POLYMER MELT FLOW IN ASYMMETRIC SUDDEN EXPANSION: A NUMERICAL STUDY

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Abstract. *The present work deals with numerical simulation of three-dimensional, laminar, incompressible and non-isothermal polymer melt flow inside channels with sudden expansions. The main goal of the paper is to study the general features of the flow, assessing the effects of channel aspect ratio on pressure drop and Nusselt number. The mathematical model comprises the mass, momentum and energy conservative laws within a framework of generalized Newtonian formulation. The non-Newtonian flow behavior is modeled by the Cross constitutive relation. The numerical simulation is performed by the commercial software ANSYS FLUENT, that was based in a Finite Volume formulation. The pressure-velocity coupling is treated by segregated solution. The results obtained are divided in two parts: i) firstly, the numerical verification indicates that the FLUENT solution presents good agreement with other literature works; (ii) afterwards, the parametric study demonstrates that pressure drop and Nusselt number are very sensitive to the channel aspect ratio.*

Keywords*: Polymer melt flow, Non-Newtonian flow, Three-dimensional sudden expansion, Numerical analysis.*

1. INTRODUCTION

Polymer melt flow in sudden expansions is frequently found in industrial operations. The flow is characterized by a high friction producing great pressure drop and a heating effect due to the viscous dissipation. In academy, the geometries of plane channels, symmetrical and asymmetrical sudden expansions have been used extensively to assess the accuracy of numerical schemes and to study flow features such as velocities and temperature profiles, pressure drop, recirculation regions and bifurcations.

With regard to the physical analysis of the problem, Pinho et al. (2003) presented a study on pressure losses and vortex length for the laminar, isothermal and non-Newtonian flow in 2D sudden expansion. Mânica and De Bortoli (2004) solved numerically laminar and incompressible flow in sudden expansion and observed that, for high Reynolds number the solution becomes time dependent and shown that some non-Newtonian fluids have behavior similar to that of Newtonian fluids in sudden expansions.

Koh et al (2004) and Zdanski et al (2008) have studied the polymer melt flow in planes channels, analyzing the effect of viscous dissipation and the behavior of the velocity and temperature profiles. In a recent work, Zdanski et al (2011) investigate aspects of forced convection heat transfer of polymer melt flow in a 2D plane channel with sudden contraction/expansion sections. The work is focused on the assessment of the local and global Nusselt numbers based upon a parametric study of the effects of the contraction/expansion aspect ratio and entrance flow velocity.

Zdanski and Vaz Jr. (2009a) presented a study about the general features of the flow field, influence of viscous dissipation in temperature rise and viscosity variations in 3D polymer melt flow in sudden expansion. In other work, Zdanski and Vaz Jr. (2009b) studied the influence of expansion ratio and inlet temperature on flow parameters, such as velocity, pressure drop and viscosity distribution on polymer melts flow in 2D sudden expansion.

Hassan et al. (2009) presented a 3D study for the effect of the gate location on the solidification and temperature distribution during the cooling of polymer material by injection molding. Three different positions for the gate location are analysed and the paper also shows that the gate position has a great effect on the temperature distribution of the injected product. In other work, Hassan et al. (2010) studied the effect of the cooling channels position and their form – circular, rectangular and square – on the heat transfer process during the cooling of polymer in injection moulding. The main conclusion indicates that the time required to completely solidifying the product decreases when using rectangular channels. Finally, it is important to mention that both works adopt the Cross constitutive relation to describe the rheological behavior of the polymer.

The main objective of the present work is to study the polymer melt flow inside 3D channels with sudden expansions. The results obtained are divided in two parts: i) firstly, the numerical verification indicates that the FLUENT solution presents good agreement with other literature works; (ii) afterwards, the parametric study demonstrates that pressure drop and Nusselt number are very sensitive to the channel aspect ratio.

2. THEORICAL FORMULATION

2.1 Governing equations

Polymer melt flow in an injection moulding can be described by the generalized Newtonian formulation, the flow is assumed non-Newtonian, laminar and incompressible. The present numerical model is based on the solution of the coupled Navier-Stokes equations for the velocity, the energy equation for the temperature and a Poisson equation for the pressure. Thus the mass, linear momentum and energy conservation laws are given by

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_j u_i)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\eta(T,\dot{\gamma})\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \tag{2}$$

and

$$\frac{\partial(\rho c T)}{\partial t} + \frac{\partial(\rho c u_i T)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right) + \eta(T,\dot{\gamma})\dot{\gamma}^2 \tag{3}$$

where *i, j = 1,…,3* represent the indicial notation, x_i and u_i are the coordinate directions and the velocity components, respectively, η is the apparent viscosity, *T* is the temperature, *p* is pressure, ρ is the specific mass, *k* is the thermal conductivity and *c* is the specific heat. The term $\dot{\gamma}$ represents the equivalent shear rate, and can be written as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2} \tag{4}$$

2.2 Polymer rheology

For generalized Newtonian flow model, the apparent viscosity of the polymer is computed as function of shear rate and temperature through the Cross constitutive relation. The model used in the present work describe the non-Newtonian behavior of commercial polymers, POM (Polyacetal).

$$\eta(T,\dot{\gamma}) = \frac{\eta_0(T)}{1+[\lambda(T)\dot{\gamma}]^{1-n(T)}} \tag{5}$$

where

$$\eta_0(T) = a_1 \exp\left(\frac{a_2}{T}\right) \tag{6}$$

$$\lambda(T) = b_1 \exp\left(\frac{b_2}{T}\right) \tag{7}$$

and

$$n(T) = c_1 \exp\left(-\frac{c_2}{T}\right) \tag{8}$$

According to Bom et al.(2000), Polyacetal (POM) in a molten state can be represented by the Cross relation with the following parameters: a_1=0.022603 Pa.s, a_2=5003.01 K, b_1=1.6425×10^{-6} s, b_2=3901.0 K, c_1=1.3574 K, c_2=653.73 K. Furthermore, for this specific polymer, the thermo-physical properties used in the simulations are ρ=1143.9 kg/m^3, *k* = 0.31 W/mK and *c* = 2420.0 J/kgK.

2.3 Numerical method

The numerical simulation is performed by the commercial software ANSYS FLUENT, that is based in finite volume method. A collocated mesh formulation is developed, i.e. all the dependent variables such as pressure and velocity are stored at the centre of control volume. The convective and diffusive terms are evaluated using second-order spatial accuracy formulae based on upwind/central differences. To avoid the numerical oscillations, checkerboarding effect, the software uses the Rhie–Chow (1982) interpolation method. The coupling between the mass and momentum conservation equations was performed using the SIMPLE algorithm by Patankar and Spalding (1972), based on the subsequent correction of the pressure field for mass conservation satisfaction.

3. RESULTS AND DISCUSSION

3.1 Problem statament

The polymer melt flow in 3D asymmetric sudden expansion is numerically investigated. The problem geometry with its mains dimensions is depicted in Fig 1.

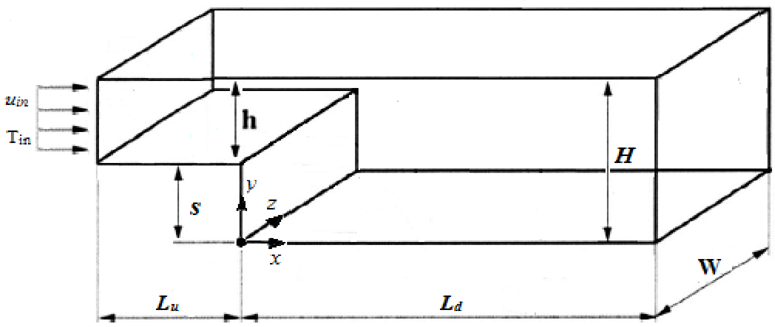

Figure 1. Channel geometry with main dimensions.

The computational domain extend $L_u=3h$ upstream of the expansion section, the channel length downstream of the expansion is assumed $L_d=40h$. The expansion ratio of the channel *(ER = H/h)* is assumed equal to 2, being the channel thickness upstream and downstream of the expansion section *h*=2mm and *H*=4mm, respectively. The channel aspect ratio *(AR = W/h)* is varied from 2 to 6.

In the present work, the boundary conditions adopted are as follows: (i) uniform velocity, u_{in}, and temperature, T_{in}, distributions at the entrance plane, u_{in}= 6 cm/s and T_{in}= 478 K, respectively; (ii) the constant inlet *x*-velocity is equal to u_{in}, being null *y* and *z*-velocity component; (iii) the pressure at the inlet section is extrapolated from inside nodes with the assumption of linear variation; (iv) parabolic condition for all variables at the exit section (null derivative); and (v) non-slip condition (null velocity) and prescribed temperature at solid walls, T_{wall} = 453 K.

3.2 Validation and verification

Verification and validation of computer simulations are the primary methods to build and quantify confidence in modeling and simulation. The process of comparing a given numerical solution against experimental data is known in literature as validation, if a numerical solution is confronted against theoretical result, the procedure is generally referred as verification (Oberkampf and Trucano, 2002).

The test for verification compares the present numerical method with the numerical solution by Zdanski and Vaz Jr., (2009a). The rheological model adopted is the cross constitutive relation and the geometry is 3D channel with asymmetric sudden expansion, similar to Fig. 1. The computational domain extends $L_u=3h$ upstream of the expansion section, the channel length downstream of the expansion is assumed $L_d=40h$. The inlet and wall temperature are T_{in} = T_{wall} = 453 K and the inlet velocity is u_{in}= 6 cm/s. The expansion ratio of the channel *(ER = H/h)* is assumed equal to 2, being the channel thickness upstream and downstream of the expansion section *h*=2mm and *H*=4 mm, respectively. For this test, the channel aspect ratio *(AR = W/h)* is assumed equal to 4. The boundary conditions at the walls correspond to non-slip velocities and null derivatives for pressure.

In the reference work, Zdanski and Vaz Jr. (2009a), the computational mesh at the region upstream of the expansion section is mapped by 21 x 31 x 101 grid points, whilst the downstream region 101 x 61 x 101 nodes. The mesh is non-uniform only at the x-direction, with point clustering near the expansion section. The maximum stretching factor used is around 4%. The full computational domain comprises 688,012 grid points.

The non-dimensional temperature profile for fully developed region at selected *z/W* stations along the channel spanwise direction is shown in the Fig 2. The agreement between both numerical solutions is quite satisfactory, being the differences credit to the methods used in each simulation. The present work use the finite volume method while Zdanski and Vaz Jr., (2009a) use the finite differences. The finite difference method evaluates $\dot{\gamma}$ and η at the mesh node, whereas finite volume accounts for fluxes at the control volume surfaces.

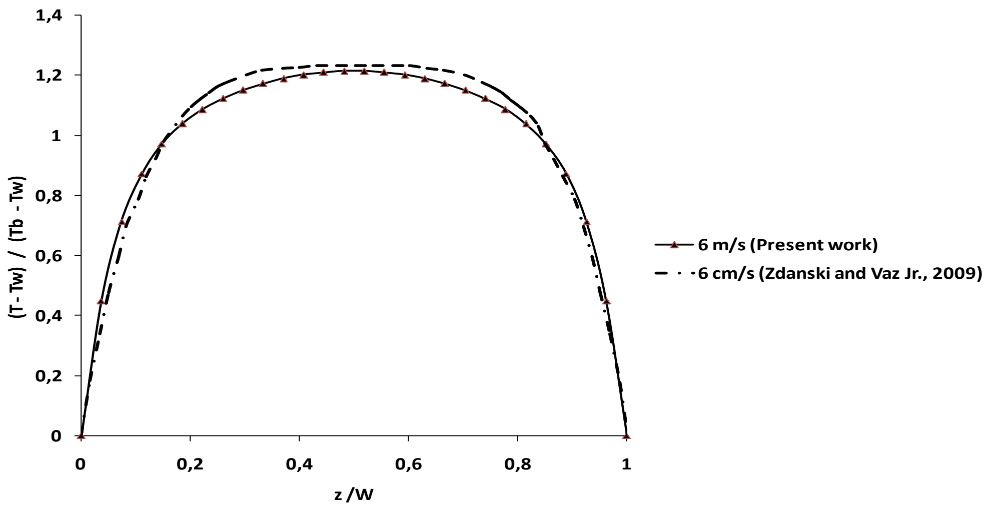

Figure 2. Non-dimensional temperature profiles for fully developed flow region (y=H/2 and x=40h).

3.3 Physical analysis of the problem

The aim of this work is to assess the influence of channel aspect ratio on pressure drop, local and average Nusselt number distributions along the channel length. It is important to mention that previous authors work (Zdanski and Vaz Jr., 2009a) studied the 3D polymer melt flow topology based on streamlines maps. Besides, the temperature, velocity, recirculation zones and viscosity contours are discussed by analyzing the 2D distributions in several x, y and z planes. However, that work did not analyze the Nusselt number and the pressure drop across the channel length.

Firstly, it is worth mentioning that the simulation performed adopt the boundary conditions described at section 3.1. The effect of the channel aspect ratio *(AR = W/h)* on the pressure drop between the entrance and exit sections is presented in Tab. 1. Both dimensional pressure drop values and dimensionless pressure loss coefficient are shown. The first represent the difference between average pressure values at the inlet and exit sections, $\Delta\overline{p}$, known as *´mean pressure drop´*. The latter is defined as (Zdanski and Vaz Jr., 2009b)

$$C_I \equiv \frac{\Delta\overline{p}}{(1/2)\ \rho\ (u_{in})^2}. \tag{9}$$

Table 1. Pressure variation though the channel.

Aspect ratio (AR)	$\Delta\overline{p}$ [MPa]	$C_I x10^{-6}$
2	4,092	1,9874
4	2,745	1,3332
6	2,426	1,1780

The results show that the pressure drop decreases with increasing the aspect ratio. The maximum pressure drop occurs for the AR = 2, equal to $\Delta\overline{p}$ = 4,092 MPa. For the values of the AR = 4 and 6, comparing with AR = 2, the pressure drop decreases 32.92 % and 40.723%, respectively. This result indicates that channel aspect ratio is an important variable for obtained the pumping power required to sustain the polymer flow.

The average Nusselt number as function of the aspect ratio is presented in Fig. 3. The following Nusselt definition is adopted,

$$\overline{Nu} = \frac{\overline{h_c}\, s}{k} \tag{10}$$

where s is equal to step height (see Fig. 1). The convective coefficient is defined as

$$\overline{h_c} = \frac{\overline{q"_s}}{(T_w - T_{in})}. \tag{11}$$

In the preceding expression $\overline{q"_s}$ is the average heat flux evaluated at the four walls for a given value of x (see Fig. 1). The distributions of the average Nusselt number along the channel length (downstream of the expansion section) are presented for AR = 2, 4 and 6.

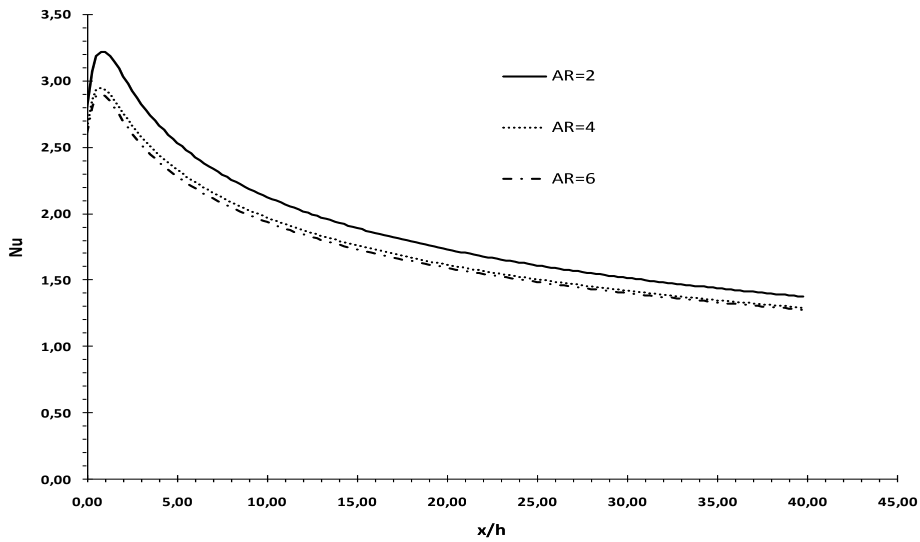

Figure 3. Nusselt number distribution along the channel length.

The simulation shows that the Nusselt number presents a maximum value 3.22 at the x/h = 0,75 for the AR=2, for the values of the AR=4 and 6 the Nusselt number decreases 8,38% and 9,94%, respectively. It is interesting to mention that narrow channel (AR=2) presents the maximum Nusselt number due to higher temperature gradient near walls.

The local Nusselt number distributions for different aspect ratios are presented in Fig. 4 (in the stepped wall). The following Nusselt definition is adopted,

$$Nu = \frac{h_c s}{k} \tag{12}$$

where s is equal to step height. The convective coefficient is defined as

$$h_c = \frac{q"_s}{(T_w - T_{in})}. \tag{13}$$

In the preceding expression $q"_s$ is the heat flux evaluated in the middle of channel (*z/W = 0.5*) at the lower stepped wall (*y/h=0*). The distributions of the local Nusselt number along the streamwise direction are presented for AR = 2, 4 and 6.

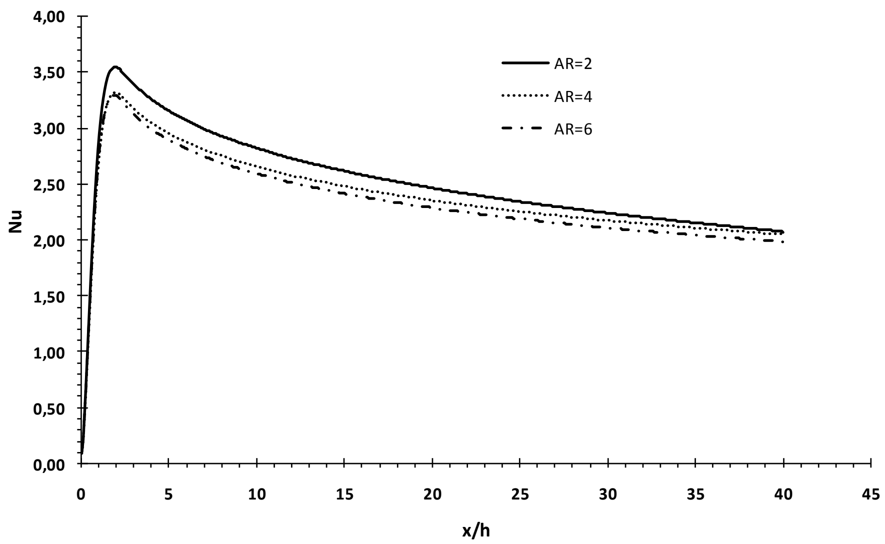

Figure 4. Local Nusselt number distribution along the streamwise direction (*z/W = 0.5*).

The simulation shows that the local Nusselt number in the middle plane (*z/W = 0.5*) has a similar behavior to the average Nusselt, i.e, the local Nusselt number presents a maximum value 3.544 at the *x/h* = 1.88 for the AR=2, for the values of the AR=4 and 6, the Nusselt number decreases 6.35 % and 7.25%, respectively.

Finally, the local Nusselt number distributions for different *z/W* planes are presented in Fig. 5. The Nusselt number definition and the convective coefficient adopted are similar to equations (12) and (13). In this case q''_s is the heat flux evaluated in three different positions (*z/W = 0.5, 0.7* and *0.9*) at the lower wall (*y/h=0*). The distributions of the local Nusselt number along streamwise direction are presented for AR = 4.

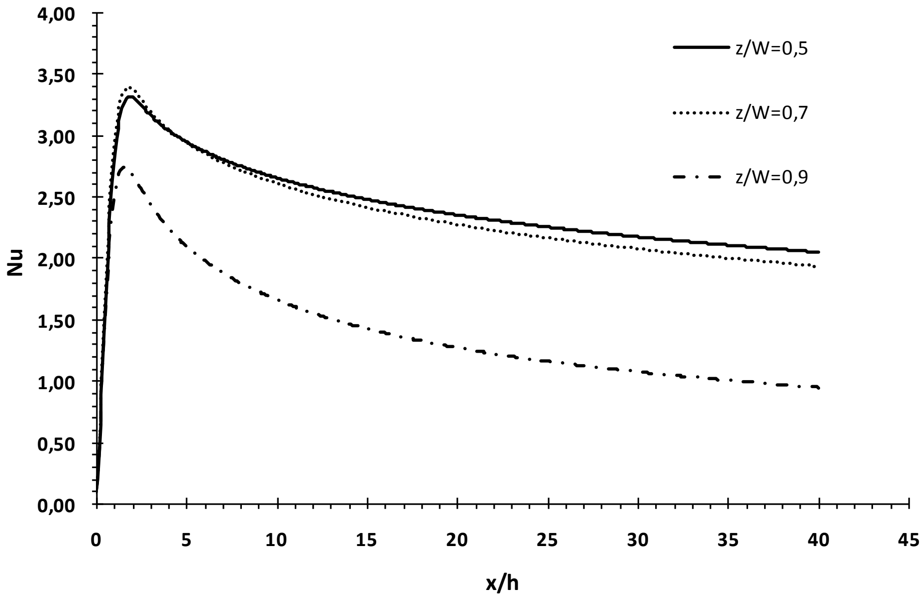

Figure 5. Local Nusselt number distribution along the streamwise direction (*AR = 4*).

The results indicate that in the vicinity of the lateral wall (*z/w = 0.9*) the Nusselt number is lower than in the channel center (*z/w = 0.5*). This result is consistent since the near wall region presents lower velocities.

4. CONCLUSION

The non-isothermal polymer melt flow in 3D asymmetric sudden expansions was studied numerically using the finite-volume method. The generalized Newtonian formulation was adopted to analyze the behavior of polymer, described by the Cross constitutive relation.

Firstly, the numerical solution was compared with other literature works and the agreement was acceptable. Afterwards, the physical analysis clearly shows that pressure losses and the Nusselt number are directly affected by the aspect ratio. Both pressure drop and the Nusselt number decrease with increasing of the AR. Notice that the aspect ratio effect is more sensitive for pressure drop than the Nusselt number.

5. REFERENCES

Bom, R. P., Hermann, M. H. and Hoays, H. S., 2000. “Análise reológica via simulador de fluxo”. SULMAT, Joinville¸ Brazil, pp. 948-957.

Hassan, H., Regnier, N. and Defaye, G., 2009. “A 3D study on the effect of gate location on the cooling of polymer by injection molding ”. International Journal of Heat and Fluid Flow, Vol. 30, pp. 1218-1229.

Hassan, H., Regnier, N., Le Bot, C. and Defaye, G., 2010. “3D study of cooling system effect on the heat transfer during polymer injection molding ”. International Journal of Thermal Sciences, Vol. 49, pp. 161-169.

Koh, Y. H., Ong, N. S., Chen, X. Y., Lam, Y. C. and Chai, J. C., 2004. “Effect of temperature and inlet velocity on the flow of a nonnewtonian fluid”. Int. Comm. Heat Mass Transfer, Vol. 31, No 7, pp. 1005-1013.

Mânica, R., and De Bortoli, A. L., 2004, “Simulation of sudden expansion flow for Power-Law fluids”. Journal of Non-Newtonian Fluid Mechanics, Vol. 121, pp. 35–40.

Oberkampf, W. L. and Trucano, T. G., 2002. “Verification and validation in computational fluid dynamics”. Progress Aerospacial Sci, Vol. 38, pp. 209-272.

Patankar, S. V. and Spalding, D. B., 1972. “A Calculation Procedure for Heat Mass and Momentum Transfer in Three Dimensional Parabolic Flows”. Int. J. Heat Mass Transfer, Vol. 15, pp. 1787-1806.

Pinho, F. P., Oliveira, P. J. and Miranda, J.P., 2003. “Pressure losses in the laminar flow of shear-thinning Power-Law fluids across a sudden axe-symmetric expansion”. International Journal of Heat and Fluid Flow, Vol. 24, pp. 747-761.

Rhie, C. M. and Chow, W. L., 1983. "Numerical Study of the Turbulent Flow Past an Airfoil with Trailing Edge Separation". AIAA Journal.21(11), pp .1525–1532.

Zdanski, P.S.B., Vaz Jr., M. and Inácio, G. R., 2008. “A finite volume approach to simulation of polymer melt flow in channels”. Engineering Computations. Vol. 25 No. 3, pp. 233-250.

Zdanski, P.S.B. and Vaz Jr., M., 2009a. “Three-dimensional polymer melt in sudden expansion: Non-isothermal flow topology”. International Journal of Heat and Mass Transfer, Vol. 52, pp. 3585-3594.

Zdanski, P.S.B. and Vaz Jr., M., 2009b. “Non-isothermal polymer melt flow in sudden expansions”. Journal Non-Newtonian Fluid Mech, Vol. 161, pp. 42-47.

Zdanski, P.S.B., Vaz Jr., M. and Dias, A. P. C., 2011. “Forced convection heat transfer of polymer melt flow inside channels with contraction/expansion sections”. International Communications in Heat and Mass Transfer, Vol. 38, pp. 1335-1339.

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