ANALYSIS OF MATHEMATICAL CORRELATION FOR INDIRECT DETERMINATION OF LEVEL IN OIL WELLS

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Abstract. *Some operations as the indirect determination of oil level in annulus of wells, and level measurement in the annulus space, between well casing and production tubing, are common tasks under the oil production engineering. Specially to obtain necessary data to design lifting systems. The SONOLOG method is devoted to this task, however it demands productions interruption, and the allocation of a considerable handwork. Those factors make this technique relatively expensive. Thereby the objective of this work is to study a theoretical methodology based in mass balance found in literature that aimed to an indirect determination of dynamic level in annulus space of oil well. With this propose, a detailed study on this model was done, using a central composite design technique and ANOVA (Analysis of Variance) to evaluate the sensitivity and influence of the model's variables in the result. This methodology also allows us to determine ranges of dependents variables that minimize or maximize the response. It also makes this work convenient to understand the limits of this models that might be applied to estimate the fluid level in the annulus of production wells. The application of experimental design methodology and statistical analysis shows that the variable of greatest effect on the well level is the gas flow rate through the annular, then the gas compressibility. It is also noted that the combined effect of these two variables is thirdly in influence on the studied model, it is noted that this effect is more significant than the effect of temperature alone.*
Keywords*: Well's level, mass balance, desing technique, statistical analysis.*

1. NOMENCLATURE

q_{m1} Mass rate of annular gas influx, lbs/min;
q_{m2} Mass rate of gas leaving the annulus, lbs/min;
m Mass of gas contained in the annulus, lbs;
t Time, minutes;
q_m Mass flow rate, lbs/min;
q Volumetric flow rate at standard conditions, cu ft/min;
q_1, q_2 Gas volumetric rates, M scf/d;
p_{sc} Standard pressure, 14.65 psia;
M Molecular weight of the gas, lbs/mole;
T_{sc} Standard temperature, 520 degrees Rankine;
R Universal gas constant;
p Pressure, psia;
V Annular gas volume, cu. ft.;
Z Gas deviation factor.

2. INTRODUCTION

Regarding the oil lift system, it is designed according to characteristics of the reservoir, completion system, and desired volume of production. One of the variables of this project is downhole pressure, used to scale or manage the lift system. Among required parameters of downhole pressure calculation process, the well level has a great importance. Thereby, well level is understood as the distance between the wellhead and the gas liquid interface, considering the well annular region filled with gas.

Figure 1 in a schematic representation of an environment of an oil well indicating its level L, gas flow rate at the wellhead valve q_2, gas flow rate that gives off from gas liquid solution q_1, and the annular space filled with gas that account a V volume.

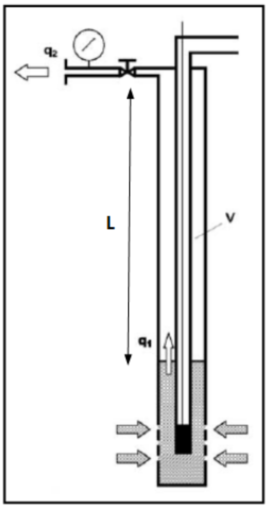

Figure 1. Simplified representation of a production well. (Modified by: TAKÁSCS, 2003)

The objective of this work is to investigate a theoretical methodology presented by Takáscs (2003) that allows the well level estimation, and to evaluate the significant effects of the variables, as well as its response sensitivity and the ranges of independent variables that maximize or minimize the solution.

3. LITERATURE REVIEW

The authors Alexander (1976), Alexander (1999) and Hasan & Kabir (1985) showed that the mass balance technique may be applied to determine the downhole pressure in oil wells.

The technique developed by Alexander (1976) is based on a mass balance of gas that enters and leaves the annular space, as shown in Fig. 1. This approach returns a theoretical value of liquid level in the well annulus. The calculation uses data from two pressures growth test performed in the annular space at the wellhead allowing the measurement of the gas leaving the annular and its temperature, as well as the others surface properties of collected fluids.

3.1. Mathematical modeling

The well model schematized above in Fig. 1 does not flow naturally, and L indicates the dynamic level in annulus space of the well, that are sometime applied to downhole pressure estimation.

In this scheme q_2 indicates the gas flow rate that leaves the annular wellhead valve, and the variable V represents the gas volume contained within the annular space. In the control volume is applied a mass balance methodology to the gas entering, leaving and remaining the annular space. Then the annulus well level has been obtained by mathematical manipulation. The dependent variable were written as function of control volume geometric properties, pressure growth rate, temperature, pressure and fluid properties, measured at wellhead condition.

In the model developed by Alexander *apud* Takács (2003) there are two main assumptions, the first one is that the gas volume at the annulus space does not suffer substantial variations within a short time interval while the wellhead surface valve is opened (shut in period), it is known as constant volume approach. Other assumption is a consideration that the downhole pressure remains constant in a short time interval.

Following, it will be exposed to the development of the mathematical model to the technique developed by Alexander (1976) and upgraded by Takács (2003) exposed on his Sucker-Rod Pumping Manual.

3.1.1 Mathematical model development

Equation (1) represents a mass balance between the gas that enters and leaves the annular space, including a temporal variation of the mass of gas contained in the annular space.

$$q_{m1} - q_{m2} = \frac{d}{dt} m \tag{1}$$

The gas flow mass has been expressed by Eq. (2) according a state equation, as seen behind:

$$q_m = \frac{p_{sc} M}{RT_{sc}} \tag{2}$$

Then the total mass of the system is determined by the following relationship:

$$m = \frac{pVM}{RTZ} \tag{3}$$

In order to determine the final form of Eq. (1), it is assumes that the temperature T, and the compressibility factor of the gas Z (deviation factor) are independent of time. Thus, substituting Eq. (2) and Eq. (3) in to Eq. (1) it is obtain the final form or mass balance of the gas in the annulus space.

$$q_1 - q_2 = \frac{50.8}{TZ}\left(V\frac{dp}{dt} + \frac{dv}{dt} \right) \tag{4}$$

Applying the assumption of constant volume in annular space it allows to neglect the change in gas annulus volume thus $\frac{dV}{dt}$ becomes zero. By closing the wellhead valve, q_2 also becomes zero, and the Eq. (4) becomes the following:

$$q_1 = \frac{50.8}{TZ} V\left(\frac{dp}{dt} \right)_1 \tag{5}$$

The term $(\frac{dp}{dt})_1$ is the pressure growth rate in the annular space near the wellhead. This test is performed as follows: first it is measured the pressure increase in the annular space near the wellhead, when the valve is closed for about 10 min. In the next step, the annulus wellhead valve is turned open, and the measurement of the pressure drop rate $(\frac{dp}{dt})_2$ begins. Applying the assumption $\frac{dV}{dt}$=0, the Eq. (4) becomes:

$$q_1 - q_2 = \frac{50.8}{TZ} V\left(\frac{dp}{dt} \right)_2 \tag{6}$$

Manipulating the Eq. (5) and Eq. (6), a mathematical relation that gives the volume of gas contained in the annular space V in cu.ft is obtained.

$$V = \frac{q_2 TZ}{50.8\left[\left(\frac{dp}{dt} \right)_1 - \left(\frac{dp}{dt} \right)_2 \right]} \tag{7}$$

Where q_2 is the gas vent rate from annulus in M scf/d, T is the gas average absolute temperature in annulus measured in degrees Rankine, Z is the average gas deviation factor in annulus volume, $(dp/dt)_1$ is the time rate of change of annulus wellhead pressure during casing shut-in in psia/minutes, and $(dp/dt)_2$ represents the time rate of change of annulus wellhead pressure during casing vent period measured in psia/min.

Since the volume of gas in the annular V is obtained by the average of pressure and average of temperature in the equation above, however, if surfaces data is used, a correction factor to the calculated volume is necessary. Takács (2003) proposes that correction, seen in Eq. (8) below.

$$V_{corrected} = V\frac{p_{surf}}{p_{avg}} \tag{8}$$

The pressure rate in Eq. (7) has been derived from the equation used to calculate the pressure distribution in a static column of gas, and it allows a correction for the volume of gas.

$$\frac{p_{surf}}{p_{avg}} = \frac{L\left[0.001877\frac{S_pG_r}{T_{avg}}\right]}{\exp\left[0.001877L\frac{S_pG_r}{T_{avg}Z_{avg}}\right]-1} \quad (9)$$

Where L is the non-corrected depth to liquid level in ft, S_pG_r is the gas specific gravity, T_{avg} absolute temperature average on the gas column in Rankine; and Z_{avg} is the gas deviation factor at average conditions.

Finally the level in the annular space is then obtained by the relationship below:

$$L = \frac{V}{5.61C} \quad (10)$$

The variable *C* is the annular space capacity in bbl/ft. According to Takács (2003), the assumption of constant downhole pressure results in a calculated volume smaller than the volume calculated with the constant volume assumption. Thereby he proposes the following correction in the correlation:

$$V' = V - 5.61p\frac{C}{grad} \quad (11)$$

Where V' is the volume of gas in cu.ft calculated with constant downhole pressure assumption, and the term grad is the hydrostatic gradient in the liquid column in psi/ft and p is the surface pressure in psia.

4. METODOLOGY

According to this study objectives, the methodology includes a sensitivity analysis of the variables in the mathematical model presented by Takács (2003), based on the methodology developed and tested by Alexandre (1976) and Alexandre (1999). Thereby the model is implemented in a spreadsheet, allowing the simulation of the annulus well level by modifying the variables.

With this purpose, it is constructed a central composite design $2^{(5-1)}$ with one central point and two blocks (BOX, Hunter & Hunter, 2015). The studied factors (independents variables) are gas flow rate leaving the exit valve Fig. 1, gas pressure, temperature, gas compressibility (deviation factor) and gas specific gravity. All these variable are measured at wellhead annulus space. Thereby, 30 simulations is performed according to the central composite delineation for the dependent variable annulus well level.

Statistic methods as the analysis are used, as the ANOVA (Analysis of Variance) table and Pareto chart to study the effects and significance of the variables isolated and/or combined in the model response (calculated level). The response surface methodology is applied according to the model two way ANOVA (Montgomery, 2009) to graphically identify the behavior, and the sensitivity of the investigated dependents variables in the response variable. It is also applied to the rages that maximize or minimize the solution. At this stage a statistical software is used to the implementation of these models.

In the delineation described above are considered as factors the following variables: gas flow rate, compressibility, specific gravity, temperature and pressure of the gas at wellhead. The independent variables and their respective calculated values of fluid levels according to the methodology presented by Takács (2003), are presented in Tab. 1.

Table 1. Experimental Design

Runs	Blocks	Flow rate [m³/d]	Z [-]	Well Head Temperature [ºC]	Well Head Pressure [KPa]	GE [-]	Liquid level [m]
1	1	50,00	0,50	21,00	101,30	0,80	167,79
2	1	50,00	0,50	21,00	400,00	0,70	167,79
3	1	50,00	0,50	100,00	101,30	0,70	214,83
4	1	50,00	0,50	100,00	400,00	0,80	192,83
5	1	50,00	1,00	21,00	101,30	0,70	343,04
6	1	50,00	1,00	21,00	400,00	0,80	321,05
7	1	50,00	1,00	100,00	101,30	0,80	437,12
8	1	50,00	1,00	100,00	400,00	0,70	415,12
9	1	250,00	0,50	21,00	101,30	0,70	863,22
10	1	250,00	0,50	21,00	400,00	0,80	841,23
11	1	250,00	0,50	100,00	101,30	0,80	1096,92
12	1	250,00	0,50	100,00	400,00	0,70	1074,92
13	1	250,00	1,00	21,00	101,30	0,80	1733,91
14	1	250,00	1,00	21,00	400,00	0,70	1711,91
15	1	250,00	1,00	100,00	101,30	0,70	2201,30
16	1	250,00	1,00	100,00	400,00	0,80	2179,30
17	1	150,00	0,75	60,10	250,65	0,75	872,09
18	1	150,00	0,75	60,10	250,65	0,75	872,09
19	2	80,94	0,75	60,10	250,65	0,75	463,14
20	2	380,94	0,75	60,10	250,65	0,75	2226,57
21	2	150,00	0,17	60,10	250,65	0,75	186,55
22	2	150,00	1,33	60,10	250,65	0,75	1557,64
23	2	150,00	0,75	-31,12	250,65	0,75	628,45
24	2	150,00	0,75	151,32	250,65	0,75	1115,73
25	2	150,00	0,75	60,10	-94,26	0,75	897,49
26	2	150,00	0,75	60,10	595,56	0,75	846,69
27	2	150,00	0,75	60,10	250,65	0,63	872,09
28	2	150,00	0,75	60,10	250,65	0,87	872,09
29	2	150,00	0,75	60,10	250,65	0,75	872,09
30	2	150,00	0,75	60,10	250,65	0,75	872,09

5. DISCUSSION AND ANALYSIS OF RESULTS

The results below are obtained through the methodology described, according to the quantitative analysis of the mass balance model and its sensitivity analysis. In Fig. 2 there is the Pareto chart that allows to show the influence of the independent variables on the dependent variable (annulus well level (Tab. 1). It is showed with a 95% confidence interval, that the most significant variable in the model of Takács (2003) is the flow rate leaving the annulus wellhead valve, followed by the gas compressibility. Furthermore, it is showed that the combined effect of the two the previous variables are more significant than the isolated effect of temperature. One obvious observation is the fact that significant responses follow a linear model, except for the flow rate, whereas for this level of significance the answer may also be represented by a quadratic model.

Analyzing the ANOVA table (Tab. 2), the conclusions are the same as those that come from Pareto chart analysis (Fig. 2). It is noted that the significant variables in the response are associated with p-values smaller than 0.05, and highest values of F-Fischer. In Tab. 2 the significant variables for a 95% confidence interval are marked in red, these variables are the most relevant in the studied model.

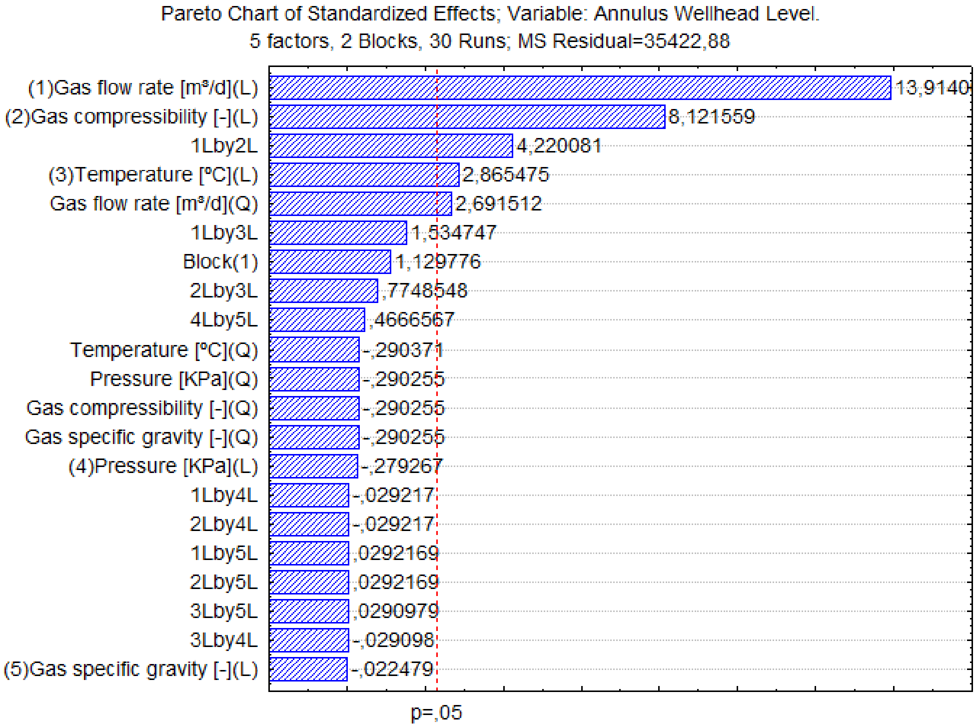

Figure 2. Standardized Effect Estimate (Absolute Value).

Table 1. ANOVA table of dependent variable annulus wellhead level.

ANOVA; Var.: Well Level [m]; R-sqr=,97404; Adj:,90588
5 factors, 2 Blocks, 30 Runs; MS Residual=35422,88

Factor	SS	df	MS	F	p
Blocks	45214	1	45214	1,2764	0,291304
(1) Gas flow rate [m³/d](L)	6857903	1	6857903	193,6010	0,000001
Gas flow rate [m³/d](Q)	256612	1	256612	7,2442	0,027432
(2)Gas compressibility [-](L)	2336483	1	2336483	65,9597	0,000039
Gas compressibility [-](Q)	2984	1	2984	0,0842	0,779003
(3)Temperature[ºC](L)	290855	1	290855	8,2109	0,020974
Temperature [ºC](Q)	2987	1	2987	0,0843	0,778917
(4)Well head pressure [KPa](L)	2763	1	2763	0,0780	0,787123
Well head pressure [KPa](Q)	2984	1	2984	0,0842	0,779003
(5)Gas specific gravity [-](L)	18	1	18	0,0005	0,982617
Gas specific gravity [-](Q)	2984	1	2984	0,0842	0,779003
1L by 2L	630849	1	630849	17,8091	0,002916
1L by 3L	83437	1	83437	2,3554	0,163393
1L by 4L	30	1	30	0,0009	0,977407
1L by 5L	30	1	30	0,0009	0,977407
2L by 3L	21268	1	21268	0,6004	0,460712
2L by 4L	30	1	30	0,0009	0,977407
2L by 5L	30	1	30	0,0009	0,977407

3L by 4L	30	1	30	0,0008	0,977499
3L by 5L	30	1	30	0,0008	0,977499
4L by 5L	7714	1	7714	0,2178	0,653190
Error	283383	8	35423		
Total SS	10914802	29			

5.1.1 Analysis via surface response

It is used the second order model proposed by Montgomery (2009, pp 604) to obtain the response surface, since it allows to include curvature. The 3D response surface is obtained to the dependent variable annulus well level as a function of: the gas flow rate, and gas compressibility (deviation factor) pouring the void (Fig. 3); the temperature at the wellhead, and gas flow rate (Fig. 4); the pressure at the wellhead, and gas flow rate (Fig. 5); and the specific gravity, and gas flow rate at the wellhead (Fig. 6).

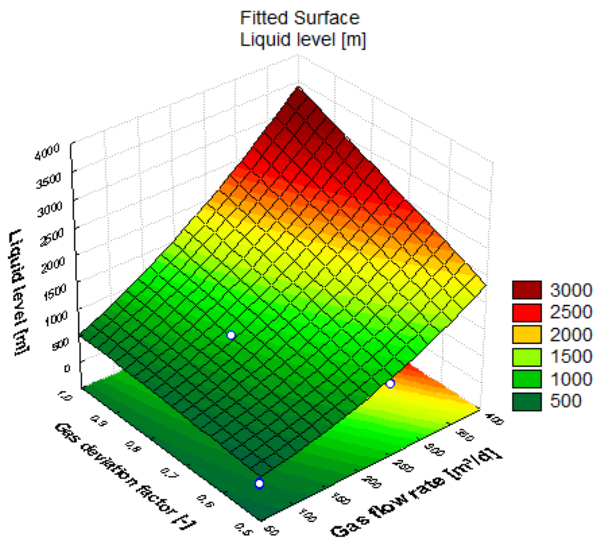

Figure 3. Gas deviation factor x gas flow rate x liquid level.

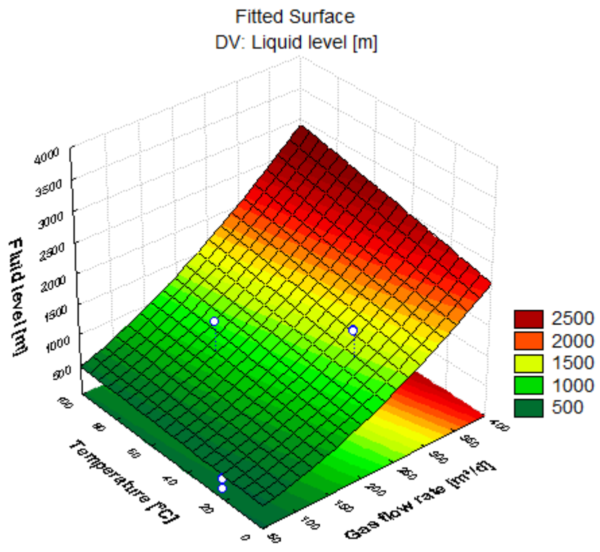

Figure 4. Temperature x gas flow rate x liquid level

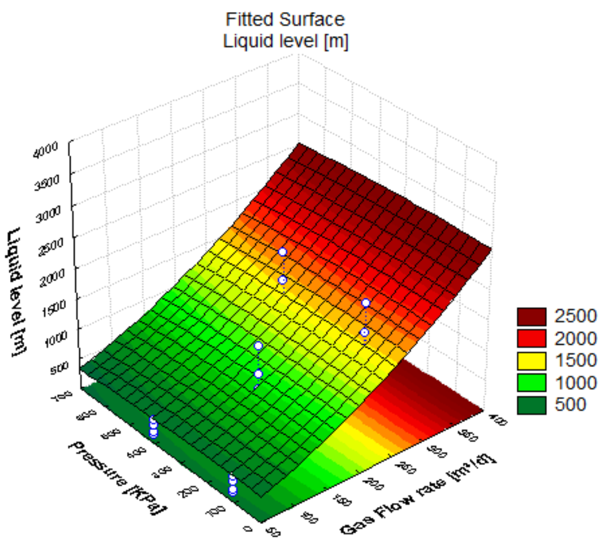

Figure 5. Pressure x gas flow rate x liquid level.

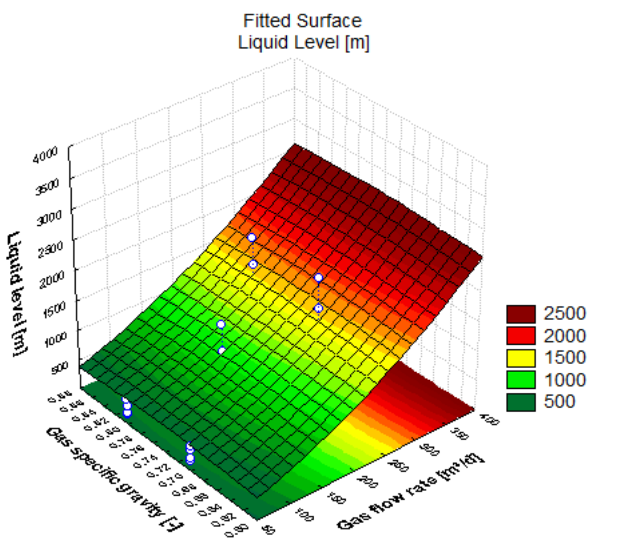

Figure 6. Specific gravity x flow rate x liquid level.

From the responses surfaces above, Fig. 3, Fig. 4, Fig. 5, and Fig. 6 may be easily exposed the great importance and effect of the variable flow rate on the response. Comparing Fig. 3 and Fig. 4, the effects of the compressibility (gas deviation factor) and the temperature has a significant impact, however the effect of temperature (Fig. 4) is shown more relevant when the flow rate becomes higher. The effects of pressure and gas specific gravity in Alexandre model *apud* Takáscs (2003) show no great significance in the solution, it is observed by the inclination of segments that are almost constant for those variables. That fact was expected, since these variables are not significant in the Pareto chart analysis and ANOVA Table (Fig. 2 and Tab. 2).

6. CONCLUSIONS

In this study are performed a qualitative and quantitative evaluation on the effects of the independent variables on the response of the model developed by Alexandre (1976) *apud* Takáscs (2003). The model describes the annulus well level from a mass balance of the amount of gas flowing through the annular space as function of the variables: flow rate, compressibility, specific gravity, temperature and pressure of the gas at the annulus space close to the wellhead, and among other variables that for this study were considered constant.

It is shown through the application of experimental design and statistical analysis, that the variable of greatest effect on the well level is the gas flow rate through annular q_2, followed by the gas compressibility. It is also noted that the combined effect of these two variables is thirdly in influence on the studied model, it is noted that this effect is more significant than the effect of temperature alone. From the statistical analysis it is shown that the effects of pressure and gas specific gravity is not significant for the calculation of the well flow level. These same conclusions is observed by analyzing the response surfaces for these variables.

7. REFERENCES

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