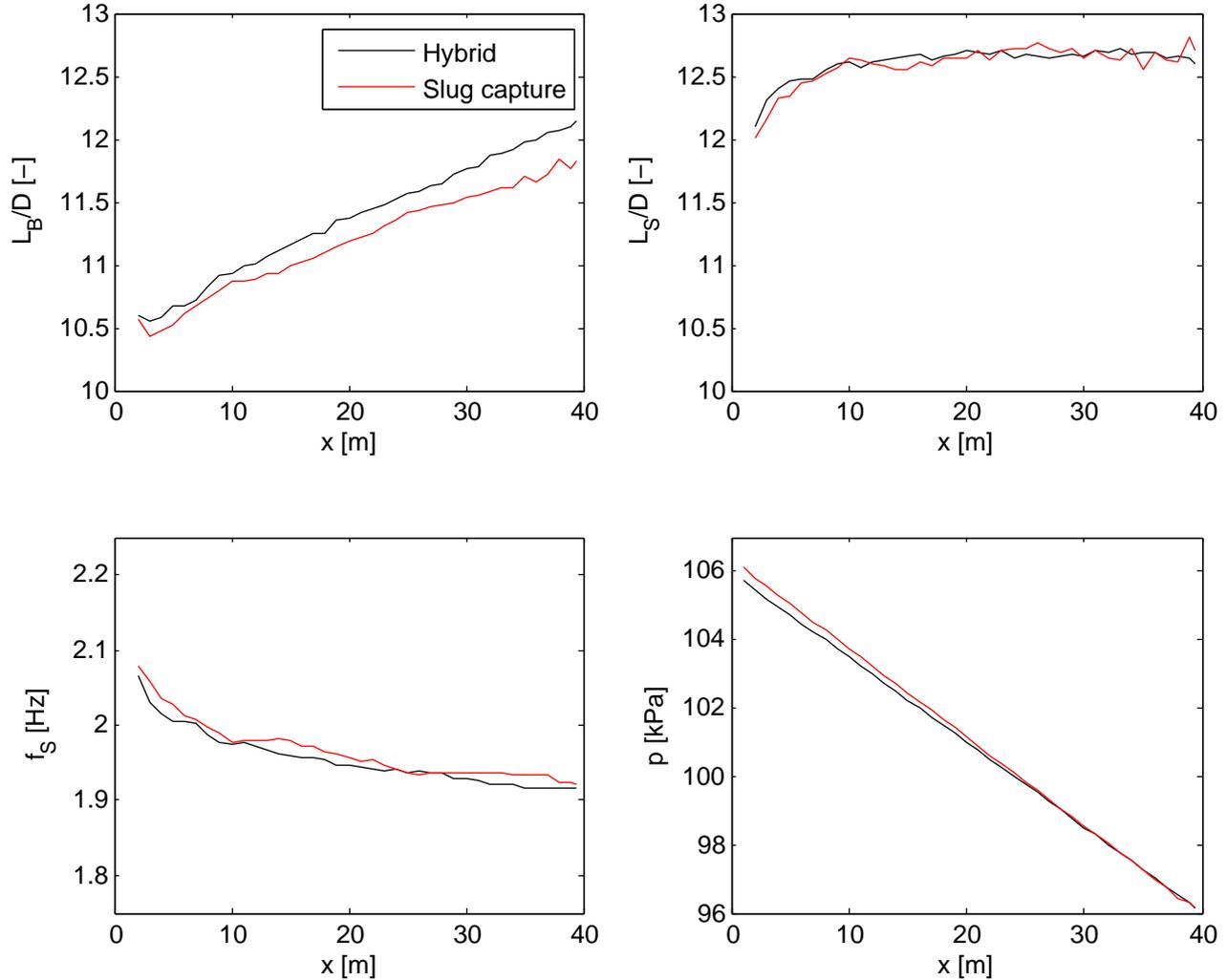
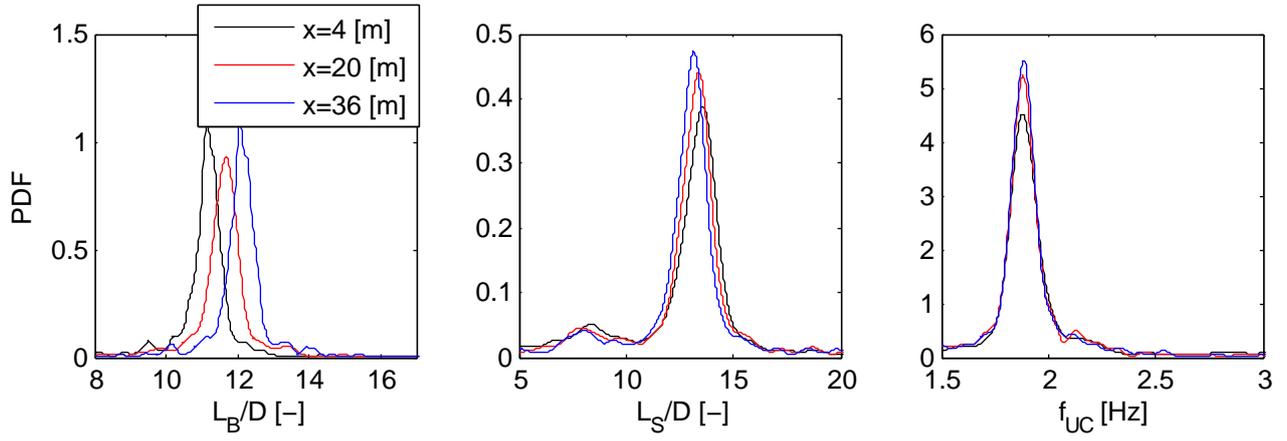


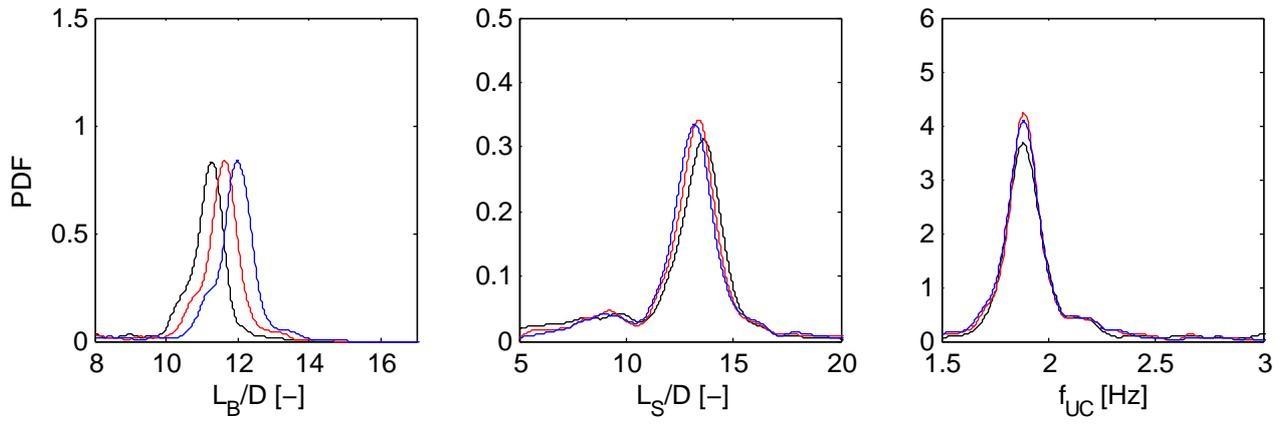
Figure 10: Average distribution of the flow properties along the pipe.

There was good agreement between the two codes. The bubbles have an average length of  $10.5D$  when it is generated near the entrance of the pipe, and it gradually increased to around  $12D$  at the outlet of the pipe. It is expected due to coalescence and gas expansion. The slug length has a slightly raise from  $12D$  to approximately  $12.6D$ . Because the unit cell length tends to increase, the slug frequency has a modestly decline along the pipe. The pressure gradient is constant. The results show that the slug tracking methodology produces very similar results when compared to the slug capturing methodology, even though it does not use a refined grid as does the two-fluid model.

To compare the results from the two methodologies, a virtual probe was placed in three positions. Figure 11 represents the probability density functions (PDF) of the properties (bubble length, slug length and unit-cell frequency). While Figure 11a shows the PDF distributions for the hybrid methodology, Figure 11b shows the distributions for the slug capturing methodology.



(a) Hybrid code



(b) Slug capture code

Figure 11: PDF distribution at the virtual probes located at  $x = 4.0$  [m],  $x = 20.0$  [m] and  $x = 36.0$  [m].

The PDF distribution shows that the bubble length increases along the pipe, as expected. The slug length and unit-cell frequency, on the other hand, have marginal changes. Again, there was reasonably good agreement among the two methodologies.

The code was implemented in Fortran 90, using object-oriented programming. A Intel(R) Core(TM) i7 4GHz computer performed the simulations. Table 1 shows the time for simulation and the number of iterations. The two simulations stopped after 600 bubbles exited the pipe, starting to count after 80 [s].

Table 1: Computational cost for simulations

	Hybrid code	Slug capturing code
Simulation time [min]	3.803	9.085
Number of iterations	78,682	78,679

The hybrid code uses the slug-tracking methodology to simulate the propagation of slugs. The hybrid code was approximately 58% faster since the slug tracking model has a very low computational cost when compared to the slug capturing model, as observed in Tab. 1. Finally, Fig. 12 shows a screenshot of the simulation at the instant  $t = 40$  [s].

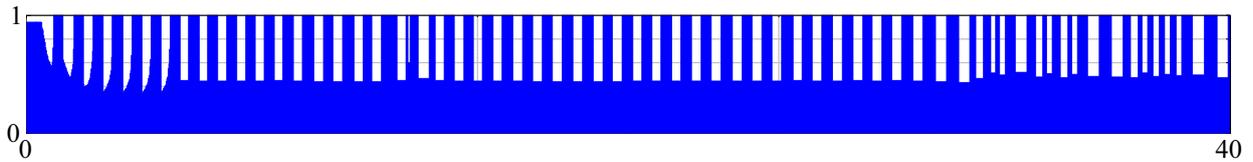


Figure 12: Liquid hold-up distribution at the instant  $t = 40$  [s].

It is important to notice that the first seven slugs near the entrance of the pipe are linked to the slug capturing model, and the remaining ones are linked to the slug tracking model.

## 5. CONCLUSIONS

In this work, it was proposed a coupling methodology for a slug capturing and a slug tracking models. Overall, the simulations have demonstrated that there is a good concordance between the hybrid code and slug capturing code. Also, using the hybrid code it was possible to significantly reduce the computational cost of the simulations without compromising the results. In future works, more simulations for larger pipes, different inclinations and pairs of velocities will be performed. Change of direction shall be implemented in the hybrid code as well.

## 6. ACKNOWLEDGMENTS

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## 8. RESPONSIBILITY NOTICE

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