CONJUGATED HEAT TRANSFER WITH SLIP FLOW IN MICROCHANNELS: SINGLE DOMAIN INTEGRAL TRANSFORMS WITH ENHANCED CONVERGENCE

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Abstract. *This paper addresses the conjugated convection-conduction heat transfer problem in circular microchannels within the slip flow regime for low Péclet numbers. Based on recent developments of the Generalized Integral Transform Technique in combination with a single domain formulation strategy, the original multi-region problem is written as a single region with space variable coefificents with abrupt shifts, including a fictitious layer to model the temperature jump at the interface. An integral balance technique for enhancing the convergence of the eigenfunctions is applied, so as to achieve more accurate results for the proposed multiscale problem. The results are employed in the coverification of a solution obtained through a dedicated finite difference code for the original multi-region problem.*

Keywords*: Forced convection, slip flow, temperature jump, integral transforms, convergence enhancement, circular microchannels*

1. INTRODUCTION

A number of previous works in the analysis of thermal microsystems, led to the observation of discrepancies between the experimental results and classical correlations or simulations for the associated heat transfer coefficients (Morini, 2004). Among other issues, these discrepancies are related to classical simplifications employed in macroscale problems. Such simplificatios need to be carefully reviewed when dealing with heat and fluid flow in microsystems, due to the very small characteristic lengths involved, which require extended modeling with respect to the most usual simplified macro-scale formulations (Cotta *et al.*, 2016). In this work we focus on incorporating three of these typical micro-scale effects: (i) the slip flow regime in opposition to the classical no-slip boundary condition, together with the thermodynamic non-equilibrium (temperature jump); (ii) the consideration of the axial conduction term in the energy equation; and (iii) the consideration of the so-called conduction-convection conjugation effect. Despite the importance of modeling these three extensions of the classical Graetz problem at the microscale, the analysis of their combined effects has been somehow overlooked in the earlier literature, with a few exceptions (Fedorov and Viskanta, 2000).

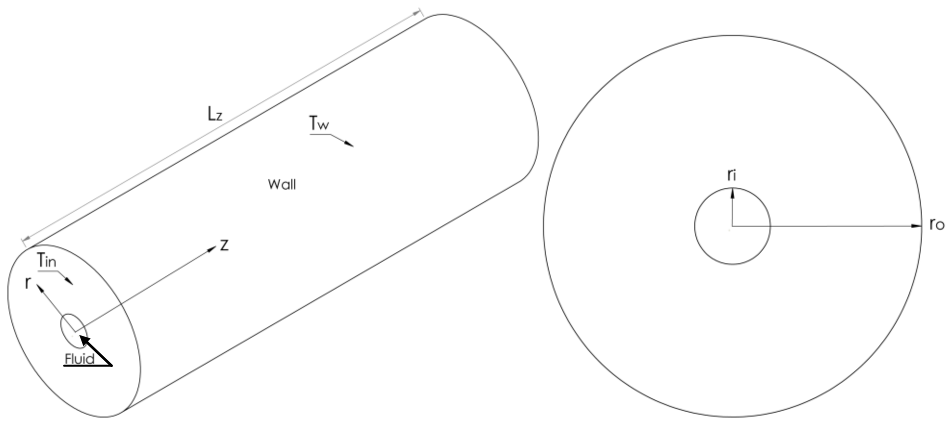
Besides the proper formulation, the use of a fast and accurate solution methodology for the derived partial differential equations is especially important when dealing with computer intensive analyses, such as inverse and optimization problems (Abreu *et al.*, 2014). In this context, the hybrid numerical-analytical approach known as the Generalized Integral Transform Technique, GITT (Cotta, 1993), has been here adopted. This methodology has been recently further advanced in combination with single domain formulations in order to rewrite multi-region problems, such as those in conjugated conduction-convection heat transfer, into single region problems with space variable thermophysical properties and source terms, so as to allow for a single integral transformation operation of the whole domain (Knupp *et al.*, 2013). This approach has also been demonstrated quite successfully in dealing with complex configurations and irregular regions (Cotta *et al.*, 2016), automatically guaranteeing heat flux and temperature continuity at the interfaces and without the need for domain decomposition schemes.

In (Knupp *et al.*, 2015b) the combination of the single domain formulation and integral transforms was employed to solve the conjugated heat transfer problem within the slip flow regime, when a fictitious solid layer between the fluid region and the actual channel wall was introduced, in order to impose the desired thermal resistance between the fluid and the wall and thus model the temperature jump at the interface. This fictitious layer needs to be kept much thinner than the characteristic length of the channel, so as to introduce a minimum perturbation to the original problem geometry. It then leads to eigenvalue problems with abrupt and multiscale variations on the governing space variable coefficients in the single domain formulation, resulting in undesirable slower convergence rates.

Based on a very recent development regarding the acceleration of the eigenfunctions convergence employing an integral balance analytical procedure (Cotta *et al.*, 2016), this work proposes the combination of the single domain formulation and integral transforms to efficiently handle the conjugated heat transfer problem within circular microchannels in the slip flow regime, including the axial conduction effect. For verification purposes, a test problem on steady or quasi-steady state hydrodynamically fully developed and thermally developing flow is addressed, and a dedicated finite difference code (Ozisik, 1994) is also implemented for the original multi-region problem, allowing for critical comparisons.

2. PROBLEM FORMULATION AND SOLUTION METHODOLOGY

Consider incompressible gas flow in a microchannel with length L_z, with circular cross-section with inner radius r_i and outer radius r_o, as illustrated in Figure 1, undergoing convective heat transfer due to a prescribed temperature T_o at the external wall, different from the inlet temperature T_{in}. The channel wall is considered to participate on the heat transfer process through transversal and axial heat conduction. Besides, the Knudsen and Péclet numbers are such that the problem falls within the region of validity of the first order slip flow modeling and in the situation when axial conduction cannot be neglected.



(a) (b)

Figure 1. (a) Schematic representation of the conjugated problem. (b) Microchannel cross-section.

The flow is here considered to be hydrodynamically fully developed, in which the gas enters the channel with a fully developed velocity profile $u_f(r)$, while a thermally developing region is established. The dimensionless formulation of the multi-region heat transfer problem, given by the coupled convection and conduction equations, is written as:

Heat convection equation (fluid):

$$U_f(R)\frac{\partial\theta_f(R,Z)}{\partial Z}=\frac{K_f}{\mathrm{Pe}^2}\frac{\partial^2\theta_f(R,Z)}{\partial Z^2}+4R_i^2\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta_f(R,Z)}{\partial R}\right),\quad 0<R<R_i,\quad 0<Z<L_Z \tag{1a}$$

$$\theta_f(R,0)=1;\qquad \left.\frac{\partial\theta_f}{\partial Z}\right|_{Z=L_Z}=0 \tag{1b,c}$$

$$\left.\frac{\partial\theta_f}{\partial R}\right|_{R=0}=0;\qquad 2R_i\beta_t\mathrm{Kn}\left.\frac{\partial\theta_f}{\partial R}\right|_{R=R_i}+\theta_f(R_i,Z)=\theta_s(R_i,Z) \tag{1d,e}$$

Heat conduction equation (solid):

$$4R_i^2\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta_s(R,Z)}{\partial R}\right)+\frac{1}{\mathrm{Pe}^2}\frac{\partial^2\theta_s(R,Z)}{\partial Z^2}=0,\quad R_i<R<1,\quad 0<Z<L_Z \tag{2a}$$

$$\theta_s(R,0)=1;\qquad \left.\frac{\partial\theta_s}{\partial Z}\right|_{Z=L_z}=0 \tag{2b,c}$$

$$K_s\left.\frac{\partial\theta_s}{\partial R}\right|_{R=R_i}=K_f\left.\frac{\partial\theta_f}{\partial R}\right|_{R=R_i};\qquad \theta_s(1,Z)=0 \tag{2d,e}$$

The temperature jump condition at the interface is modeled by Eq. (1e) and the heat flux continuity is modeled by Eq. (2d). The following dimensionless groups have been employed in Eqs. (1a-e) and (2a-e):

$$Z = \frac{z/(2r_i)}{\mathrm{Re}\,\mathrm{Pr}};\;\; R = \frac{r}{r_o};\;\; U_f = \frac{u_f}{u_{av}};\;\; \theta = \frac{T - T_o}{T_{in} - T_o};\;\; K = \frac{k}{k_f};\;\; \mathrm{Re} = \frac{u_{av}(2r_i)}{\nu};$$
$$\mathrm{Pr} = \frac{\nu \rho_f c_{p,f}}{k_f};\;\; \mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr} = \frac{u_{av}(2r_i)\rho_f c_{p,f}}{k_f};\;\; \beta_t = \frac{(2-\alpha_t)}{\alpha_t}\frac{2\gamma}{(\gamma+1)}\frac{1}{\mathrm{Pr}};\;\; \mathrm{Kn} = \frac{\lambda}{2r_i} \tag{3a-j}$$

where the subscript f denotes quantities related to the fluid, s denotes quantities related to the solid wall, and o denotes quantities at the outer wall; α_t is the thermal accommodation coefficient, $\gamma = c_{p,f}/c_{v,f}$ is the specific heat ratio, and λ is the molecular mean free path. The dimensionless velocity profile is given by (Larrodé *et al.*, 2000):

$$U_f(R) = \frac{3\left[1 - \left(R/R_i\right)^2 + 4\mathrm{Kn}\beta_v\right]}{2(1 + 6\mathrm{Kn}\beta_v)} \quad \text{with} \quad \beta_v = \frac{2-\alpha_m}{\alpha_m} \tag{4a,b}$$

where β_v is the wall velocity slip coefficient and α_m is the tangential momentum accommodation coefficient.

In order to write the conjugated problem given by Eqs. (1a-e) and (2a-e) into a single domain formulation, while still accounting for the temperature jump interface condition, it is proposed a fictitious thin layer between the fluid stream and the channel wall, in such a way that this fictitious layer can be chosen with dimensionless thickness and thermal conductivity L_{fic} and K_{fic}, respectively, so as to impose the desired thermal resistance (Knupp *et al.*, 2015b). Considering that this fictitious layer undergoes steady state heat conduction in the radial direction only, and considering the boundary condition given by Eq. (1e), a thermal resistance analysis yields the following relation:

$$K_{fic} = \mathrm{Log}\left(\frac{R_i + L_{fic}}{R_i}\right)\frac{1}{2\beta_t \mathrm{Kn}} \tag{5}$$

One should observe that even though any arbitrary value could be set for L_{fic} in Eq. (5) yielding the desired thermal conductivity for the fictitious layer, according to the required thermal resistance, this thickness should be ideally chosen with $L_{fic} \to 0$, to interfere as minimum as possible in the original geometry and the overall heat transfer process.

The conjugated problem simulating the temperature jump interface condition can then be written as a single domain formulation with space variable coefficients. Considering a pseudo transient form in a such a way that the problem can be treated as a parabolic problem with the convection term considered as a source term (Knupp *et al.*, 2013), we have:

$$\frac{\partial \theta^*}{\partial t} = 4R_i^2 \frac{1}{R}\frac{\partial}{\partial R}\left(RK(R)\frac{\partial \theta^*}{\partial R}\right) + \frac{K_{ax}(R)}{\mathrm{Pe}^2}\frac{\partial^2 \theta^*}{\partial Z^2} - U(R)\frac{\partial \theta^*}{\partial Z},\quad 0 < R < 1 + L_{fic},\quad 0 < Z < L_Z \tag{6a}$$

$$\theta^*(R,0,t) = 1;\qquad \left.\frac{\partial \theta^*}{\partial Z}\right|_{Z=L_Z} = 0 \tag{6b,c}$$

$$\left.\frac{\partial \theta^*}{\partial R}\right|_{R=0} = 0;\qquad \theta^*(1+L_{fic}, Z, t) = 0 \tag{6d,e}$$

$$\theta^*(R,Z,0) = f(R,Z) \tag{6f}$$

where the initial condition $f(R,Z)$ can be any feasible function, preferably a steady state solution estimate, since in this work we are only interested in the steady state solution, $t \to \infty$. In this work it is considered the simplest case, $f(R,Z) = 0$. The space variable coefficients in the single domain formulation in Eq. (6a), are defined as:

$$U(R) = \begin{cases} U_f(R), & \text{if } 0 \le R < R_i \\ 0, & \text{if } R_i \le R \le 1 + L_{fic} \end{cases},\;\; K(R) = \begin{cases} 1, & \text{if } 0 \le R < R_i \\ K_{fic}, & \text{if } R_i \le R < R_i + L_{fic} \\ k_s/k_f, & \text{if } R_i + L_{fic} \le R \le 1 + L_{fic} \end{cases},\;\; K_{ax}(R) = \begin{cases} 1, & \text{if } 0 \le R < R_i \\ 0, & \text{if } R_i \le R < R_i + L_{fic} \\ k_s/k_f, & \text{if } R_i + L_{fic} \le R \le 1 + L_{fic} \end{cases} \tag{6g-i}$$

It is worth noting that $K_{ax}(R)$ is introduced with zero thermal conductivity at the fictitious layer, so that this layer participates through heat conduction along the radial direction only, allowing for the thermal resistance analysis performed to derive Eq. (5).

After the problem given by Eqs. (6a-i) is solved, the desired temperature distribution can be readily obtained by considering the steady state solution, with t sufficiently high, and removing the fictitious layer from the solution:

$$\theta(R,Z) = \begin{cases} \theta^*(R,Z,t\to\infty), & \text{if } 0 \le R \le R_i \\ \theta^*(R+L_{fic},Z,t\to\infty), & \text{if } R > R_i \end{cases} \tag{7}$$

So, whereas θ^* is defined for $0 \le R \le 1+L_{fic}$, θ is defined for $0 \le R \le 1$, the original problem domain before introducing the fictitious layer. As demonstrated in a number of previous works (Knupp *et al.*, 2013; Knupp *et al.*, 2015a; Cotta *et al.*, 2016) for the Cartesian coordinates system, the problem given by Eqs. (6a-i) can be solved through the proposition of an eigenvalue problem containing the information regarding the different domains in its governing coefficients. For the problem addressed in this work we have:

$$\frac{d}{dR}\left(RK(R)\frac{d\psi_i(R)}{dR}\right) + \mu_i^2\psi_i(R)R = 0 \tag{8a}$$

$$\left.\frac{d\psi_i}{dR}\right|_{R=0} = 0; \quad \psi_i(1+L_{fic}) = 0 \tag{8b,c}$$

which allows for the definition of the following transform-inverse pair:

$$\text{transform: } \bar{\theta}_i^*(Z,t) = \int_0^{1+L_{fic}} R\tilde{\psi}_i(R)\theta^*(R,Z,t)dR \tag{9a}$$

$$\text{inverse: } \theta^*(R,Z,t) = \sum_{i=1}^{\infty}\tilde{\psi}_i(R)\bar{\theta}_i^*(Z,t) \tag{9b}$$

with the normalized eigenfunctions calculated as:

$$\tilde{\psi}_i(R) = \frac{\psi_i(R)}{N_{\psi i}^{1/2}}, \quad N_{\psi i} = \int_0^{1+L_{fic}} R\psi_i^2(R)dR \tag{10a,b}$$

Operating on Eq. (6a) with $\int_0^{1+L_{fic}} \tilde{\psi}_i(R)(\cdot)dR$ and making use of the boundary conditions, we obtain the following one-dimensional transformed partial differential system:

$$\frac{\partial\bar{\theta}_i^*(Z,t)}{\partial t} + \mu_i^2\bar{\theta}_i^*(Z,t) = \bar{g}_i(Z,t,\bar{\theta}_n^*), \quad i=1,2,..., \quad n=1,2,... \tag{11a}$$

where

$$\bar{g}_i(Z,t,\bar{\theta}_n^*) = -\sum_{n=1}^{\infty}\frac{\partial\bar{\theta}_n}{\partial Z}\int_0^{1+L_{fic}} U(R)\tilde{\psi}_i\tilde{\psi}_n dR + \sum_{n=1}^{\infty}\frac{\partial^2\bar{\theta}_n}{\partial Z^2}\int_0^{1+L_{fic}}\tilde{\psi}_i\tilde{\psi}_n dR \tag{11b}$$

with the following transformed boundary and initial conditions, obtained after operating on Eqs. (6b,c) and Eq. (6f) with $\int_0^{1+L_{fic}} R\tilde{\psi}_i(R)(\cdot)dR$:

$$\bar{\theta}_i^*(0,t) = \int_0^{1+L_{fic}} R\tilde{\psi}_i(R)dR; \quad \left.\frac{\partial\bar{\theta}_i^*}{\partial Z}\right|_{Z=L_Z} = 0 \tag{11c,d}$$

$$\bar{\theta}_i^*(Z,0) = \bar{f}_i(Z) = \int_0^{1+L_{fic}} R\tilde{\psi}_i(R)f(R,Z)dR \tag{11e}$$

System (11), after truncation to an order N, can be numerically solved to provide results for the transformed temperatures $\bar{\theta}_i^*(Z,t)$. The *Mathematica* software, for instance, provides the routine *NDSolve* which is able to solve this kind of system under automatic absolute and relative error control.

Hence, the main task in this procedure is related to the solution of the eigenvalue problem given by Eqs. (8a-c), which does not allow for an explicit analytic solution. Although the GITT itself can be readily employed to solve this eigenvalue problem (Knupp *et al.*, 2013; Knupp *et al.*, 2015a; Cotta *et al.*, 2016), the convergence rates can become impractical, due to the multiscale behaviour of the governing coefficients, requiring the implementation of a convergence enhancement technique for computational savings, such as the integral balance procedure, which can be employed to derive analytical expressions to accelerate the eigenfunctions expansion convergence (Cotta *et al.*, 2016).

Starting with successive integration of the original eigenvalue problem from the boundaries to any arbitrary point in the domain, analytical expressions for the eigenfunction and its derivative are obtained, with dependence on the boundary values of both the eigenfunction and the associated derivative, and explicitly accounting for the space variable coefficients in the eigenfunction representation. Making use of the available boundary conditions in the original eigenvalue problem, the boundary values are eliminated from the derived expressions for the eigenfunction and its derivative. Following this procedure in solving the eigenvalue problem given by Eqs. (8a-c), as described in details for a general problem in (Cotta *et al.*, 2016 In Press), the following expressions with enhanced convergence are obtained for the eigenfunctions $\psi(R)$ and the derivatives $d\psi(R)/dR$:

$$\psi(R) = \mu^2 \int_R^{1+L_{fic}} \frac{1}{K(R')} \int_0^{R'} R''\psi(R'')\, dR''dR' \tag{12a}$$

$$\frac{d\psi(R)}{dR} = -\frac{\mu^2}{K(R)}\left[\int_0^R R'\psi(R')\, dR'\right] \tag{12b}$$

Then, consider an expansion representation for the original eigenfunctions, $\psi(R)$, appearing on the RHS of Eqs. (12a,b):

$$\text{transform: } \bar{\psi}_{in} = \int_0^{1+L_{fic}} R\tilde{\Omega}_n(R)\psi_i(R)dR \tag{13a}$$

$$\text{inverse: } \psi_i(R) = \sum_{n=1}^{\infty} \tilde{\Omega}_n(R)\bar{\psi}_{in} \tag{13b}$$

which employs a simpler basis, such as provided by the following eigenvalue problem, with explicit analytic solution for the eigenfunctions $\Omega(R)$, and the corresponding eigenvalues λ:

$$\frac{d}{dR}\left(R\frac{d\Omega_i(R)}{dR}\right) + \lambda_i^2\Omega_i(R)R = 0 \tag{14a}$$

$$\left.\frac{d\Omega_i}{dR}\right|_{R=0} = 0; \quad \Omega_i(1+L_{fic}) = 0 \tag{14b,c}$$

The normalized auxiliary eigenfunctions are calculated as:

$$\tilde{\Omega}_i(R) = \frac{\Omega_i(R)}{N_{\Omega i}^{1/2}}, \quad N_{\Omega i} = \int_0^{1+L_{fic}} R\Omega_i^2(R)dR \tag{15a,b}$$

Substituting the expansion representation given by Eq. (13b) into the RHS of Eqs. (12a,b) one obtains:

$$\psi(R) = \mu^2 \sum_{n=1}^{\infty} \bar{\psi}_{in} IB_n(R) \tag{16a}$$

$$\frac{d\psi(R)}{dR} = -\frac{\mu}{K(R)}^2 \sum_{n=1}^{\infty} \bar{\psi}_{in} IA_n(R) \tag{16b}$$

with

$$IA_n(R) = \int_0^R R'\tilde{\Omega}_n(R')dR', \quad IB_n(R) = \int_R^{1+L_{fic}} \frac{1}{K(R')} IA_n(R')dR' \tag{16c,d}$$

The integral transformation of Eq. (8a) can be achieved by operating on this equation with $\int_0^{1+L_{fic}} (\cdot)\tilde{\Omega}_m(R)dR$. Then, substituting the integral balance expressions given by Eqs. (16a,b) for $\psi(R)$ and $d\psi(R)/dR$ appearing in this expression, yields:

$$(\mathbf{A}-\mu^2\mathbf{B})\bar{\boldsymbol{\psi}}=0\ , \text{ with } \quad A_{n,m}=\int_0^{1+L_{fic}} IA_n(R)\frac{d\tilde{\Omega}_m(R)}{dR}dR\ , \quad B_{n,m}=-\int_0^{1+L_{fic}} RIB_n(R)\tilde{\Omega}_m(R)dR \tag{18a}$$

where $IA_n(R)$ and $IB_n(R)$ are given by Eqs. (16c,d). This algebraic eigenvalue problem can be numerically solved to provide results for the eigenvalues μ and eigenvectors $\bar{\psi}_{in}$, which can be readily employed back in Eqs. (16a,b) to obtain analytical expressions for the eigenfunctions and related derivatives, $\psi_i(R)$ and $d\psi_i(R)/dR$, respectively.

Besides the GITT solution with integral balance for the single domain formulation just described, this work also reports some results obtained without the convergence acceleration technique, for comparison purposes. Moreover, a dedicated finite difference solution (second-order accurate in R and first-order accurate in Z) is implemented for the original multi-region problem given by Eqs. (1a-e) and Eqs (2a-e) in order to provide a comparative analysis against the single domain integral transforms solution.

3. RESULTS AND DISCUSSION

As test case, the flow of air ($k_f=0.0271$ W/m°C) inside an acrylic microchannel ($k_s=0.2$ W/m°C) is considered, resulting in $K_s=k_s/k_f=7.38$. In the examples presented, it has been adopted $\beta_v=1.5$, $\beta_t=2.0$ and $\text{Kn}=0.025$, which are representative values for these parameters (Naveira-Cotta *et al.*, 2010; Naveira-Cotta, 2016).

Results regarding the solution of the eigenproblem given by Eqs. (8a-c) are first reported, employing the GITT with integral balance for convergence acceleration in comparison with the traditional solution (Cotta, 1993). In both cases it has been considered $L_{fic}=10^{-3}$ resulting in $K_{fic}=0.04988$. Tables 1(a,b) present the convergence behavior of the first five eigenvalues. It is worth observing that in Table 1(a) the eigenvalues seem to be converged with three to four significant digits for a truncation order $M=50$, for the case employing the integral balance, whereas in Table 1(b) a very high truncation order of $M=8000$ is needed in order to achieve an agreement of two to three significant digits with Table 1(a), demonstrating the remarkable convergence acceleration effect obtained with the integral balance approach. Figure 2 depicts the comparison of the spatial behavior of the first two calculated eigenfunctions, $\psi_1(R)$ and $\psi_2(R)$, with $M=50$ in both cases, where it can be noticed that the profiles calculated with the integral balance technique present a clear abrupt transition at the interface, $R=0.2$, as result of the solution accurately capturing the fictitious layer which is located from $R=0.2$ to $R=0.2+L_{fic}=0.201$ in this case. On the other hand, this multiscale character of the problem poses a remarkable difficulty for the solution without the convergence acceleration technique, yielding smooth profiles, even at $R=0.2$. In fact, in order to accurately capture the thin fictitious layer without any kind of convergence acceleration, auxiliary eigenfunctions with much higher frequency would be needed, which would require an impractical high truncation order M, as demonstrated in Table 1.

Table 1. Convergence behavior of the first five eigenvalues in problem (8) solved via GITT: (a) with integral balance; (b) without integral balance.

(a)

M	μ_1	μ_2	μ_3	μ_4	μ_5
10	2.544	4.305	6.910	9.776	11.665
20	2.544	4.297	6.909	9.639	11.444
30	2.544	4.296	6.909	9.627	11.436
40	2.543	4.296	6.909	9.623	11.433
50	2.543	4.295	6.909	9.620	11.431

(b)

M	μ_1	μ_2	μ_3	μ_4	μ_5
500	2.566	4.589	6.923	10.015	11.737
1000	2.565	4.571	6.922	9.994	11.714
2000	2.557	4.455	6.916	9.841	11.577
4000	2.549	4.361	6.912	9.709	11.483
8000	2.546	4.324	6.910	9.656	11.450

Tables 2(a,b) present the convergence behavior of the calculated temperature field via GITT with integral balance for L_{fic} = 10^{-2} and L_{fic} = 10^{-3}, respectively, considering the case with $\text{Pe}=10$ with $N=10$ to $N=50$ as the truncation order in Eq. (9b), employing M = 100 terms in the solution of the eigenproblem with space variable coefficients. In both tables the numerical solution of the original multi-region problem is also presented, obtained via the Finite Difference Method (FDM) employing a second-order accurate scheme in *R* and first-order accurate scheme in *Z*. The FDM solution presented was obtained with 1200 nodes in the longitudinal direction and 140 nodes in the radial direction (70 in each region, fluid and solid), yielding convergence of the three digits shown, confirmed in a grid independence test.

In both cases, L_{fic} = 10^{-2} and L_{fic} = 10^{-3}, the GITT solution is converged to five significant digits for the truncation order N = 50 and practically verifying the FDM solution of the original multi-region problem. Although smaller values of L_{fic} should in principle lead to better results comparing to the original multi-region problem, it should be remembered that the lower this value, the more difficult becomes the convergence of the eigenvalue problem with multiscale variable coefficients.

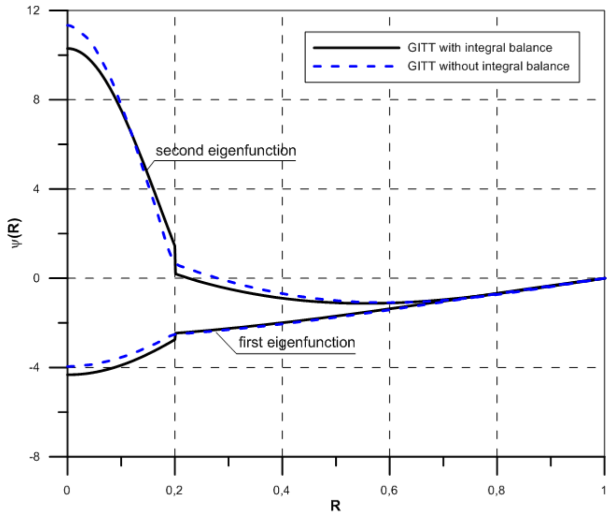


Figure 2. First two eigenfunctions calculated via GITT with and without integral balance employing L_{fic} = 10^{-3}.

Table 2(a). Convergence behavior for the GITT solution (M = 100) with (a) L_{fic} = 10^{-2}; (b) L_{fic} = 10^{-3}.

(a)

N	$\theta(R, Z = 0.05)$ Pe = 10			
	$R = 0.1$	$R_f = 0.2$	$R_s = 0.2$	$R = 0.6$
10	0.90548	0.77921	0.74928	0.62869
20	0.90612	0.77957	0.74997	0.62806
30	0.90611	0.77950	0.74995	0.62807
40	0.90611	0.77949	0.74995	0.62806
50	0.90611	0.77949	0.74995	0.62806
FDM	**0.906**	**0.780**	**0.750**	**0.627**

(b)

N	$\theta(R, Z = 0.05)$ Pe = 10			
	$R = 0.1$	$R_f = 0.2$	$R_s = 0.2$	$R = 0.6$
10	0.90627	0.77951	0.74962	0.62812
20	0.90681	0.77992	0.75035	0.62749
30	0.90681	0.77990	0.75035	0.62750
40	0.90681	0.77989	0.75035	0.62750
50	0.90681	0.77989	0.75034	0.62750
FDM	**0.906**	**0.780**	**0.750**	**0.627**

Figures 3(a,b) show temperature profiles at different longitudinal positions for $\mathrm{Pe} = 1$ and $\mathrm{Pe} = 10$, respectively, confirming the good agreement between the numerical and the proposed hybrid solution, while comparing the GITT solutions with and without convergence acceleration of the eigenvalue problem solution. It is interesting to confirm that the most impressive effect of the convergence acceleration technique refers to the interface, allowing for the hybrid solution with the single domain formulation to accurately capture the temperature jump. Hence, even though the GITT solution without convergence acceleration is able to yield quite good results at the solid region, it would require much larger truncation orders to accurately treat the interface, without error propagation to the fluid region. It can also be observed that this effect is more pronounced for $\mathrm{Pe} = 10$, where a more significant temperature jump occurs.

4. CONCLUSIONS

Conjugated convection-conduction heat transfer is analyzed in circular microchannels within the slip flow regime and for low Péclet numbers, i.e. considering the axial conduction effect, by means of the Generalized Integral Transform Technique in combination with a single domain reformulation strategy. The original multi-region problem is written in a single domain with space variable coefificents with abrupt shifts, including a fictitious layer to model the temperature jump at the interface, leading to a multiscale problem. In order to obtain accurate results within sufficiently low truncation orders, an integral balance technique for enhancing the convergence of the eigenfunction, recently developed for the solution of eigenvalue problems with multiscale and abrupt transitions, has been employed. Critical comparisons against the traditional solution (without convergence acceleration of the eigenvalue problem solution) and a dedicated finite difference code for the original multi-region problem, demonstrated the enhanced convergence behavior and improved accuracy achieved by the novel methodology.

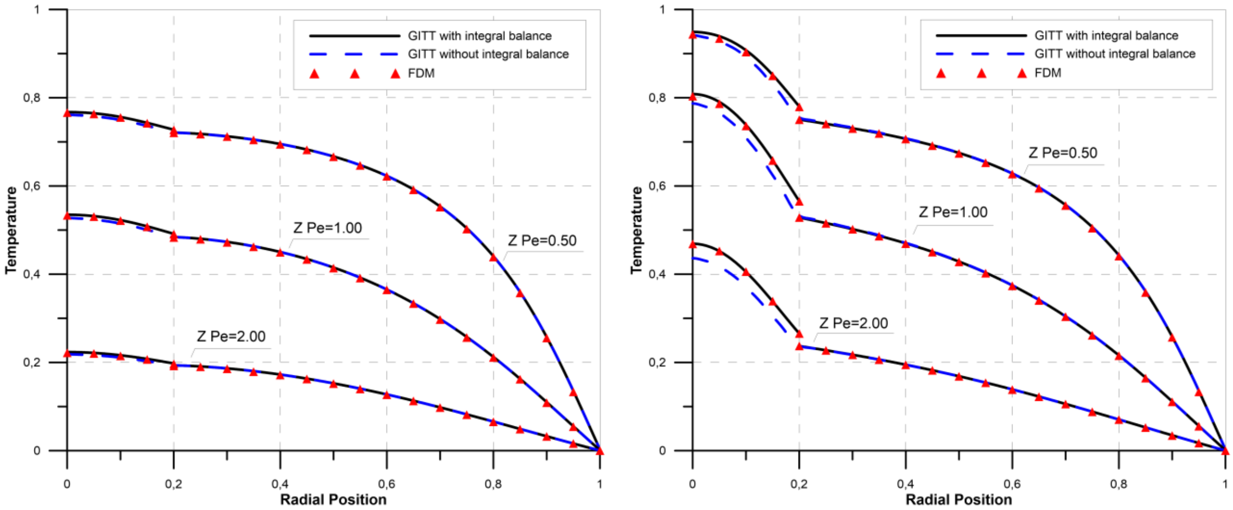


Figure 3. Temperature profiles at different longitudinal positions for (a) $\mathrm{Pe}=1$ and (b) $\mathrm{Pe}=10$.

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