CONSTRUCTAL DESIGN OF FINNED ASSEMBLIES WITH RECTANGULAR SHAPE TO IMPROVE THE HEAT REMOVAL BY CONVECTION

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Abstract. *This numerical study applies Constructal Design to find the shape of a finned assembly that provides the most effective heat removal by means of convection. Here a rectangular heat generating medium is cooled by four finned assemblies with 3, 5, 7 and 9 number of rectangular fins. Both, the assembly of fins and the heat-generating medium, are made of cooper and the numerical solution assumes a perfect thermal contact between them. In order to find the configuration that reduces most the overall thermal resistance of the whole system a numerical code based on finite elements and developed in MATLAB environment is used to solve the temperature field in the domain for all geometrical configurations studied. In this context, the area of the heat-generating medium and the total area of the system, as well as the thermal conductivities are kept constant.*

Keywords*: Constructal Design, Convection, Finned assemblies, Heat removal, Overall thermal resistance.*

1. INTRODUCTION

Constructal Design, the method used to optimize flow systems, animated or inanimated, is based on a physics principle, the Constructal Law: "For a finite size flow system to persist in time (to survive) it must evolve in such a way that it provides easier access to the imposed currents that flow through it". Thus, based on Constructal Law, Constructal Design has been widely used to optimize flow systems in the engineering field and its aplication on extended surfaces is recent on the literature. This method is based on the exhaustive search for the optimal geometry, varying freely the degrees of freedom considering the constraints of the problem. In Bejan and Almogbel (2000), Constructal Design was applied in the optimization process of fins with a "T" shape in order to obtain the maximization of the overall thermal conductance of the whole system considering the imposed volume and materials constraints of the fins. Bonjour et al. (2004) performed an analytical and numerical study concerning the geometric optimization of fins with a radial and "ramified" shape inserted into heat exchangers with co-axial flows. It was analysed the relation between performance and architecture of fins. Matos et al. (2004) conducted a tridimensional study to optimize staggered arrangements of heat exchangers with circular and eliptical tubes. Lorenzini et al., (2009) proposed the geometric optimization of fins with "T-Y" shape, i.e., a configuration where exist a cavity between the two branches of the fins. Again, the objective was the minimization of the global thermal resistance of the system concerning the total volume and the constraints. The results showed that the cavity with a lower volume and fins with a higher volume increase the performance of the finned assembly. Lorenzini and Rocha, (2006) minimized the thermal global resistance considering the total volume and materials constraints of the fins aiming the optimization of finned assemblies with an "Y" shape. This was a complete optimization study performed with the exhaustive search, i.e., all degrees of freedom were optimized and the global thermal resistance as well as the optimal geometry were correlated by power laws.

In this work, Constructal Design is applied to find the best configuration of finned assembly aiming the highest heat removal by convection. Here, the cooling of a heat generating medium with rectangular shape is investigated by finned assemblies with N = 3, 5, 7 and 9 fins with rectangular shape. It's assumed for both, the finned assembly and the heat generating medium, the thermal conductivity of cooper and the numerical solution considers a perfect thermal coupling between them. In order to find the configuration that reduces most the global thermal resistance of the whole system, a numerical code based on finite elements is used to determine the temperature field of the whole mathematical domain. In this context, the area of heat generating medium and the total area of the system, as well as the thermal conductivities and the dimensionless heat transfer coefficient, λ, are considered constants.

2. MATHEMATICAL MODEL

Consider the body shown in Fig. 1. The configuration is two-dimensional with the third dimension (*W*) sufficiently long when compared with the other three heights H_0, H_1 e H_2 and the lengths L_0, L_1 and L_2. The rectangular heat generating element has a thermal conductivity *k* and the finned assembly has a thermal conductivity k_F. The rectangular element generates heat uniformly in a volumetric rate $q'''(W/m^3)$. The lower and side surfaces are insulated and the heat generated current ($q'''AW$) is first removed by conduction and then convection through the fins.

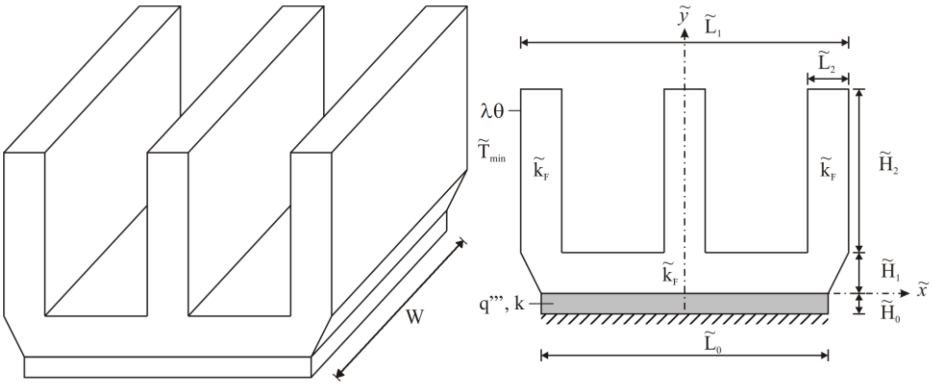


Figure 1. Rectangular heat generating element cooled by a finned assembly with three cooper fins.

This work consists in calculating the maximum excess dimensionless temperature $(T_{max}-T_{min})/(q'''A/k)$ and see which geometry (L_1/H_0, L_2/H_2, H_1, N) best removes the generated heat. It is assumed a steady state problem with uniform heat generation. The equation for heat diffusion to the heat generating medium is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{q'''}{k}=0 \tag{1}$$

Equation (1) can be put in dimensionless form using the following dimensionless groups.

$$\theta=\frac{T-T_\infty}{q'''A/k} \tag{2}$$

$$\tilde{x},\tilde{y},\tilde{k},\tilde{k}_p,\tilde{L}_0,\tilde{H}_0,\tilde{L}_1,\tilde{H}_1,\tilde{L}_2,\tilde{H}_2=\frac{x,y,k,k_p,L_0,H_0,L_1,H_1,L_2,H_2}{A^{1/2}} \tag{3}$$

where *A* in Eq. (3) represents the total area of the mathematical domain. This analysis provides the excess dimensionless temperature as a function of the geometry of the finned assembly solving the heat diffusion equation with uniform heat generation for the rectangular element placed at the base of the considered domain. The heat diffusion equation in dimensionless form is represented by

$$\frac{\partial^2 \theta}{\partial \tilde{x}^2}+\frac{\partial^2 \theta}{\partial \tilde{y}^2}+1=0 \tag{4}$$

Assuming no heat generation in the finned assembly, the heat diffusion for this region is

$$\frac{\partial^2 \theta}{\partial \tilde{x}^2}+\frac{\partial^2 \theta}{\partial \tilde{y}^2}=0 \tag{5}$$

The maximum excess of dimensionless temperature, θ_{max}, is defined as

$$\theta_{max}=\frac{T_{max}-T_\infty}{q'''A/k_F} \tag{6}$$

where k_F is fins thermal conductivity of the finned assembly. Evaluating the thermal resistance, Eqs. (4) and (5) are solved using the boundary conditions given considering the finned assembly subjected to convection, i.e.:

$$\frac{\partial \theta}{\partial n} = -\lambda \theta \tag{7}$$

where λ is given by Eq. (8) as

$$\lambda = \frac{hA^{1/2}}{k} \tag{8}$$

The sides as the bottom of the heat generating medium are considered adiabatic. These boundary conditions in dimensionless terms are given by

$$\frac{\partial \theta}{\partial n} = 0 \tag{9}$$

The next step is to achieve optimization, i.e., minimizing the thermal resistance given by Eq. (6). In this context it is considered two constraints of area. The first constraint is the total area of the mathematical domain

$$A = NL_2H_2 + \frac{1}{2}(L_1 + L_0)H_1 + L_0H_0 \tag{10}$$

and the second constraint is the area occupied by the rectangular heat generating element.

$$A_p = L_0H_0 \tag{11}$$

Dividing Eq. (11) by Eq.(10) yields the fraction of area, ϕ_o, of the heat generating element

$$\phi_p = \frac{A_p}{A} \tag{12}$$

which can be expressed in dimensionless terms as

$$\phi_p = \tilde{L}_0\tilde{H}_0 \tag{13}$$

From Eq. (13) it is possible to express Eq. (10) in dimensionless form::

$$1 - \phi_p = N\tilde{L}_2\tilde{H}_2 + \frac{1}{2}(\tilde{L}_1 + \tilde{L}_0)\tilde{H}_1 \tag{14}$$

In the search for the optimal solution, the problem considers four degrees of freedom: H_1, L_1/H_1, L_2/H_2 N, and five constants: $\phi_p = 0.15$, $\lambda = 1$, $L_0/H_0 = 10$, $k_F = k = 300$. From equations presented it is computationally possible to vary the parameters presented above and thus to obtain the geometry that facilitates the flow of heat out of finned assembly.

3. NUMERICAL MODEL

The numerical method was developed for configurations involving 3, 5, 7 and 9 fins for finned assembly. The function defined by Eq. (6) is determined by numerical solution of the heat conduction equation given by Eqs. (4) and (5) for the temperature field of each configuration assumed, depending on the chosen degrees of freedom. The mesh used was obtained through successive refinements until the criterion $\left|(\theta^j_{max} - \theta^{j+1}_{max})/\theta^j_{max}\right| \le 5\times10^{-4}$ is satisfied, where θ^j_{max} is the maximum excess of dimensionless temperature calculated using the actual mesh, θ^{j+1}_{max} represents the maximum excess of dimensionless temperature calculated from the next mesh refinement, when the number of elements increases four times. The numerical solution of the heat conduction equation was obtained with the implementation of a finite element code based on triangular elements and developed using MATLAB. Table 1 shows tests performed to obtain mesh independence from the number of elements.

Table 1. Mesh refinements to obtain results independent of elements number, considering: N = 3, L_1/H_1 = 106, L_2/H_2 = 0.015 and H_1 = 0.01.

Number of nodes	Number of triangular elements	θ^j_{max}	θ^{j+1}_{max}	$\left\|(\theta^j_{max} - \theta^{j+1}_{max})/\theta^j_{max}\right\| \le 5\times10^{-4}$
1193	2015	0.008301	0.008306	5,9 x 10-4
4400	8060	0.008306	0.008307	1,7 x 10 -4
16859	32240	0.00837	-	-

Thus the mesh, considering the proposed independence criterion, consists of 32240 triangular elements in case the number of fins N = 3. Similar tests were carried out for the case of N = 5, 7 and 9 obtained independent meshes with numbers 13936, 33408 and 36064 triangular elements. In the next section are performed a series of simulations, varying the degrees of freedom of the problem, in order to obtain the optimal geometry.

4. RESULTS

At first, it was found the maximum excess of dimensionless temperature of finned assemblies considering a range of values for L_2/H_2. Figure 2 shows the optimization of the only one degree of freedom L_2/H_2 considering L_1/H_1 = 6, H_1 = 0.08 and N = 3.

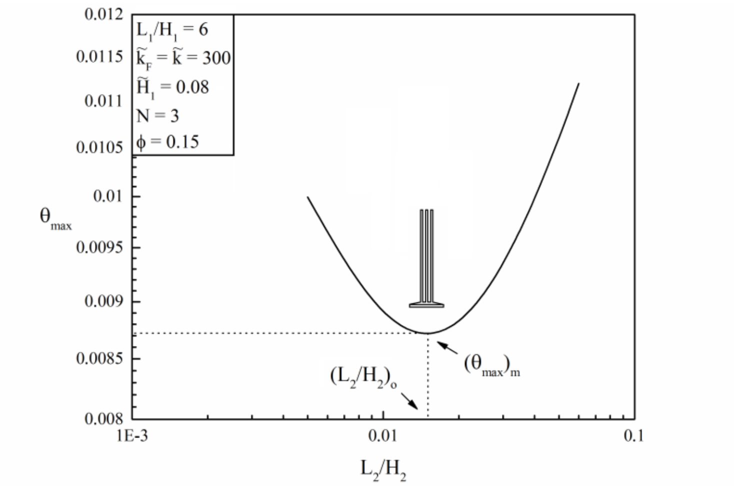
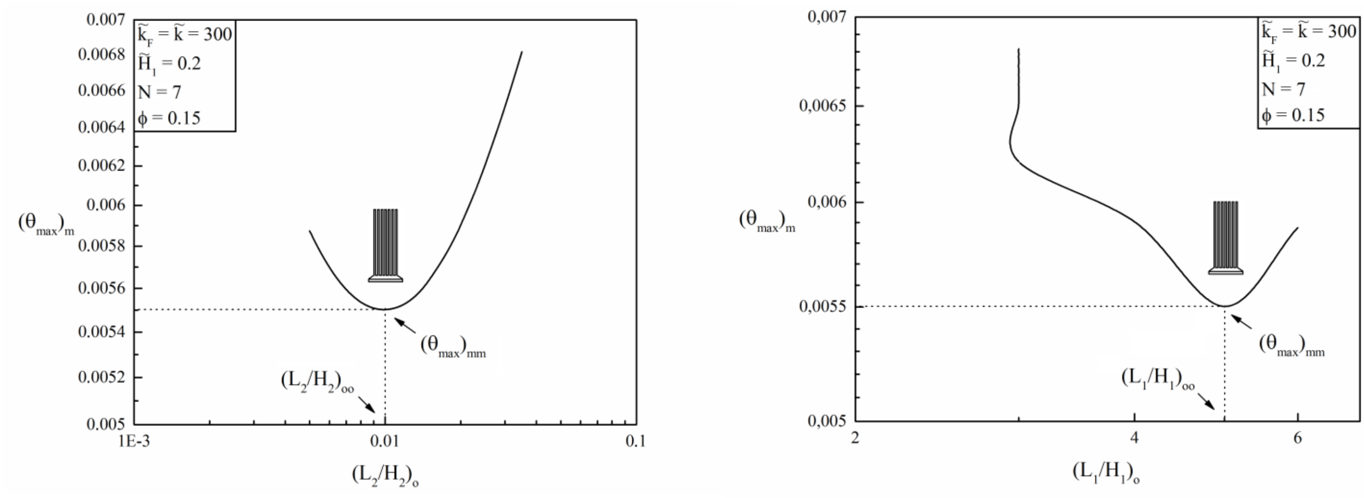


Figure 2. Maximum excess of dimensionless temperature, θ_{max}, for a range of values of L_2/H_2.

Considering the geometry optimization of the degree of freedom L_2/H_2 shown in Fig. 2, the minimum value $(\theta_{max})_m$ = 0.00872 is achieved for $(L_2/H_2)_o$ = 0.015. This optimum value found for L_2/H_2 is repeated for most simulations and their constancy is reason for further investigation.

In a second moment the geometry is doubly optimized with respect to two ratios L_1/H_1 and L_2/H_2. The other constant arbitrated above remain the same. The graphs of Fig. 3 shows the thermal optimizations for finned assemblies with the number of fins N = 7. The numerical values for the maximum excess of dimensionless temperature $(\theta_{max})_m$, of the following example were obtained by varying $(L_1/H_1)_o$ from 2 to 6 and values of $(L_2/H_2)_o$ between 0.001 and 0.035.

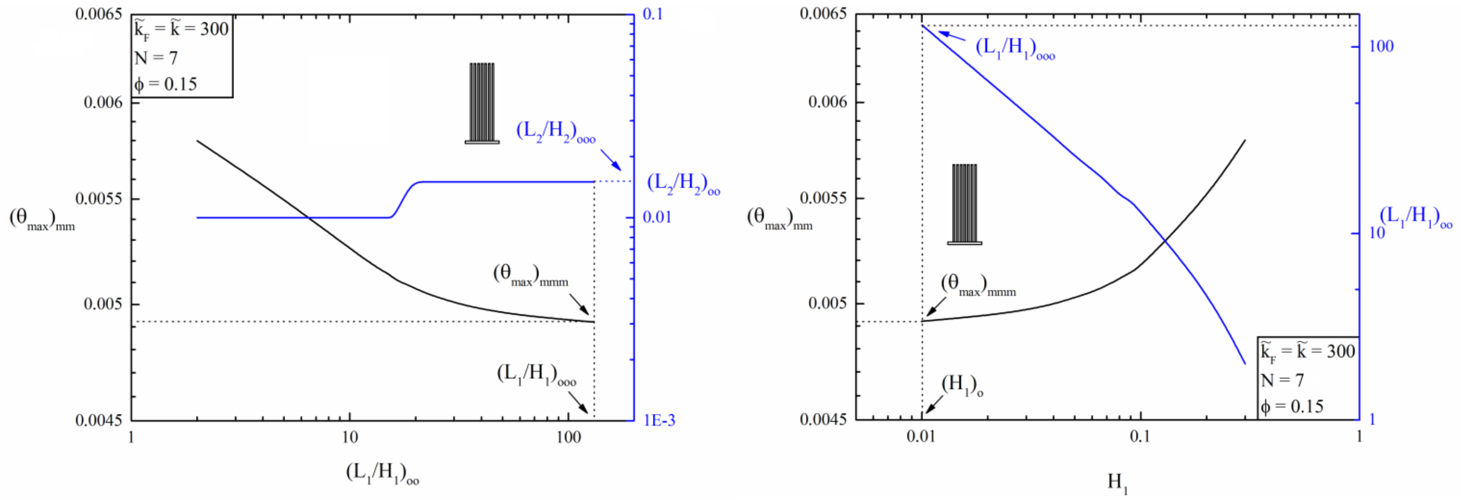
The analysis of graphics (a) and (b) of Fig. 3 reveal doubly optimized values for the geometries $(L_2/H_2)_{oo}$ = 0.01 and $(L_1/H_1)_{oo}$ = 5 considering a number of fins, N = 7, of the finned assembly and H_1 = 0.2. For the value of H_1 = 0.2 the copper layer between the fins and the rectangular heat generating element is considerably thick which contributes to the optimum values of the ratio L_1/H_1 are smaller in relation to other configurations with lower H_1 values .



(a) (b)

Figure 3. Maximum excess of dimensionless temperature, $(\theta_{max})_m$: (a) for a range of values of $(L_2/H_2)_o$, (b) for a range of values of $(L_1/H_1)_o$.

In the third stage are considered the simultaneous optimization of geometries, L_1/H_1, L_2/H_2 and H_1 of the finned assembly. In the example shown in Fig. 4 below, it was taken again a value $N = 7$ for the number of fins, and as in the previous analysis, it was optimized ratios L_1/H_1 and L_2/H_2, but in this case it was used different values of H_1 and not a single fixed value. The figures showed that lower values of H_1 tend to reduce the maximum excess of dimensionless temperature, which can be checked in graph (b) of Fig. 4, where the value $(H_1)_o = 0.01$. Similarly, lower values of H_1 tend to provide optimal high values of the ratio L_1/H_1, which is observed in the graph (a) of Fig. 4 the three times optimized ratio $(L_1/H_1)_{ooo} = 130$.



(a) (b)

Figure 4. Maximum excess of dimensionless temperature, $(\theta_{max})_{mm}$: (a) for a range of values of $(L_1/H_1)_{oo}$, (b) for a range of values of H_1.

In the last step it was taken the optimization process considering the simultaneous optimization of four degrees of this problem, that is, the ratios L_1/H_1, L_2/H_2 and height H_1 and the number of fins, N, of the finned assembly. The analysis showed an optimal ratio L_2/H_2 ranging values from 0.01 to 0.015, low values for the optimized dimensionless height H_1 and hence high values for the optimal ratio L_1/H_1.

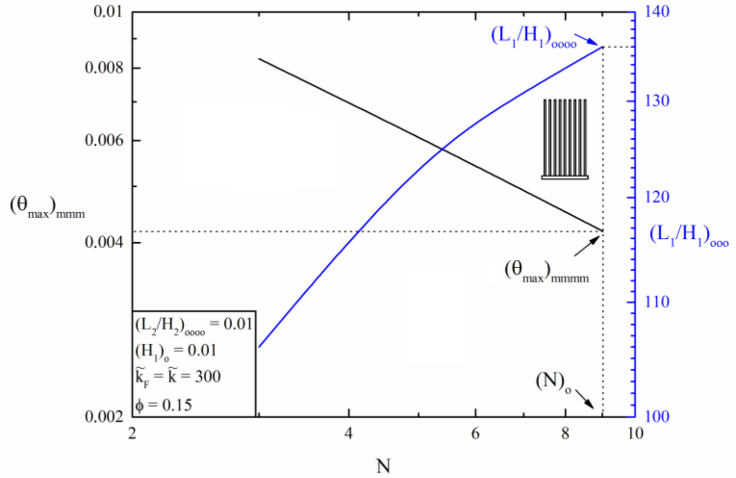


Figure 5. Maximum excess of dimensionless temperature, $(\theta_{max})_{mmm}$ for different values of *N*.

In the last analysis it was verified the relevance of number of fins, *N*, of the finned assemblies, considering simulations with values of *N* = 3, 5, 7 and 9 fins. All procedures previously carried in the optimization process were performed again for each value of *N* and it was found that the reduction of maximum excess of dimensionless temperature, θ_{max}, is obtained for values of $(L_1/H_1)_{oooo} = 136$, $(L_2/H_2)_{oooo} = 0.01$, $H_1 = 0.01$ and $N = 9$ which generated a dimensionless temperature four times minimized, $(\theta_{max})_{mmmm} = 0.0041$. Figures (6 – 9) present a visual analysis of optimized geometries, considering different values of *N* and H_1. For each situation is presented the optimal values of L_1/H_1 and L_2/H_2.

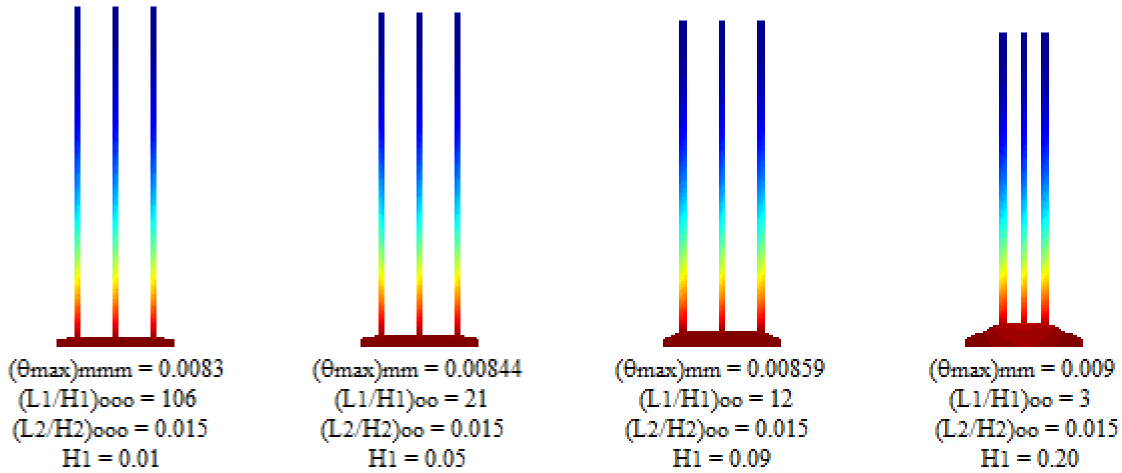


Figura 6. Geometries optimized with respect to H_1 and ratios L_1/H_1 and L_2/H_2 for a number of fins, $N = 3$.

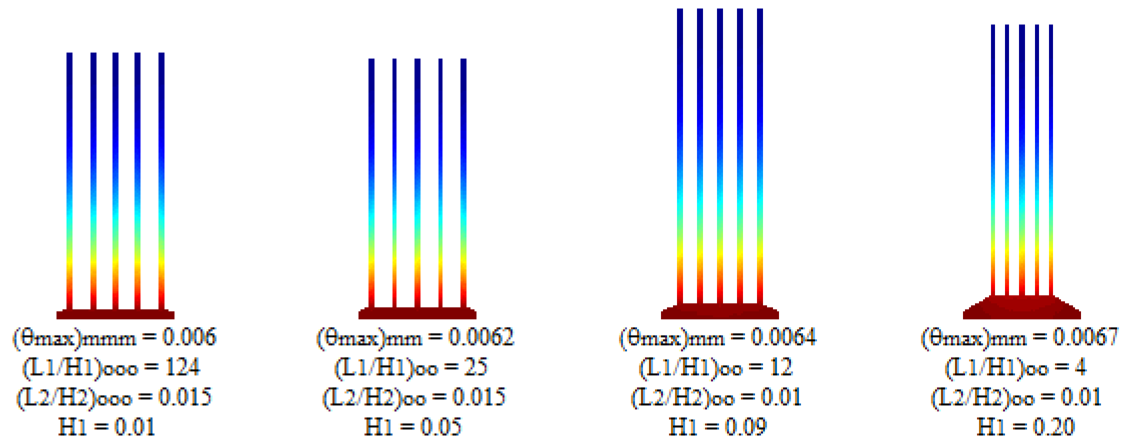


Figure 7. Geometries optimized with respect to H_1 and ratios L_1/H_1 and L_2/H_2 for a number of fins, $N = 5$.

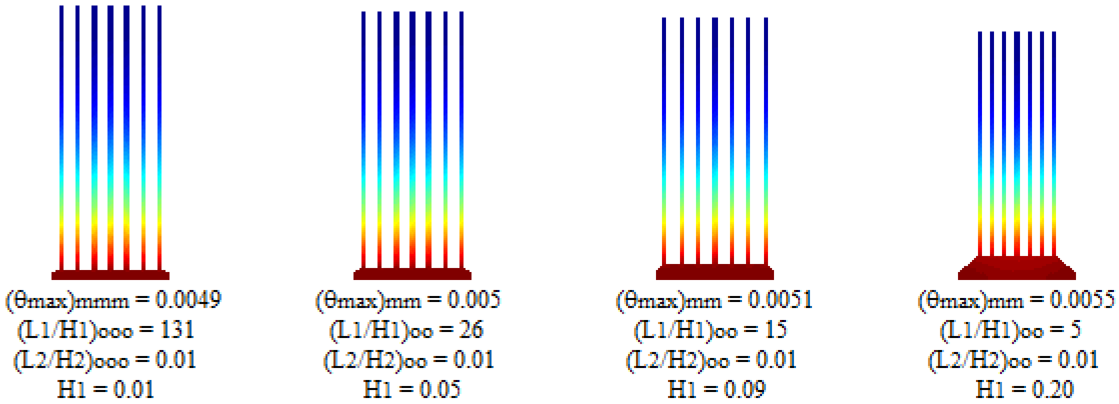


Figure 8. Geometries optimized with respect to H_1 and ratios L_1/H_1 and L_2/H_2 for a number of fins, N = 7.

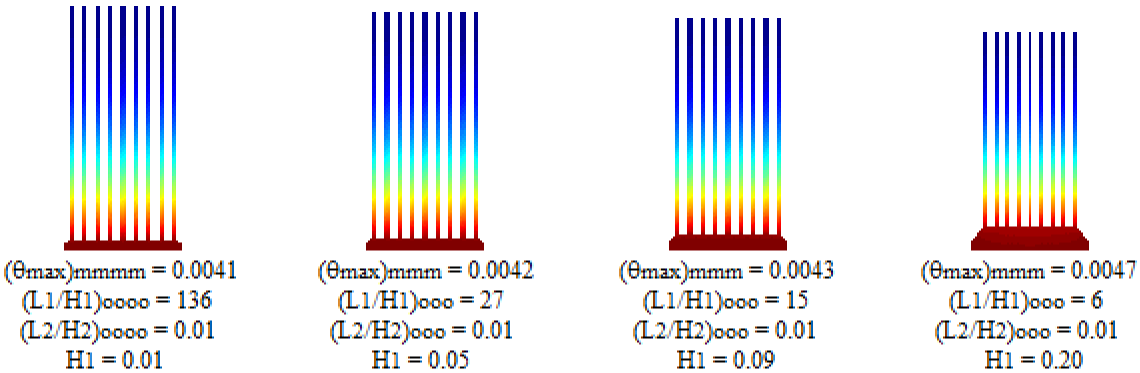


Figure 9. Geometries optimized with respect to H_1 and ratios L_1/H_1 and L_2/H_2 for a number of fins, N = 9.

5. CONCLUSIONS

A numerical work involving Constructal Design was developed, seeking the optimal geometry of a finned assembly to maximize the heat transfer between a rectangular heat generating element and fins. In this process different values for L_1/H_1 and L_2/H_2 ratios were investigated, in the same way as the height H_1 and the number of fins, N, of the finned assembly. From the simulations it was possible to state some conclusions about the optimization process. The numerical results obtained in the first step of the optimization process, when it was optimized a single degree of freedom yield the optimal ratio $(L_2/H_2)_o$ = 0.015 and this value was repeated in most simulations including other degrees of freedom. The fact that the optimal ratio L_2/H_2 = 0.015 occurs in most simulations, as well as the value of L_2/H_2 = 0.01 is subjected to be verified in further studies and may be associated with thermal conductivity values of the finned assembly, k and k_F, and the value λ assigned to this work. The simultaneous optimization of three degrees of freedom showed that the optimization process goes toward the decreasing of values of H_1, since every value assumed for this dimensionless height serves as an insulator with respect to the heat generating element diminishing the removal of heat through the fins. Therefore, the geometry three times optimized was obtained for the value $(H_1)_o$ = 0.01. The ultimately analysis found that increasing the number of fins increases the heat removal from the finned assembly. Visual analysis of the optimized geometries also reveals that the increasing of N, in the finned assemblies, has the greatest relevance on reducing the maximum excess of dimensionless temperature as compared to changes in other degrees of freedom.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Bejan, A., 1997. Constructal-theory network of conducting paths for cooling a heat generating volume, Int. J. Heat Mass Transf. 40 (4) 799-816.

Bejan, A.; Almogbel, M. Constructal, 2000, T-shaped Fins, International Journal of Heat and Mass Transfer, v. 43(12-15), p.2101-2115.

Bejan, A., Lorente, S., 2008. Design with Constructal Theory, Wiley, New Jersey.

Bejan, A., Zane, J. P., 2012. Design in Nature: How the Constructal Law Governs Evolution in Biology, Physics, Technology, and Social Organization, Random House LLC, New York.

Biserni, C., Rocha, L. A. O., Stanescu, G., Lorenzini, G. 2007. Constructal H-shaped cavities according to Bejan's theory, Int. J. Heat Mass Transf. 50 2132 – 2138.

Boichot, R., Luo, L., Fan, Y., 2009. Tree-network structure generation for heat conduction by cellular automaton, Energy Convers. Manage. 50 376–386.

Bonjour, J.; Rocha, L. A. O.; Bejan, A.; Meunier, F., 2004, Dendritic Fins Optimization for a Coaxial Two-Stream Heat Exchanger, International Journal of Heat and Mass Transfer, v.47 (1) p. 111-124.

Da Silva, A.K., Vasile, C., Bejan, A., 2004. Disc cooled with high-conductivity inserts that extend inward from the perimeter, Int. J. Heat Mass Transf. 47 4257–4263.

Dirker, J., Meyer, J. P., 2009a. Heat removal from power electronics in two direction sets using embedded solid state cooling layers – a proposed non-numerical calculation method, Heat Transfer Eng. 30 (6) 452-465.

Dirker J., and Meyer J.P., 2009b. Thermal characterization of embedded heat spreading layers in rectangular heat-generating electronic modules, Int. J. Heat Mass Transf. 52 1374-1384.

Ghalambor, S., Agonafer, D., Haji-Sheikh, A. 2013. Analytical Thermal Solution to a Nonuniformly Powered Stack Package with Contact Resistance, J. Heat Transfer, 135 111015-1 (9 pages).

Ghodoossi, L. Egrican, N., 2004. Conductive cooling of triangular shaped electronics using constructal theory, Energy

Hajmohammadi, M. R., Shariatzadeh, O. J., 2014. Phi and Psi shaped conductive routes for improved cooling in a heat generating piece. Int. J. Therm. Sci. 77 66–74.

Horbach, C. S., Dos Santos, E. D., Isoldi, L. A., Rocha, L. A. O., 2014. Constructal design of Y-shaped pathways for cooling a heat-generating body, Defect Diffus. Forum 348 (1) 245-260.

Kuddusi, L., Denton, J. C., 2007. Analytical solution for heat conduction in composite slabs and its implementation in constructal solution for cooling of electronics, Energy Convers. Manage. 48 1089-1105.

Ledezma, G. A., Bejan, A., Errera, M. R., 1997. Constructal tree networks for heat transfer, J. Appl. Phys. 82 (1) 89–100.

Li, Q., Steven, G. P., Xie, Y. M., Querin, O. M., 2004. Evolutionary topology optimization for temperature reduction of heat conducting fields, Int. J. Heat Mass Transf. 47 (23) 5071–5083.

Lorenzini, G., Rocha, L. A. O. 2006. Constructal Design of Y-Shaped Assembly of Fins, Int. J. Heat Mass Transf. 49 4552-4557.

Lorenzini, G.; Rocha, L. A. O., 2009, Constructal Design of T-Y Assembly of Fins for an Optimized Heat Removal, International Journal of Heat and Mass Transfer, v. 52, p.1458-1463.

Lorenzini, G., Biserni, C., Rocha, L. A. O. 2013. Constructal design of non-uniform X-shaped conductive pathways for cooling, Int. J. Thermal Sci. 71 140-147.

MATLAB, 2000. user's guide, version 6.0.088, release 12, The Mathworks Inc.

Matos, R.S.; Laursen, T.A.; Vargas, J.V.C.; Bejan, 2004, A. Three-Dimensional Optimization of Staggered Finned Circular and Elliptic Tubes in Forced Convection, International Journal of Thermal Sciences, v. 43 p. 477-487.

Rocha, L.A.O., Lorente, S., Bejan, A., 2002. Constructal design for cooling a disc-shaped area by conduction, Int. J. Heat Mass Transf. 45 1643-1652.

Rocha, L. A. O., Lorente, S., Bejan, A., 2006. Conduction tree networks with loops for cooling a heat generating volume, Int. J. Heat Mass Transf. 49 2626-2635.

Steven, G. P., Li, Q., Xie, Y. M., 2000. Evolutionary topology and shape design for general physical field problem, Comput. Mech. 26 (2) 129–139.

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