

PREDICTION OF GAS LEAKAGE IN THE PISTON-CYLINDER CLEARANCE OF RECIPROCATING COMPRESSORS

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Abstract. *The leakage of gas through the piston-cylinder clearance can significantly reduce the volumetric and isentropic efficiencies of reciprocating compressors. This paper presents a model based on the Reynolds equation for compressible fluid flow to predict such a leakage in a coupled manner with a model that simulates the compression cycle. Some parameters associated with flow geometry and piston velocity are investigated in order to determine their effects on the gas leakage and volumetric and isentropic efficiencies. The results reveal that the piston velocity plays an important role in the gas leakage, an aspect that was not observed in previous incompressible flow models available in the literature.*

Keywords: *Refrigeration compressor, Reciprocating compressor, Piston-cylinder clearance, Leakage*

1. INTRODUCTION

Gas leakage is a major source of inefficiencies in refrigeration compressors. Not only does it lower the mass flow rate provided by the compressor, reducing its volumetric efficiency, but also gives rise to outflux of energy due to gas leakage, decreasing the isentropic efficiency. According to Silva and Deschamps (2015), a gap of 1 μ m between the valve and its seat can reduce the volumetric and isentropic efficiencies by 2.7% and 4.4%, respectively. With respect to reciprocating compressors, the two main sources of gas leakage are: (i) the gap between the valve and the valve seat and (ii) the piston-cylinder clearance.

Two aspects bring about the flow in the piston-cylinder clearance: (i) the difference of pressure between the compression chamber and the internal environment of the compressor shell, (ii) the motion of the piston. In most reciprocating compressors, such clearance is filled by a mixture of refrigerant gas and lubricating oil. Besides its lubricating function, the oil also acts as sealing element for the piston-cylinder clearance. Therefore, the leakage in the piston-cylinder clearance is more detrimental to oil-less compressors. Moreover, leakage is more critical in low-capacity compressors, since it may be comparable to the mass flow rate of such compressors. The reduction of the clearance results in higher manufacturing costs and may considerably increase the energy loss due to friction in the clearance.

The leakage in the clearance between the cylinder and the piston has been the object of several studies. Zuk and Smith (1969) proposed an analytical model to estimate leakage in small gaps between parallel plates, applying the Reynolds equation for compressible flow with static boundaries. Ferreira and Lilie (1984) presented an analytical model to assess the leakage in the piston-cylinder gap with a moving piston, by assuming an incompressible flow formulation and disregarding inertia forces. Another analytical model was proposed by Yuan et. al (1992), considering inertia and viscous forces and gas compressibility effects, but without taking into account the piston velocity. The authors observed that inertia effects are negligible for clearances smaller than 6 μ m.

Rigola et al. (2009) applied a numerical model based on the two-dimensional incompressible Reynolds equation including the piston velocity. They reported an exponential behavior between the size of the gap and the leakage rate, and analyzed different compressors and refrigerant fluids. Lohn and Pereira (2014) developed a three-dimensional numerical model for predictions of leakage in the piston-cylinder clearance, which were compared with the results of two analytical models: (i) Zuk and Smith (1969) and (ii) Ferreira and Lilie (1984). Lohn and Pereira (2014) verified that the mass flow rates obtained via a compressible flow formulation were considerably different from those calculated with the incompressible flow formulation, pointing out that the compressible formulation is more suitable for this type of flow. They also showed that the piston velocity significantly affects the gas leakage rate when the pressure difference is kept constant.

There are other related studies focused on the fluid film lubrication in the piston-cylinder clearance, introducing the presence of lubricating oil (Couto, 2001; Hülse, 2008; Dias, 2012). In such situations, the leakage in the clearance is often regarded as consumption of oil. Additionally, there are also studies addressing leakage of gas in other types of compressor, for which leakage is an even bigger issue, such as rotary compressors (Pandeya and Soedel, 1978; Costa et al., 1990; Teh and Ooi, 2008; Bell et al., 2012) and axial flow compressors (Berdanier, 2015).

Although recent studies have shed some light on the effects of piston velocity on the gas leakage rate, the models available in the literature are either accurate and computationally too expensive, such as two and three-dimensional numerical models, or too simple analytical models that do not take into account compressibility effects or the piston velocity. The present paper reports a numerical model based on the one-dimensional compressible Reynolds equation, including the piston motion. The model considers only the refrigerant fluid in the clearance and can predict the gas leakage

at a low computational cost, allowing its use in a coupled manner with a simulation model for the compression cycle. Thus, the leakage in the piston-cylinder clearance is calculated for different piston positions and pressure differences during the compression cycle. The role of the piston motion on the gas leakage rate is investigated throughout the entire compression cycle.

2. NUMERICAL MODEL AND SOLUTION PROCEDURE

The flow geometry addressed herein is illustrated in Fig. 1. The region between the piston and the cylinder of length L and clearance δ represents the solution domain. The gas in the compression chamber has a pressure higher than the gas in the compressor internal environment for most of the compression cycle. This pressure difference and the piston motion bring about the flow in the piston-cylinder clearance. When the piston moves towards the cylinder head, gas is dragged into the compression chamber and vice-versa when the piston moves in the other direction.

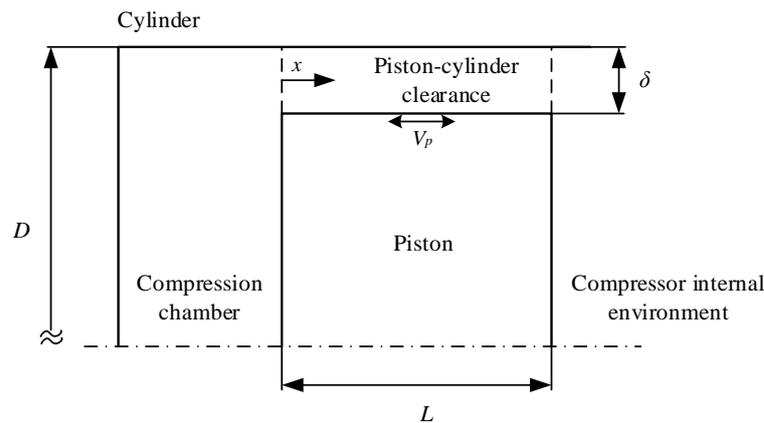


Figure 1. Flow geometry.

The compressible flow in the piston-cylinder clearance can be modeled by Eq. (1), which is a simplified form of the Reynolds equation, with p and μ representing the static pressure and the fluid dynamic viscosity, respectively, and V_p the piston velocity. It should be noted that if the term on the right-hand side of Eq. (1) is eliminated, an analytical solution can be obtained, as indicated by Zuk and Smith (1969).

$$\frac{d}{dx} \left(p \frac{dp}{dx} \right) = \frac{6\mu V_p}{\delta^2} \frac{dp}{dx} \quad (1)$$

The following hypotheses are assumed in Eq. (1): i) inertia effects are negligible; ii) the pressure varies only in the x direction; iii) the piston-cylinder gap is much smaller than the piston and cylinder diameters, so the annular gap can be treated as a flat channel; iv) the gas behaves as an ideal gas; v) constant dynamic viscosity; vi) isothermal flow and vii) piston and cylinder are concentric, so δ is constant.

Equation (1) is a differential equation that cannot be solved analytically and, for this reason, we decided to apply the finite volume method (FVM) for the numerical solution. The solution domain is divided into equally spaced control volumes to form a computational mesh. The differential equation is discretized in each control volume, resulting a system of algebraic equations.

The pressures at the boundaries of the volumes are estimated via arithmetic mean of the pressures at the neighboring points, allowing the pressure field to be calculated by solving the system of algebraic equations via the solver provided by the Eigen library (Eigen, 2016). Since Eq. (1) is non-linear, an initial pressure field is required to determine the coefficients of the algebraic equations. An iterative procedure is then applied, with the coefficients being updated and the calculations repeated until the convergence criterion is achieved.

Once the pressure field is known, the mass flow rate \dot{m}_{leak} can be obtained at any point of the solution domain by using Eq. (2), with D and ρ being the diameter of the cylinder and the gas density. The first and second terms inside the parentheses are the contributions to the leakage promoted by the pressure difference and piston motion; the latter being also known as the Couette term. We evaluate Eq. (2) numerically, estimating the pressure derivative by a second order approximation.

$$\dot{m}_{leak} = \pi D \left(-\frac{\rho \delta^3}{12\mu} \frac{dp}{dx} + \frac{\rho V_p \delta}{2} \right) \quad (2)$$

Two boundary conditions for pressure are needed in the numerical model. The first one is the pressure in the compressor internal environment, which is constant throughout the compression cycle. The second is the pressure in the compression chamber (P_{cc}), which is evaluated at each crank angle (wt) with a simulation model described in details by Ussyk (1980). This simulation model also evaluates the piston velocity, dynamic viscosity and density used in Eq. (2), throughout the compression cycle. In turn, the model for the flow in the piston-cylinder clearance gives the leakage required to evaluate the compression cycle. Figure 2 illustrates the coupling between these two models.

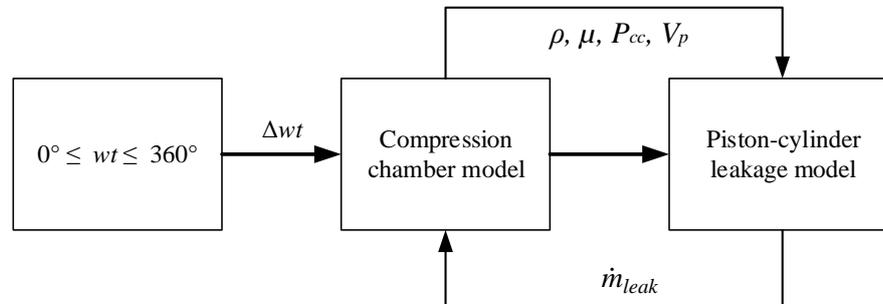


Figure 2. Schematic of the solution procedure.

3. RESULTS AND DISCUSSION

The developed model was used to investigate the effects of several parameters on the gas leakage, such as compressor operating conditions, compressor speed (represented by the drive frequency), and clearance dimensions. Additionally, compressibility effects were also assessed by comparing the results obtained with formulations of compressible and incompressible flows.

The results presented hereafter were obtained for a low-capacity reciprocating compressor with displacement of 3 cm³ operating with isobutane (R600a). The compression cycle was solved for small increments of crank angle ($\Delta wt = 0.001^\circ$). A total of 250 control volumes were used to discretize the solution domain, with a difference less than 0.1% between predictions and estimates via the Richardson extrapolation.

It should be mentioned that the model employed to simulate the compression cycle was simplified to eliminate the influence of valve dynamics and heat transfer at the compression chamber walls on the compressor efficiency.

The model was initially validated through comparisons between its predictions and the results of a three-dimensional model developed with a CFD commercial code, considering $c = 14 \mu\text{m}$ and different piston velocities. Figure 3 shows the results for the dimensionless leakage (\dot{m}_{leak}^*), which was calculated by dividing the leakage in each case by the leakage associated to the case in the simplified model where $V_p = -4 \text{ m/s}$. Figure 3 reveals that both models are in good agreement, with differences within 10%. Positive velocities represent that the piston is moving towards the top dead center, and the opposite for negative values.

The compressor operating conditions are defined by the evaporating and condensing temperatures of the refrigeration cycle, which defines the compressor suction and discharge pressures. Reference operating conditions used to test refrigeration compressors are the low-back pressure (LBP), medium-back pressure (MBP) and high-back pressure (HBP), as shown in Tab. 1. The effect of such operating conditions on gas leakage is also investigated.

The results for $c = 9 \mu\text{m}$ shown in Fig. 4 reveal that the gas leakage increases with the evaporating pressure (P_{evap}). This may seem unexpected since the pressure difference decreases as the evaporating pressure increases, keeping the same condensing pressure (7.62 bar). Nevertheless, as indicated by Eq. (2), the leakage mass flow rate also depends on the gas density, which is highest for the HBP condition, as shown in Fig. 5, because the gas temperature in the discharge process reaches its lowest value. This aspect compensates for the smaller pressure difference of the HBP condition.

Table 1. Compressor operating conditions.

Denomination	Evaporating temperature	Evaporating pressure	Condensing temperature	Condensing pressure
LBP	-23.3 °C	0.629 bar		
MBP	-6.7 °C	1.229 bar	54.4 °C	7.62 bar
HBP	7.2 °C	2.011 bar		

The piston velocity is a direct consequence of the drive frequency. The model was applied with different values of frequency in order to investigate its effects on gas leakage throughout the compression cycle. Figure 6 shows the results

of simulations for $c = 5 \mu\text{m}$ and LBP condition. The ‘no velocity’ curve represents the results of Eq. (2) without the Couette term. Figure 6 shows that leakage is greatly affected by piston velocity, being reduced in the compression and discharge processes and decreased in the expansion and suction processes as the piston velocity is increased. In the compression process, the piston is moving towards the top dead center and therefore it carries gas to the compression chamber through viscous effects, reducing the gas leakage. The opposite effect occurs when the piston moves towards the bottom dead center.

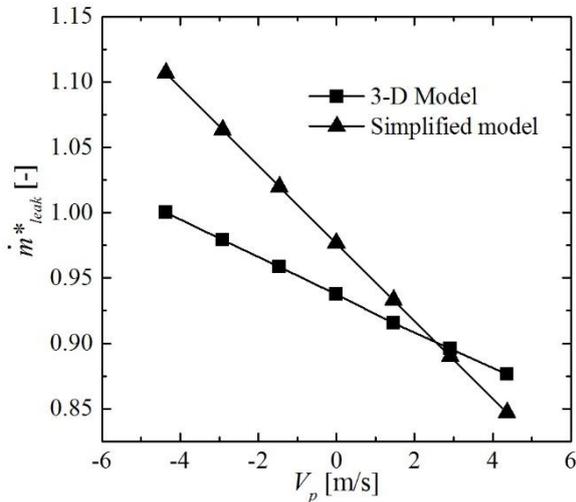


Figure 3. Comparison between simplified and 3-D models.

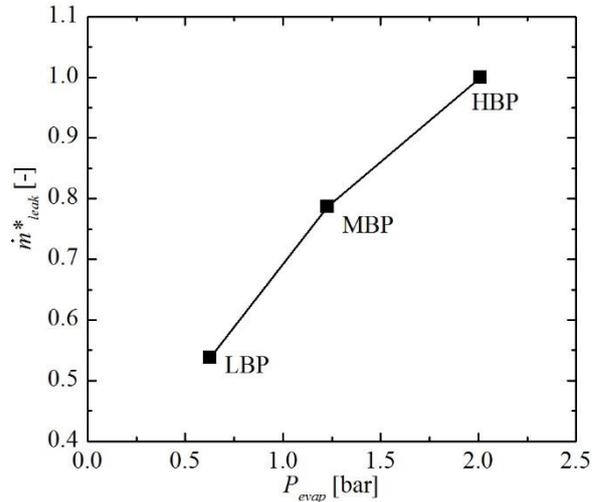


Figure 4. Gas Leakage for different compressor operating conditions.

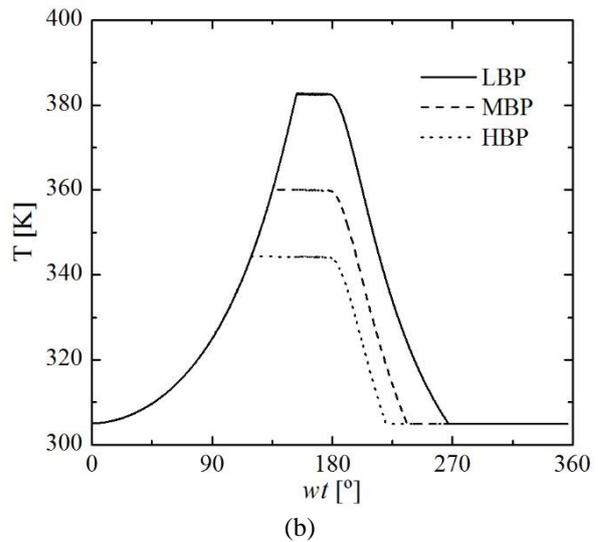
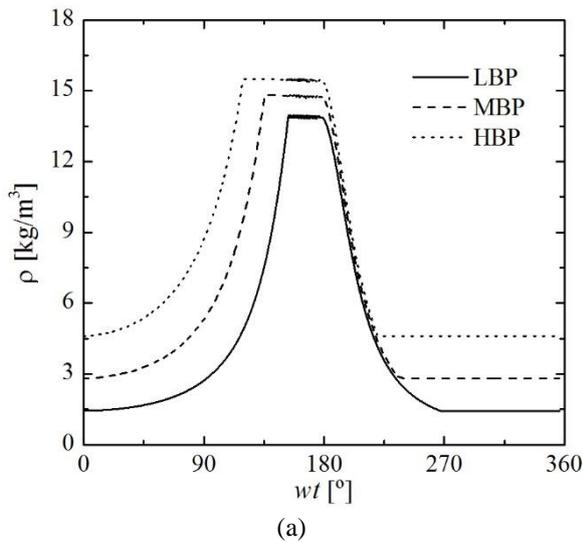


Figure 5. Gas density (a) and temperature (b) throughout the compression cycle.

The most significant difference between the results of leakage for the three velocity conditions occurs at the crank angle around 135° , when the second term on the right hand size of Eq. (2) associated with the piston velocity (Couette term) reaches its maximum value, being positive according to the convention adopted herein. The total leakage in the cycle can be obtained by integrating the curves in Fig. 6, showing that the total leakage is reduced by 8% when $f = 90$ Hz.

Gas leakage for two diametric clearances (c) are shown in Fig. 7. The simulations were carried out with a frequency of 60 Hz and LBP condition. As can be seen, the influence of the piston velocity represented by the Couette term is more significant in the smaller clearance. The results also suggest that the pressure term in Eq. (2) becomes increasingly important as the clearance is increased.

In order to investigate compressibility effects on gas leakage, we carried simulations for the conditions analyzed in Fig. 6, but this time using the incompressible fluid flow model presented by Ferreira and Lilie (1984). The calculations considered different frequencies of operation, $c = 5 \mu\text{m}$ and LBP condition. The results shown in Fig. 8 indicate that the incompressible fluid model fails to predict the effect of the piston velocity on the leakage. Therefore, as also observed by Lohn and Pereira (2014), a compressible fluid formulation is required to leakage in the piston-cylinder clearance.

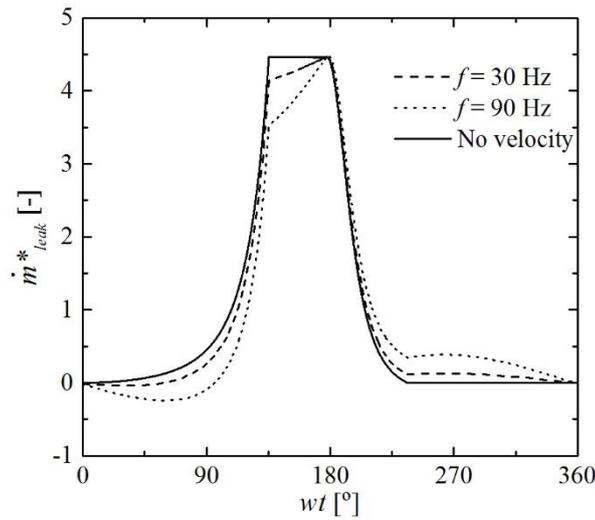


Figure 6. Gas leakage during the compression cycle for different compressor speeds; $c = 5 \mu\text{m}$ and LBP condition.

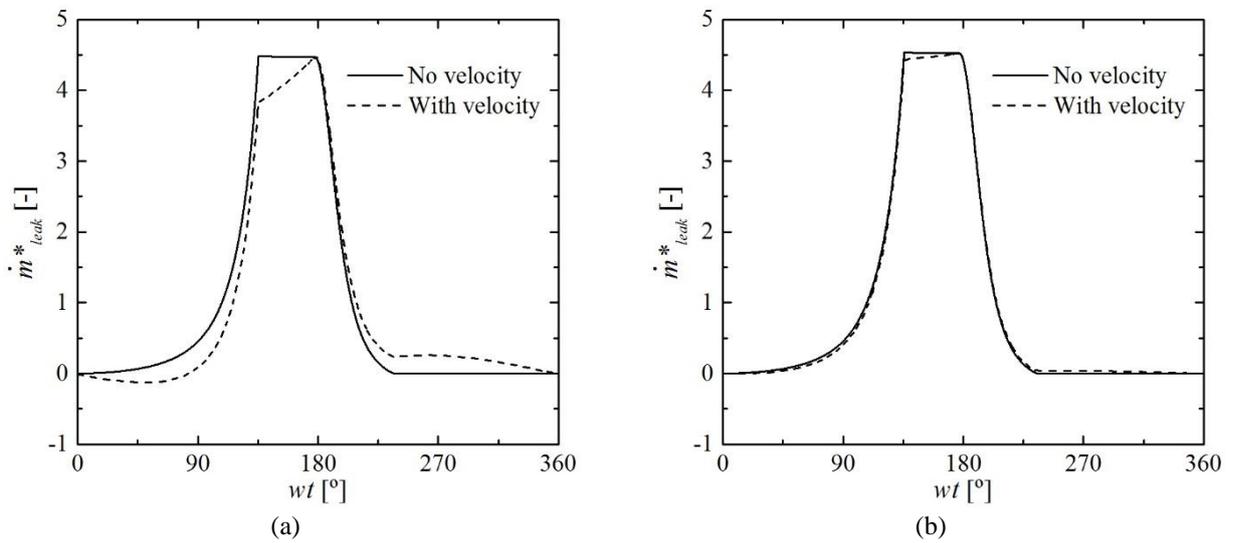


Figure 7. Gas leakage for for different clearances c ; $f = 60 \text{ Hz}$ and LBP condition: (a) $c = 5 \mu\text{m}$; (b) $c = 13 \mu\text{m}$.

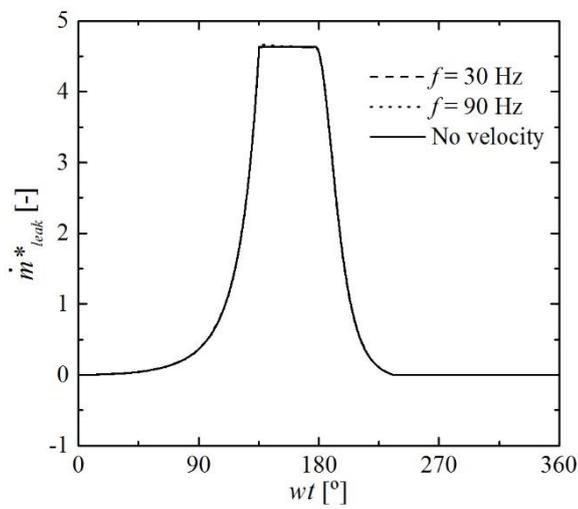


Figure 8. Predictions of gas leakage of incompressible fluid model for different frequencies ($c = 5 \mu\text{m}$ and LBP condition).

The volumetric efficiency and isentropic efficiency are two parameters commonly used to evaluate the compressor performance. The first is defined as the ratio between the actual mass flow and the ideal mass flow rate, as indicated in Eq. (3). The ideal mass flow rate is obtained for the hypothetical case with no gas leakage, cylinder clearance volume, flow restriction and backflow in valves and suction gas superheating.

$$\eta_v = \frac{\dot{m}}{\dot{m}_{th}} \quad (3)$$

The isentropic efficiency is defined as the ratio between the compression power required by an isentropic process and the actual indicated power, i.e.:

$$\eta_s = \frac{\dot{W}_{th}}{\dot{W}} \quad (4)$$

Figure 9 shows the results of efficiency reduction due to leakage, considering different frequencies of operation and clearances for the LBP condition. The efficiency reduction is evaluated as the difference between the compressor efficiencies with and without gas leakage, as indicated by Eq. (5). Figure 9 reveals that gas leakage is more detrimental to both efficiencies for smaller drive frequencies and larger clearances. In the worst case, the efficiency reduction can reach more than 4% for the volumetric efficiency and more than 10% for the isentropic efficiency.

$$\Delta\eta_v = |\eta_v - \eta_{v, no\ leak}| \quad ; \quad \Delta\eta_s = |\eta_s - \eta_{s, no\ leak}| \quad (5)$$

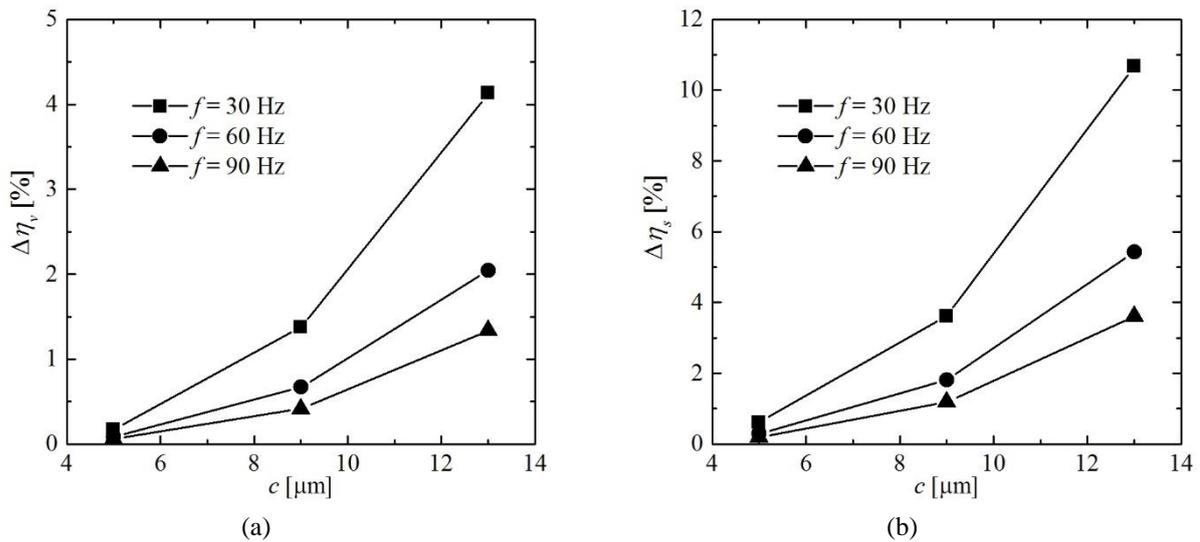


Figure 9. Curves of reduction of volumetric efficiency (a) and isentropic efficiency (b) for different frequencies of operation.

4. CONCLUSION

A simulation model based on the compressible fluid flow formulation of the Reynolds equation was developed to predict gas leakage in the piston-cylinder clearance. The model was validated by comparing its predictions with results obtained with a three-dimensional model. The leakage model was then coupled to a simulation model of the compression cycle. The leakage in the piston-cylinder clearance was then analyzed throughout the compression cycle for different parameters, such as operating conditions, compressor speed, piston-cylinder clearance and compressibility effects. We found that leakage is increased with the evaporating pressure when the condensing pressure is kept constant, because the the density of the gas inside the compression chamber also increases. We also observed that the piston velocity plays an important role, with a significant reduction of leakage as the velocity is increased. However, the influence of piston velocity becomes less important as the clearance is increases, when the pressure difference is the main drive force of the flow. The present study also showed that a compressible fluid formulation is required in any simulation model developed to predict the gas leakage in the piston-cylinder clearance. As far as the effect on the compressor performance, the gas leakage was found to be more detrimental to the volumetric and isentropic efficiencies in the case of small drive frequency and large clearances. For the compressor under analysis, reductions of up to 4% and 10% were predicted for the volumetric and isentropic efficiencies, respectively.

5. ACKNOWLEDGEMENTS

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